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CHIRAL SYMMETRY AND THE BAG MODEL:
A NEW STARTING POINT FOR NUCLEAR PHYSICS

A.W. Thomas

TRIUMF, 4004 Wesbrook Mall, Vancouver, B.C., Canada V6T 2A3

and

Division TH, CERN, 1211 Geneva 23, Switzerland[*]

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1. INTRODUCTION

"They must often change who would be constant in happiness and wisdom."

Classical nuclear theory deals with a many body system of neutrons and protons interacting non-relativistically through two-body potentials. It has, of course, long been realized that there must be corrections to this simple picture — for example the meson exchange effects which preclude a simple interpretation of the magnetic moment of the deuteron in terms of d-state probability. Nevertheless the availability of beams of pions, and the consequent ability to study the excitation of real isobars in nuclei, has been critical in the realization that for many problems one must develop a theoretical model which explicitly includes pion and isobar degrees of freedom (see for example the proceedings of recent topical conferences Cat+ 82, MT 80).

While these developments have been taking place in intermediate energy physics, and particularly since the discovery of the J/ψ, our colleagues in high energy physics have become thoroughly convinced of the quark model of hadron structure. This approach to the structure of hadrons began in the early 1960's. On the basis of symmetry considerations Gell-Mann, Ne'eman and Zweig suggested that all hadrons might be made from more elementary components — the quarks (or aces) (GN 64). These constitute the fundamental representation of the group SU(3). All of the low mass hadrons were found to fall into low-dimensional representations of SU(3). In the case of the mesons they could be thought of as being made of quark-anti-quark, while for the baryons three quarks were required. Nevertheless at that stage it was not clear whether the quarks were real particles or simply a mathematical trick.

One of the initial problems of the quark model was that, for example, the $J_Z = +3/2$ state of the Δ^{++} would necessarily be made from three identical up-quarks in the same spin and spatial state. Since the quarks should be fermions this would violate Fermi statistics. In order to overcome this difficulty the quarks were assigned a new, unobserved property called "colour"—each quark having three possible colours. (A somewhat older, but equivalent explanation involved parastatistics.) This apparently ad hoc explanation became a great strength of the model when it was realized that one could build a theory of strong interactions (quantum chromodynamics or QCD) based on a gauge theory of colour—the symmetry group again being SU(3) (AL 73, MP 78).

It was soon established that because of the non-abelian nature of the theory it had two novel features. First, at short distances, or high momentum transfer, the interactions become weaker—"asymptotic freedom". This realization was crucial in the identification of partons—the elementary, apparently free constituents of the proton observed in deep inelastic e- and ν-scattering—with quarks (Clo 79). Second, it seems that at large distances the interaction grows stronger. This property is generally believed, though not yet proven, to lead to confinement of the quarks into colour-singlet objects—hence three-quark baryons, and no free quarks.

At the present time a great deal of theoretical effort is being devoted to attempts to solve the QCD equations—mainly by brute force using Monte Carlo techniques. In the absence of exact solutions, we must either abandon all hope of tackling nuclear problems, or rely on phenomenological models. Fortunately, we have at our disposal a variety of successful, phenomenological models which incorporate the features expected from QCD. Of all these models the MIT bag model is perhaps the most attractive. As we shall see, it incorporates the facts that quarks are confined, pointlike and essentially massless. The model is therefore relativistic, and can be summarised in a relatively concise Lagrangian formalism. This feature has proven essential in the recent developments involving chiral symmetry, which we shall review in Sections 5 and 6.

We shall see that in the bag model, as in any other quark model of nucleon structure, the nucleon is far from pointlike. Its radius is about one fermi, so that at the average internucleon separation of 1.8 fm at nuclear matter density ($\rho_0 \sim 0.17$ fm^{-3}) the nucleon bags overlap! This is a rather different state of affairs from that envisaged in most modern N-N potential models. As Baym has discussed (Bay 79) (see also Section 7), with a bag radius of 1.0 fm one would expect to find considerable linking of different bags in a nucleus (and hence free flow of colour current between bags), even at half nuclear matter density! In that case even the independent particle shell model behaviour of valence nucleons is mysterious.

By lowering the bag size just a little—e.g. to $R \sim (0.8\text{-}0.9)$ fm, as in the cloudy bag model—the critical density can be made about the same as nuclear matter density. In this way the problem with the independent particle shell model becomes less severe. Nevertheless it seems inevitable that there should be considerable linking of bags in finite nuclei. Thus we are forced to suggest that a precise description of many phenomena in nuclear physics may require the explicit inclusion of the quarks themselves. This seems to us the natural extension of the developments involving isobars to which we referred earlier. Such a suggestion deserves urgent theoretical (and, when the questions are clearly formulated, experimental) attention in the next few years. (Incidentally there has been some discussion of quark degrees of freedom in nuclei by Robson (Rob 78), who derived effective many-body forces on the basis of a non-relativistic quark model. Our approach will be rather different.)

One of the defects of the MIT bag model from the nuclear physics point of view is the absence of any mechanism for long range N-N interactions. In fact this is just one indication of a fundamental problem in the model, namely that it badly violates chiral symmetry. Since chiral symmetry is a property of QCD itself, this is quite worrying. The chiral bag models have been developed in response to this difficulty. At the present stage of the phenomenology the pion appears as fundamental as the quarks, although eventually this must be improved. Recent work which suggests that the pion exists as a consequence of dynamical symmetry breaking in short distance QCD will be discussed and related to the chiral bag ideas.

In summary, we shall see that whereas a great deal of progress has been made towards understanding single hadron properties, we are just beginning to make progress on the problem of two or more interacting hadrons. We have little doubt that for the next five to ten years this will be one of the major areas of research in nuclear physics (if not the major one). With this in mind the time is right for a graduate level introduction to the concepts and models that will be used. We hope that this review may help to provide such a bridge between the high energy and nuclear communities.

In general the tone of the first major sections (Sections 2-5) is quite pedagogical. Full details of the algebra are often given in order that the reader can concentrate on the ideas and concepts being presented. After studying these sections carefully, the keen student should have a fairly good working knowledge of the MIT bag model, as well as a degree of familiarity with chiral symmetry. By the end of Section 6 which is more in the nature of a review, he will be essentially au courant with all published chiral bag models, and particularly the cloudy bag model. Section 7 is of quite general interest and in it we attempt to set the stage for future work in the physics of many-body systems of composite nucleons.

This review will have succeeded if a good number of its readers decide to take part in this fascinating new approach to a very old subject. Needless to say we welcome all constructive comments concerning anything said here.

2. THE BASIC BAG MODEL

In order to have a sound basis for the later developments of direct relevance to nuclear physics, we must first describe the original MIT bag model. The discussion in Section 2.1 is meant to lay this basis in considerable detail. It follows closely the pedagogical approach of Hey (77), to which we refer for more discussion of excited state spectroscopy. Section 2.2 deals with the application of the model to the mass spectrum of the low-lying hadrons. In Section 2.3 we briefly review some recent attempts to justify the bag model starting from QCD. Finally in Section 2.4 we discuss the *relationship* to the popular, non-relativistic quark models.

2.1. The MIT Bag Model

2.1.1. *Bogolioubov*

The MIT bag model actually had its beginnings in the late 60's in the attempts to describe phenomenologically a system of confined, relativistic quarks. In particular, Bogolioubov (Bog 67) considered the simplest possible case of a massless Dirac particle moving freely inside a spherical volume of radius R, outside of which there was a scalar potential of strength m. Clearly by taking the limit $m \to \infty$ we can confine the quarks to the spherical volume.

Let us therefore begin with the Dirac equation for a particle of mass m

$$H\psi = i\frac{\partial\psi}{\partial t} , \tag{2.1}$$

with the Hamiltonian

$$H = \underline{\alpha} \cdot \underline{p} + \beta m . \tag{2.2}$$

(Our convention for Dirac matrices is summarized in Appendix I.) There are two operators which commute with H, and can therefore be used to classify its eigenstates. These are,

$$\vec{j} = \vec{\ell} + \vec{\sigma}/2 \ , \tag{2.3}$$

where, when necessary we have

$$\vec{\sigma} = \begin{bmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{bmatrix} , \tag{2.4}$$

and the relativistic analogue of the operator k described in Appendix I. The analogue K is

$$K = \beta(\underline{\sigma}\cdot\underline{\ell} + 1) \ . \tag{2.5}$$

With these definitions it is straightforward to prove that

$$[\vec{j},K] = 0 = [H,\vec{j}] = [H,K] \ , \tag{2.6}$$

and

$$K^2 = \beta^2\left(1 + (\underline{\sigma}\cdot\underline{\ell})^2 + 2\underline{\sigma}\cdot\underline{\ell}\right) = \vec{j}^2 + \frac{1}{4} \ . \tag{2.7}$$

Clearly K has eigenvalues κ, where

$$\kappa = \pm\left(j + \frac{1}{2}\right) . \tag{2.8}$$

In the case of a central, scalar field W(r), the Dirac equation becomes

$$H\psi(\underline{r}) = \left(\underline{\alpha}\cdot\underline{p} + \beta\left(m + W(r)\right)\right)\psi(\underline{r}) = E\psi(\underline{r}) \ , \tag{2.9}$$

where

$$\psi(\underline{r},t) = \psi(\underline{r})\ e^{-iEt} \ , \tag{2.10}$$

$$\vec{j}^2\psi_\kappa^\mu = j(j+1)\psi_\kappa^\mu \ ; \ j_z\psi_\kappa^\mu = \mu\ \psi_\kappa^\mu \ ; \tag{2.11}$$

and

$$K\psi_\kappa^\mu = -\kappa\psi_\kappa^\mu \ . \tag{2.12}$$

Let the solution of Eq. (2.9) have the form

$$\psi = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} , \tag{2.13}$$

then the structure of K,

$$K = \begin{bmatrix} k & 0 \\ 0 & -k \end{bmatrix} , \tag{2.14}$$

implies that ψ can be written, without loss of generality, as

$$\psi_\kappa^\mu = \begin{bmatrix} g(r) & \chi_\kappa^\mu \\ i\ f(r) & \chi_{-\kappa}^\mu \end{bmatrix} . \tag{2.15}$$

Then, using

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} - i\,\frac{\hat{r}}{r}\times\vec{\ell} , \tag{2.16}$$

we can write the kinetic energy piece of H as

$$\begin{aligned} \underline{\alpha}\cdot\underline{p} &= -i\ \underline{\alpha}\cdot\hat{r}\,\frac{\partial}{\partial r} + \frac{i}{r}\,\underline{\alpha}\cdot\hat{r}\ \underline{\sigma}\cdot\underline{\ell} , \\ &= -i\ \underline{\alpha}\cdot\hat{r}\,\frac{\partial}{\partial r} + \frac{i}{r}\,\underline{\alpha}\cdot\hat{r}(\beta K-1) . \end{aligned} \tag{2.17}$$

Substituting Eq. (2.15) and (2.17) into the Dirac equation (2.9) it becomes

$$\begin{aligned} (E-W-m)\ g &= -\left(\frac{df}{dr} + \frac{f}{r}\right) + \frac{\kappa f}{r} , \\ (E+W+m)\ f &= \left(\frac{dg}{dr} + \frac{g}{r}\right) + \frac{\kappa g}{r} . \end{aligned} \tag{2.18}$$

Bogolioubov's simple model of confinement (Bog 67) corresponded to the scalar potential

$$\begin{aligned} W(r) &= -m \quad r \leqslant R , \\ W(r) &= \ \ 0 \quad r > R . \end{aligned} \tag{2.19}$$

If we now define

$$U = m + W , \tag{2.20}$$

Eq. (2.18) becomes

$$\begin{aligned} \frac{df}{dr} &= \frac{\kappa-1}{r}\,f - (E-U)g , \\ \frac{dg}{dr} &= (E+U)f - \frac{\kappa+1}{r}\,g , \end{aligned} \tag{2.21}$$

Consider the case $\kappa = -1$, which is the $s_{1/2}$ level. Eq. (2.21) implies

$$f = (E+U)^{-1}\,\frac{dg}{dr} , \tag{2.22}$$

so that defining $g = u/r$, the equation for the upper component of ψ_κ^μ is

$$\frac{d^2u}{dr^2} + (E^2-U^2)\ u = 0 . \tag{2.23}$$

Inside the scalar potential well this means

$$\ddot{u} + E^2 u = 0 \ , \tag{2.24}$$

and hence

$$u(r) = A \sin Er \ . \tag{2.25}$$

Outside the scalar well u(r) satisfies

$$\ddot{u} - (m^2 - E^2)\, u = 0 \ , \tag{2.26}$$

and hence

$$u(r) = A(\sin ER)\, e^{-\sqrt{m^2-E^2}(r-R)} \ . \tag{2.27}$$

This is an eigenvalue problem because u (and of course g) must be continuous at r = R. If we also demand that f(r) [defined by Eq. (2.22)] be continuous, we obtain the matching condition

$$\cos(ER) + \frac{\sqrt{1-(E/m)^2}}{1+(E/m)} \sin ER = \frac{\sin ER}{ER}\left[1 - \frac{E}{E+m}\right] . \tag{2.28}$$

Clearly in the limit $m \to \infty$ (corresponding to confinement) this becomes

$$(\sin ER)/(ER) = \left(\frac{\sin ER}{ER} - \cos ER\right)/ER \ , \tag{2.29}$$

and hence

$$j_o(ER) = j_1(ER) \ . \tag{2.30}$$

This is the appropriate boundary condition for massless, confined quarks.

If we parametrise the energy levels as

$$E_{n\kappa} = \omega_{n\kappa}/R \ , \tag{2.31}$$

where n is the principal quantum number, we find $\omega_{1-1} = 2.04$, $\omega_{2-1} = 5.40$ and so on. The solution has the form

$$\psi^{\mu}_{ns_{1/2}} \equiv \psi^{\mu}_{n\kappa\equiv-1} = N_{n,-1} \begin{bmatrix} j_o\left(\frac{\omega r}{R}\right) \chi^{\mu}_{-1} \\ -i j_1\left(\frac{\omega r}{R}\right) \chi^{\mu}_{1} \end{bmatrix} , \tag{2.32}$$

and using Eq. (I.15) from Appendix I this may be written as

$$\psi_{n,-1}(\underline{r}) = \frac{N_{n,-1}}{\sqrt{4\pi}} \begin{bmatrix} j_o\left(\frac{\omega r}{R}\right) \\ i\ \underline{\sigma}\cdot\hat{r}\ j_1\left(\frac{\omega r}{R}\right) \end{bmatrix} \chi^{\mu}_{1/2} \ , \tag{2.33}$$

with $\chi^{\mu}_{1/2}$ a Pauli spinor, and the normalisation constant given by

$$N^2_{n,-1} = \frac{\omega^3_{n,-1}}{2R^3(\omega_{n,-1}-1)\sin^2(\omega_{n,-1})} \ . \tag{2.34}$$

The density of quarks is readily calculated as

$$j^o \equiv \rho = \bar{\psi}\,\gamma^o\,\psi \sim \left[j_o^2\left(\frac{\omega r}{R}\right) + j_1^2\left(\frac{\omega r}{R}\right)\right]\theta(R-r) \ , \tag{2.35a}$$

where

$$\theta(x) = 1 \quad x \geq 0$$
$$= 0 \quad x < 0 \ . \tag{2.35b}$$

Thus the density [Eq. (2.35a)] certainly does not vanish at $r = R$. Clearly, although the lower component is suppressed for small r, it does make a sizeable contribution near the surface of the bag. Of course it is *natural* to ask whether this is not unusual in comparison with non-relativistic experience, where $\psi(R)$ would be zero. However, such a solution would not be consistent with the linear Dirac equation. What counts is that there should be no current flow through the surface of the confining region. For example, in the MIT bag model it is required that (we use $q(x)$ for the quark wave function in the MIT model, but it is identical to $\psi(x)$ in the static, spherical case)

$$n_\mu \bar{q}\,\gamma^\mu q = 0 \ , \tag{2.36}$$

at the surface—where n^μ is a unit four vector normal to the surface of the confining region.

In the MIT bag model (Cho+ 74, DeG+ 75, Joh 75, JJ 77, HK 78) the condition (2.36) is imposed through a linear boundary condition

$$i\ \gamma\cdot n\ q = q \ , \tag{2.37}$$

at the surface. This implies

$$q^+ = -i\ q^+\ \gamma^+\cdot n \ , \tag{2.38}$$

and hence

$$\bar{q} = -i\ \bar{q}\ \gamma\cdot n \ , \tag{2.39}$$

because

$$\gamma^\mu = \gamma^o\ \gamma^{\mu+}\ \gamma^o \ . \tag{2.40}$$

Consider now the normal flow of current through the bag surface,

$$i\, n_\mu j^\mu = i\, n_\mu\, \bar{q}\, \gamma^\mu\, q \ ,$$

which from Eq. (2.39) and (2.37) respectively is

$$\begin{aligned} i\, n_\mu j^\mu &= (\bar{q}\, i\gamma{\cdot}n) q = \bar{q}(i\gamma{\cdot}nq) \ , \\ &= -\bar{q}\, q \ = \ \bar{q}\, q \ , \\ &= 0 \ . \end{aligned} \tag{2.41}$$

Thus it is not the density, but $\bar{q}\, q$ which should vanish at the boundary in a relativistic theory. If we now return to the model of Bogolioubov, Eq. (2.33) implies that

$$\bar{\psi}\, \psi\big|_{r=R} = \left(j_0(\omega), i\, \underline{\sigma}{\cdot}\hat{r}\, j_1(\omega) \right) \cdot \begin{pmatrix} j_0(\omega) \\ i\, \underline{\sigma}{\cdot}\hat{r}\, j_1(\omega) \end{pmatrix}$$

$$= j_0^2(\omega) - j_1^2(\omega) = 0 \ , \tag{2.42}$$

by Eq. (2.30). That is, the matching condition of Bogolioubov is exactly equivalent to the linear boundary condition (*l.b.c.*) for the static spherical MIT bag

$$i\, \gamma{\cdot}n\, \psi = -i\, \underline{\gamma}{\cdot}\hat{r}\, \psi = \psi \ , \tag{2.43}$$

where

$$n^\mu = (0, \hat{r}) \ . \tag{2.44}$$

Suppose we now demand that the lowest energy state of Bogolioubov's model reproduce the nucleon mass. Just as in the independent particle shell model, we add the energies of each of the three quarks in the 1s level giving

$$M_N = \frac{3\omega_{1-1}}{R} \ . \tag{2.45}$$

Using $\omega_{1-1} = 2.04$ we find that the radius of the nucleon bag is

$$R_N = 1.3 \text{ fm} \ . \tag{2.46}$$

Then the first excited state of the nucleon, namely the Roper resonance, in which one quark is simply raised from the $1s_{1/2}$ to the $2s_{1/2}$ state, should have a mass M_R where

$$\frac{M_R}{M_N} = \frac{2\omega_{1-1}+\omega_{2-1}}{3\omega_{1-1}} = \frac{4.08+5.40}{6.12} = 1.55 \ . \tag{2.47}$$

This is in remarkable agreement with the experimental ratio 1.56 — to quote Bogolioubov, "une telle coincidence est un peu surprenante".*

2.1.2. *Energy-momentum conservation*

Up to this point there is little practical difference between the bag model of the MIT group and that of Bogolioubov. Although the MIT model is decently covariant, and this will be put to use in later sections, in practical calculations one is forced to work with the static spherical case. Nevertheless, the more rigorous formal approach did help the MIT group to recognise a fundamental problem in the Bogolioubov model. In order to see this we consider the energy momentum tensor for that model, $T^{\mu\nu}_{Bog}$,

$$T^{\mu\nu}_{Bog} = T^{\mu\nu}_{D} \ \theta_V \ , \tag{2.48}$$

where θ_V defines the bag volume,

$$\begin{aligned} \theta_V &= 1 \text{ inside,} \\ &= 0 \text{ outside,} \end{aligned} \tag{2.49}$$

and $T^{\mu\nu}_D$ is the familiar energy-momentum tensor for a *free* Dirac field,

$$T^{\mu\nu}_D = \frac{i}{2} \bar{q}(x) \ \gamma^\mu \overleftrightarrow{\partial}^\nu q(x) \ . \tag{2.50}$$

(As usual we have defined

$$\overleftrightarrow{\partial}^\nu = \overrightarrow{\partial}^\nu - \overleftarrow{\partial}^\nu \ , \tag{2.51}$$

where the first and second terms on the r.h.s act to the right and left respectively.) The condition for overall energy and momentum conservation is that the divergence of the energy momentum tensor should vanish, and this is certainly true for $T^{\mu\nu}_D$, as is easily proven from the free Dirac equation ($i \not\partial q(x) = 0$, for a massless quark)

$$\partial_\mu T^{\mu\nu}_D = 0 \ . \tag{2.52}$$

*Such a 'coincidence' is a little surprising.

However, the fact that these quarks move freely only inside the restricted region of space V, leads to problems. Indeed,

$$\partial_\mu \theta_V = n_\mu \Delta_S \ , \tag{2.53}$$

where Δ_S is a surface delta function,

$$\Delta_S = -n\cdot\partial(\theta_V) \ . \tag{2.54}$$

In the static spherical case [see Eq. (2.44)] we find that Δ_S is simply $\delta(r-R)$. Putting all this together we obtain

$$\partial_\mu T^{\mu\nu}_{Bog} = \frac{i}{2}\,\bar{q}\,\gamma\cdot n \overleftrightarrow{\partial}^\nu q\,\Delta_S \ , \tag{2.55}$$

and using the *l.b.c.* [Eq. (2.37)]

$$\partial_\mu T^{\mu\nu}_{Bog} = -\frac{1}{2}\,\partial^\nu[\bar{q}\,q]\,\Delta_S = -P_D\,n^\nu\,\Delta_S \ , \tag{2.56}$$

where P_D is the pressure exerted on the bag wall by the contained Dirac gas

$$P_D = -\frac{1}{2}\,n\cdot\partial(\bar{q}\,q)\,|_{surface} \ . \tag{2.57}$$

Clearly the model of Bogolioubov violates energy-momentum conservation! Furthermore, this violation is an essential result of the confinement process. We shall see in Section 4 that a similar problem arises in connection with the axial current.

The resolution of this problem is possible only if we are willing to add a new ingredient. In particular, we simply add a phenomenological energy density term "$-B\,\theta_V$" to the Lagrangian density (see Section 4). *Then (since $T^{\mu\nu}$ involves $-\mathcal{L}g^{\mu\nu}$) the new energy-momentum* tensor, $T^{\mu\nu}_{MIT}$, has the form

$$T^{\mu\nu}_{MIT} = (T^{\mu\nu}_D + B g^{\mu\nu})\theta_V \ . \tag{2.58}$$

Therefore, the divergence of the energy-momentum tensor is

$$\partial_\mu T^{\mu\nu}_{MIT} = (-P_D + B)n^\nu\,\Delta_S \ , \tag{2.59}$$

which will vanish if

$$B = P_D = -\frac{1}{2}\,n\cdot\partial[\bar{q}(x)q(x)]_{surface} \ . \tag{2.60}$$

Equation (2.60) involves the square of the quark fields, and hence is referred to as the *non-linear boundary condition (n.l.b.c.)* of the MIT bag model. Because of this condition the introduction of a constant B involves no new parameters.

By taking the explicit solutions of the free Dirac equation (massless case),

$$\psi_\kappa^\mu = N_\kappa \begin{pmatrix} j_\ell(\omega r/R)\chi_\kappa^\mu \\ \pm i \ j_{\ell\mp 1}(\omega r/R)\chi_{-\kappa}^\mu \end{pmatrix}, \tag{2.61}$$

[where the upper (lower) sign refers to κ positive (or negative)], it is easily verified that only $\kappa = 1$ leads to an angle independent result on the r.h.s. of Eq. (2.60). Thus, only states with $j = 1/2$ can satisfy the *n.l.b.c.* as given. In fact, for applications to excited state spectroscopy the strict *n.l.b.c.*, Eq. (2.60), must be relaxed in favour of an angle averaged version (Reb 76, DJ 76, DeG 76). We shall not pursue this topic further, because it is of little direct relevance to the low-lying baryons of interest in nuclear physics.

The meaning of this addition to $T^{\mu\nu}$ can be clarified by considering the total energy of the bag state

$$P^0 = \int d^3x \ T^{00}(x) \ , \tag{2.62}$$

which we shall label E(R) rather than M(R) as a precursor to our discussion of c.m. corrections later,

$$E(R) = \frac{3\omega_{1-1}}{R} + \frac{4\pi}{3} R^3 B \ . \tag{2.63}$$

The first term is the kinetic energy, which also appeared in Bogolioubov's model, while the second is a volume term. Essentially it implies that it costs an energy BV to make this bubble in the vacuum within which the quarks move freely. It should be intuitively clear that energy-momentum conservation is related to pressure balance at the bag surface, so that a small change in radius should not significantly increase E(R). Nevertheless, it is a recommended exercise for the reader to show explicitly that the *n.l.b.c.* implies that

$$\frac{\partial E}{\partial R} = 0 \; . \tag{2.64}$$

In concluding this section we wish to stress that it is an *assumption* of the model that B should be constant for all hadrons (see Section 2.3.1). Furthermore, as all hadronic bags have radii in the region (0.8-1.1) fm, this assumption has really not been severely tested. For example, Hasenfratz and Kuti (HK 78) show that a surface tension (or some linear combination of the two) can produce similar results. Clearly any simple phenomenological device like B is a crude representation of the complicated QCD mechanism which leads to confinement, and one must be cautious about taking it too seriously outside the limited range where it has been used so far.

2.2. The Spectroscopy of Low-Lying States

2.2.1. *General features — massless quarks*

We have seen that the only change in the calculation of the energy in the MIT bag model with respect to Bogolioubov is the addition of a volume term, BV. It is *assumed* that B is a universal constant, chosen to fit one piece of data. Once B is chosen, because of the *n.l.b.c.* (which as we have seen implies $\partial E/\partial R = 0$) the radius of the bag is uniquely determined for each hadron. Generalising Eq. (2.63) to include excited states, the *n.l.b.c.* implies

$$\frac{\partial E(R)}{\partial R} = \frac{-\sum_i \omega_i}{R^2} + 4\pi \, R^2 \, B = 0 \; , \tag{2.65}$$

and hence

$$R^4 = \sum_i \omega_i / (4\pi \; B) \; . \tag{2.66}$$

Using Eq. (2.66) we can then simplify the expression for E(R),

$$E(R) = \frac{4}{3}\left(\sum_i \omega_i\right)/R = \frac{4}{3}\left(\sum_i \omega_i\right)^{3/4} (4\pi B)^{1/4} . \qquad (2.67)$$

Clearly the remarkable result obtained by Bogolioubov for the mass of the Roper resonance was indeed a coincidence! In the absence of spin-dependent corrections to be discussed below, M_R/M_N is $(1.55)^{3/4}$ = 1.39, which is still not bad. If once again we choose the average of nucleon and delta masses to set the mass scale we now find

$$R_N \simeq 1.46 \text{ fm} \qquad (2.68)$$

$$B^{1/4} \simeq 113 \text{ MeV}, \; B = 21 \text{ MeV/fm}^3 . \qquad (2.69)$$

2.2.2. *Hyperfine structure*

Since the N and Δ are split by about 300 MeV one is straining the atomic label of 'hyperfine' a little. Nevertheless, it is generally accepted that the degeneracy between these two baryons is broken by the spin-spin interaction. Within the context of QCD DeRujula *et al.* (DeR+ 75) were the first to observe that one-gluon exchange could provide just this kind of interaction. The ideas of DeRujula, Georgi and Glashow have been developed over the last few years into a tremendously successful phenomenological description of hadronic properties using a harmonic oscillator basis and non-relativistic quarks—most notably by Isgur, Karl and collaborators (IK 77, 78, 79, Gut+ 79, For 81, and the whole proceedings of the Baryon '80 Conference, Isg 80). In essence these calculations involve a diagonalisation of the one-gluon exchange interaction in a very limited harmonic oscillator basis.

The philosophy of the bag model is rather different. It is hoped that the bag itself provides a suitable, phenomenological description of the non-perturbative gluon interactions—including gluon-self-coupling. All that remains is the (hopefully weak) one-gluon exchange interaction, which is first order in the colour coupling constant (α_c). Thus there is no diagonalisation procedure in the bag model. One simply uses first order perturbation theory to estimate the energy shifts. As far as this procedure has been tested (which is really not far because of the technical complexity!), the use of perturbation theory does seem justified. For example, Close and Horgan (CH 80,81), and more recently Maxwell and Vento (MV 81), have shown that the admixture of higher configurations in the usual $(1s_{1/2})^3$ nucleon ground state is very small. This is quite different from the large effects found in the non-relativistic models, and shows up most dramatically in the attempts to understand the negative charge radius of the neutron—as we shall discuss in detail in Section 6.2.

If we keep only terms of order α_c the problem reduces to the evaluation of the graphs shown in Fig. 2.1, where both transverse and Coulombic gluons are included. Let us identify ($\vec{E}^a_i$, $\vec{B}^a_i$: a ε 1, ..., 8) as the colour electric and magnetic fields generated by the i^{th} quark. Since the non-perturbative vacuum outside the bag is supposed to be a perfect colour-dia-electric medium (e.g. Cho+ 74a, Lee 79) the appropriate boundary conditions are (for a static spherical bag)

$$\hat{r} \cdot \left(\sum_i \vec{E}^a_i\right) = 0 \; ; \quad \hat{r} \times \left(\sum_i \vec{B}^a_i\right) = 0 \; , \tag{2.70}$$

at the surface. These fields obey the Maxwell equations

$$\underline{\nabla} \times \underline{B}^a_i = \underline{j}^a_i \; ; \quad \underline{\nabla} \cdot \underline{B}^a_i = 0 \; , \tag{2.71}$$

and

$$\underline{\nabla} \cdot \underline{E}^a_i = j^{oa}_i ; \quad \underline{\nabla} \times \underline{E}^a_i = 0 \; , \tag{2.72}$$

inside the bag volume. Here the quark colour current is simply

$$j^{\mu a}_i(x) = g\, \overline{q}_i(x)\, \gamma^\mu\, \lambda^a\, q_i(x) \; , \tag{2.73}$$

with λ^a the usual Gell-Mann SU(3) matrices and g the strong QCD coupling constant. Having solved this rather straightforward problem in classic electromagnetism the one-gluon exchange contribution to the energy, ΔE_g, is

$$\Delta E_g = \alpha_c \sum_{a=1}^{8} \frac{1}{2} \int_{Bag} d\underline{x}[\underline{E}^a \cdot \underline{E}^a - \underline{B}^a \cdot \underline{B}^a] \ , \tag{2.74}$$

$$= \Delta E_g^E + \Delta E_g^M . \tag{2.75}$$

There is one rather shady feature of the present discussion. That is, the quark self-energy shown in Fig. 2.1b should be included as part of the renormalisation of the quark mass and therefore, treated separately. For the magnetic term this is in fact what is done. That is, instead of Eq. (2.74) and (2.75) one actually uses

$$\Delta E_g^M = -\alpha_c \sum_{a=1}^{8} \sum_{i<j} \frac{1}{2} \int d\underline{x} \ \underline{B}_i^a \cdot \underline{B}_j^a \ . \tag{2.76}$$

This is possible because *each* $\underline{B}_i^a$ *separately* satisfies the boundary condition (2.70). On the other hand, for a uniformly charged sphere the electric field is necessarily in the radial direction, in fact

$$\underline{E}_i^a(\underline{r}) \sim \hat{r} \int_0^r \overline{q}_i(\underline{x}) \gamma^0 \ \lambda^a \ q_i(\underline{x}) \ d^3x \ . \tag{2.77}$$

Thus it is possible to satisfy the boundary condition (2.70) only for a colour singlet, for which

$$\sum_i \lambda_i^a = 0 \ . \tag{2.78}$$

Therefore, in order to preserve the boundary conditions we are forced to keep those self-energy terms [Fig. 2.1b] which involve Coulomb-like gluons.

For hadrons in which all quarks have the same mass and are in the same orbit the radial distributions will be identical. From Eq. (2.78) the total colour electric contribution to the energy will then be zero. Even in the case of strange hadrons, in which case $m_s \neq m_{u,d}$ (see Section 2.2.3), the colour electric contribution is of the order 5 MeV or less and is usually ignored (DeG+ 75). Finally then the one-gluon exchange quark-quark interaction gives a spin-spin contribution to the energy. Solving Eq. (2.71) for $\underline{B}_i^a$ and substituting in Eq. (2.76) one finds

$$\Delta E_g^M = \frac{-3\alpha_c}{R} \sum_{a=1}^{8} \sum_{i<j} (\underline{\sigma}_i \lambda_i^a)(\underline{\sigma}_j \lambda_j^a) \times M(m_i, m_j, R) \ , \tag{2.79}$$

where M is a function of the masses of quarks i and j, and the bag radius, which can be obtained in closed form (DeG+ 75).

Using the fact that physical hadrons are colour singlets, so that

$$\sum_i \lambda_i^a = 0 \ , \tag{2.80}$$

and the property of the SU(3) matrices

$$\sum_a (\lambda_i^a)^2 = 16/3 \ , \tag{2.81}$$

one easily finds ($i \neq j$)

$$\begin{aligned} \sum_a \lambda_i^a \lambda_j^a &= -8/3 \quad \text{baryons} \ , \\ &= -16/3 \quad \text{mesons} \ . \end{aligned} \tag{2.82}$$

Thus, the one-gluon exchange interaction is

$$\Delta E_g^M = \frac{\lambda\alpha_c}{R} \sum_{i<j} \widetilde{M}(m_i, m_j, R) \underline{\sigma}_i \cdot \underline{\sigma}_j \ , \tag{2.83}$$

where $\lambda = 1$ for baryons and 2 for mesons. The fact that the sign of the force is the same for both baryons and mesons is a direct consequence of using a non-Abelian theory. Clearly the effect of this interaction is to move m_N down and m_Δ up, because the Δ contains only triplet states ($\underline{\sigma}_i \cdot \underline{\sigma}_j = +1$). In fact in this case the amounts up and down are equal and, of course, proportional to α_c. This splitting essentially determines α_c, and in the old MIT fits it was 2.2.

As another example consider the Λ and Σ. Because $m_s \neq m_{u,d}$ these will also be split. Basically in the Λ the u and d are in a spin singlet state so $\underline{\sigma}_u \cdot \underline{\sigma}_d = -3$, and $\underline{\sigma}_s \cdot (\underline{\sigma}_u + \underline{\sigma}_d) = 0$. In the Σ the u and d have $S = 1$, so $\underline{\sigma}_u \cdot \underline{\sigma}_d = +1$, and $\underline{\sigma}_s \cdot (\underline{\sigma}_u + \underline{\sigma}_d) = -4$. Consequently

$$\Delta E_g^M(\Lambda) = -3\alpha_c \overline{M}(0,0) , \tag{2.84}$$

(where $\overline{M}(0,0)$ refers to the masses of the u and d quarks) while

$$\Delta E_g^M(\Sigma) = +1\alpha_c \overline{M}(0,0) - 4\alpha_c \overline{M}(0,m_s) . \tag{2.85}$$

Clearly $\Delta E_g^M(\Lambda) = \Delta E_g^M(\Sigma)$ if $m_s = 0$, but with $m_s > 0$, $\overline{M}(0,m_s)$ is somewhat suppressed and hence m_Λ is less than m_Σ.

2.2.3. *Non-zero quark masses*

If the strange quark was massless the other members of the nucleon octet, namely the Σ, Λ and Ξ, would all be degenerate with the nucleon. From pre-bag phenomenology one might expect that giving the strange quark a mass would solve the problem and that is indeed the case. We might add that there is presently no understanding of the quark-mass problem, these can only be regarded as free parameters of the theory (see however, CT 74, Fri 77, Gun+ 77, Wei 77).

In the case where the mass of a quark is not precisely zero inside the bag, all of the formalism of Section 2.1.1 can again be applied—we need only change the form of W(r) in the Dirac equation (2.9). Inside the bag we then have

$$(-i \,\underline{\gamma}\cdot\underline{\nabla} + \gamma^0 E + m)q(\underline{r}) = 0 , \tag{2.86}$$

with the boundary condition

$$-i \,\vec{\gamma}\cdot\hat{r}\, q = q \text{ at } r = R . \tag{2.87}$$

This has the solution for $\kappa = -1$ (i.e. an $s_{1/2}$ level),

$$q(\underline{r}) = \frac{N(x)}{\sqrt{4\pi}} \begin{bmatrix} \left(\frac{E+m}{E}\right)^{1/2} & j_0\left(\frac{xr}{R}\right) \\ \left(\frac{E-m}{E}\right)^{1/2} & i\vec{\sigma}\cdot\hat{r}\; j_1\left(\frac{xr}{R}\right) \end{bmatrix} \chi , \tag{2.88}$$

where

$$E(m,R) = \frac{1}{R}\left[x^2 + (mR)^2\right]^{1/2} , \tag{2.89}$$

and the normalisation constant is

$$N^{-2}(x) = R^3\, j_0^2(x)\, \frac{2E(E - 1/R) + m/R}{E(E - m)} . \tag{2.90}$$

The eigenfrequency, x, resulting from the imposition of the *l.b.c.* satisfies

$$\tan x = \frac{x}{1 - mR - [x^2+(mR)^2]^{1/2}} \,. \tag{2.91}$$

Obviously we have x = 2.04 when m = 0, as before. It rises to π as $mR \to \infty$. At mR = 1.5, corresponding approximately to the best fit for the strange quark mass, $m_s \sim 300$ MeV (DeG+ 75), we obtain x = 2.5, and (ER) = 2.92. Thus, forgetting about the *n.l.b.c.* for the moment, we see that replacing one up or down quark by a strange quark raises the mass of the hadron by (2.92-2.04)/R, or about 170 MeV—which by construction agrees with the observed Λ-N mass splitting.

2.2.4. *Other corrections to hadronic masses*

There are two other possible contributions to the mass of the hadron which have been discussed in the literature. As always when one quantises a radiation field there is some infinite zero-point term. However, when the quantisation is carried out in a finite cavity there will in general be additional, finite pieces which depend on the size of the cavity. It has not yet proven possible to calculate the finite remainder for a spherical cavity. (See however, Section 2.3.1 for a recent discussion by Johnson (Joh 79) based on an analogy with QED.) For phenomenological simplicity it has been parametrized (Cho+ 74, DeG+ 75) as a constant Z_0 divided by the bag radius R.

$$\Delta E_Z^{(o)} = -Z_0/R \,. \tag{2.92}$$

The constant was determined to be $Z_0 = 1.8$ in the fit of DeGrand and co-workers (DeG+ 75).

The second correction would be the most obvious to nuclear physicists. That is we have adopted the equivalent of the "independent particle shell model" for a three-quark hadron. For ^{3}He every nuclear theorist would recognise that there would be a sizeable spurious contribution to the energy from motion of the centre of mass. To estimate the size of this effect let us assume that the (relativistic) energy of the bag E(R) is related to the mass, M(R), by

$$E^2(R) \simeq \langle \underline{p}_{cm}^2 \rangle + M^2(R) \ , \tag{2.93}$$

so that

$$M(R) \simeq E(R) - \langle \underline{p}_{cm}^2 \rangle / 2E(R) \ . \tag{2.94}$$

But the total c.m. momentum is

$$\langle \underline{p}_{cm}^2 \rangle = \left\langle \left(\sum_i \underline{p}_i \right)^2 \right\rangle \ ,$$

$$\simeq \sum_i \langle p_i^2 \rangle \ . \tag{2.95}$$

Using the fact that for a massless quark $\langle p_i^2 \rangle = \omega_i^2/R^2$, we find for the nucleon [using Eq. (2.67), (2.94) and (2.95)]

$$\Delta E_{cm} \simeq \frac{-3\omega^2/R^2}{(8\omega/R)} = -\frac{3}{8}\frac{\omega}{R} \simeq -0.75/R \ . \tag{2.96}$$

For a radius of 1.4 fm this is of the order 110 MeV, which is a sizeable correction. It becomes even more important as R decreases.

From the phenomenological point of view we notice that Eq. (2.96) has the same dependence on R as Eq. (2.92) and is about one half as big. Thus a good part of the original "zero point energy" can be understood as a correction for spurious c.m. motion. Of course, our derivation should make it obvious that Z_0 should not be strictly constant, and this has been approximately taken into account in recent fits (DJ 80, Myh+ 81) by multiplying Eq. (2.96) by m_N/m_B, with m_B the physical mass of the appropriate baryon.

For the ground state baryons this c.m. correction is simply an inconvenience and requires some correction to the energy. However, when we deal with excited states it becomes critical. In particular, the first applications of the bag model to negative parity baryons found many more states than are seen experimentally. Some of these, explicitly the members of the (56, 1^-) representation of SU(4) × O(3), correspond to translation of the centre of mass of the bag and are spurious (DJ 76, DeG 76, Reb 76, Hey 77, DR 78). The detailed discussion of how to eliminate spurious c.m. motion for excited bag states is technically very complicated (unlike the non-relativistic harmonic oscillator calculations!). Moreover, the numerical results (DeG 76) are really rather poor—possibly because the MIT bag overestimates the spin-orbit force splitting of the $p_{1/2}$ and $p_{3/2}$ levels (DeG 80). We refer the interested reader to the literature already cited and particularly to the proceedings of Baryon '80 (Isg 80) for further discussion.

2.2.5. *Summary*

The complete mass formula for the original MIT bag model can then be summarised as

$$M(R) = \sum_i \frac{\omega_i}{R} + \frac{4\pi}{3} B\ R^3 + \Delta E_g^M - Z/R\ , \qquad (2.97)$$

with the spin-dependent one-gluon exchange interaction given by Eq. (2.83). The last term in Eq. (2.97) is now understood to include both c.m. and zero point energies. There are four adjustable parameters in this mass formula, namely m_s, B, α_c and Z and the radius R is determined for each hadron by the *n.l.b.c.* (the requirement of stability)

$$\frac{\partial M}{\partial R} = 0\ . \qquad (2.98)$$

The original fit by DeGrand and co-workers (DeG+ 75) is shown in Fig. 2.2. It really gives an excellent description of the lowest baryon octet and decuplet, as well as the two lowest meson octets. The only exception is the pion which is too heavy. However, it should be obvious from the discussion of Section 2.2.4 that the approximate correction for spurious c.m. motion will be meaningful only for fairly heavy states. For the pion it is not inconceivable that the entire bag model mass is spurious (DJ 80)! In particular, as we shall discuss further in Section 5, considerations of chiral symmetry strongly suggest that in the limit $m_u = m_d = 0$, the pion mass should also vanish. In this case it would be a true Goldstone boson associated with dynamically broken chiral symmetry (Gel+ 68, Pag 75, CJ 80, HG 81, GH 81, Joh 79).

2.3. Attempts to Derive a Bag Model

The proof of quark confinement on the basis of QCD has not yet been achieved. Thus there is no derivation of a bag or its properties or anything like it from a fundamental theory. Nevertheless there have been a number of very suggestive arguments which lead one to believe that the MIT bag model may not be far from the truth. A strictly personal collection of those arguments which the author finds most compelling will be briefly reported here.

2.3.1. *The bubbly vacuum*

Johnson recently presented some rather simple considerations (Joh 79) which suggest that the most stable vacuum configuration in QCD should be a collection of bubbles of size R of order Λ^{-1} (with Λ the QCD scale parameter).

The starting point for this work is the recent solution of a long-standing problem in QED. Suppose one has a cavity of radius R with conducting walls—that is with the boundary condition

$$\hat{r} \times \underline{E} = 0 = \hat{r} \cdot \underline{B} , \qquad (2.99)$$

at the surface. Then the piece of the total energy which depends on R is (Mil+ 78, BD 78, Boy 68)

$$E_{QED} = a_{QED}/R, \quad a_{QED} = 0.04618 . \qquad (2.100)$$

That this answer is finite is the result of a natural high frequency cut-off arising from the cancellation of small wavelength effects just inside and outside the conducting boundary. It must be stressed that the nature of the boundary is critical.

For QCD the analogous boundary conditions are given in Eq. (2.70), but since Maxwell's equations are invariant under $\underline{E} \to \underline{B}$ and $\underline{B} \to -\underline{E}$, we can take this result over. Now of course there are eight gluon fields and we assume that R is small enough to permit the use of perturbative QCD. To lowest order we then find

$$E^{(o)}_{QCD} = a_{QCD}/R = 8\ a_{QED}/R = 0.369/R\ . \tag{2.101}$$

The difference in QCD is, of course, that the gluons have self-interactions. Interactions of the sort shown in Fig. 2.3 are known to be attractive for the colour singlet state. Thus there is a pairing-type force which tends to favour colour singlets. Furthermore, this attraction should grow rapidly with R.

Johnson parametrises the higher order non-Abelian effects in terms of a running coupling constant

$$\alpha_c(\Lambda R) = \frac{1}{(9/2\pi)\ \ell n\left((\Lambda R)^{-1}+1\right)}\ . \tag{2.102}$$

The total energy of the bubble of radius R would then be

$$E^{(1)}_{QCD}(R) = a_{QCD}/R - \frac{b}{R}\,\alpha_c(\Lambda R)\ , \tag{2.103}$$

with b(>0) an unknown constant. Clearly as R grows, eventually $\alpha_c(\Lambda R)$ will be greater than a_{QCD}/b and the bubble has an energy density below the non-interacting case. Finally E(R) eventually vanishes as R goes to infinity. We therefore expect the most stable bubble at some finite radius R_o which can be found by minimising the energy density

$$\partial(E^{(1)}_{QCD}/V)/\partial R\,|\,R=R_o = 0\ . \tag{2.104}$$

The QCD vacuum tends to break spontaneously into a set of bubbles of size R_o!

By extending this argument to include quark degrees of freedom, Johnson was able to derive a formula for hadronic masses very close to the static MIT bag model. In particular, the bag constant (B) is simply the energy per unit volume of the empty bags surrounding the hadron. From a simple phenomenological analysis he found $R_0 \simeq 0.5$ fm with $\Lambda =$ 500 MeV. While this picture is very much simplified—for example it is not Lorentz invariant—it has many suggestive features. Most importantly there is a volume energy, the hadron is stable [see e.g. Eq. (2.98)], perturbative QCD is permitted inside the bag, and there is a very rapid phase transition at the surface. Of course the physical nature of the surface which would provide the colour-dia-electric boundary conditions is beyond the scope of this treatment.

2.3.2. *Soliton bag models*

Many groups have proposed that bag formation should be associated with a phase transition. In the presence of the strong colour fields inside the bag the vacuum is very simple and the quarks are essentially free. However, at some critical field strength there is a phase transition to a highly complicated vacuum state with colour dielectric constant $\kappa \to 0$, thus confining colour fields. In the Princeton picture the pion appears as an essential part of this process (Cal+ 78, Cal+ 79). As we shall discuss further in Sections 4 and 5, in their picture it is a Goldstone boson associated with the breaking of chiral symmetry in the complicated vacuum outside the bag. It contributes to the bag pressure.

Goldman and Haymaker (GH 81, HG 81) have recently demonstrated how pion and sigma (scalar-isoscalar) fields can appear as a result of dynamical symmetry breaking in a model of the Jona-Lasinio Nambu type

(NJ 61). Although it was not strictly QCD the model was sufficiently realistic to be highly suggestive. We shall return to the need for the pion again in Section 5. The appearance of an effective σ- field interacting with the quarks is, however, directly relevant here. In particular Friedberg and Lee have shown that it is possible to obtain bag-like states as soliton solutions of a relativistic, local field theory containing just q and σ (Bar+ 75, IM 75, Cre 74, CS 75, FL 77, FL 78, Lee 79).

A complete discussion of soliton models of elementary particles is far beyond the scope of the present review. The interested reader should refer first to the recent text by Lee (Lee 81) and then to the references therein. For our purposes it is sufficient to summarise the recent discussion of Goldflam and Wilets (GW 82), which has by far the most detailed numerical results for the soliton bag.

Consider the following Lagrangian density for interacting σ and quark fields,

$$\mathcal{L}(x) = i\,\bar{q}\,\not{\partial}\,q - g\,\bar{q}\,\sigma\,q + \frac{1}{2}(\partial_\mu\,\sigma)^2 - U(\sigma) \quad . \qquad (2.105)$$

The first and third terms are standard kinetic energy operators and the second is the simplest possible coupling. The existence of soliton-like solutions is a consequence of the non-linear form of the potential $U(\sigma)$

$$U(\sigma) = \frac{c}{24}\,\sigma^4 + \frac{b}{6}\,\sigma^3 + \frac{a}{2}\,\sigma^2 + p \ , \qquad (2.106)$$

whose general form is illustrated in Fig. 2.4. (Equation (2.106) is the most general self-coupling permitted in a renormalisable field theory.) The energy of the σ-field alone will be a minimum at the minimum of $U(\sigma)$ (recall $T^{oo} \sim -\mathcal{L} \sim +U$), namely

$$\sigma_V = \frac{3}{2c}\left[-b + \sqrt{b^2 - 8/3\ ac}\,\right]. \qquad (2.107)$$

(It is usual to choose p so that $U(\sigma_V) = 0$.) In the absence of a coupling to quark fields the lowest energy state would be simply a constant classical field $\sigma = \sigma_V$ throughout space.

However, suppose that there is a non-zero quark density at some point in space, which we can choose to be $\underline{r} = 0$. (Strictly we want $\overline{q}q \neq 0$.) The second term on the r.h.s. of Eq. (2.105) is then linear in σ as shown in Fig. 2.4. Clearly if either g or $\overline{q}q$ is large enough, it is possible that the minimum energy will occur at $\sigma = 0$ rather than $\sigma = \sigma_V$. In this region the quark and sigma fields obey coupled linear equations

$$(\underline{\alpha}\cdot\underline{p} + g\,\gamma^0\,\sigma_0)\psi_k = \varepsilon_k\,\psi_k\ ,$$
$$-\nabla^2\,\sigma_0 + U'(\sigma_0) = -g\sum_k \overline{\psi}_k\,\psi_k\ , \qquad (2.108)$$

where σ_0 is the time-independent, mean σ-field. Some typical solutions of these equations are plotted in Fig. 2.5.

In all cases $\overline{\psi}\psi$ eventually vanishes as $r \to \infty$ so that asymptotically σ returns to its usual vacuum expectation value. Inside, however, σ is very small and the quarks are essentially free ($\sigma(o) \approx 0$). That is the quarks "dig a hole" in the complicated vacuum represented by large σ_V within which things are simple. Case 1 in Fig. 2.5 represents an MIT-bag type of solution where the quarks are distributed through the bag volume, while case 2 is a SLAC-bag (Bar+ 75) with its strongly surface-peaked quark distribution.

Many other intermediate solutions are possible depending on the choice of parameters (a,b,c). However, the bag like properties, namely that the quarks are essentially free inside and that the transition region from inside to outside is quite sharp is true in all confining solutions. That is, the transition is sharp in all solutions where $(g\sigma_V)$ [the quark mass outside the bag from Eq. (2.105)] is chosen to be extremely large. Finally, we note that as discussed by Lee the colour dielectric constant κ is

$$\kappa = \left(1 - \frac{\sigma}{\sigma_V}\right), \tag{2.109}$$

and therefore vanishes outside the bag (Lee 79). It will be close to one inside, and the gluon fields therefore essentially free, if $\sigma \ll \sigma_V$ in that region. In that case a perturbation expansion of hadronic properties in powers of the colour coupling constant α_c should make sense. That is precisely the philosophy of the phenomenological bag model which we have discussed!

2.4. Relationship to the Non-Relativistic Quark Models

Although it is not our purpose to review the non-relativistic quark model here, it is so widely used and generally regarded as being so successful that some comments must be made about the relationship to the bag model. Some of the comments found here have also been made by Thomas DeGrand (DeG 80).

While the identification is not so straightforward, it may be helpful to consider the bag model quarks with essentially zero mass (for u and d) to be what is usually referred to as "current quarks". It is these objects that are confined in an infinite scalar potential as we have seen. The result of this confinement is an energy level of the scale of typical hadronic masses. This eigenfunction can be thought of as a "constituent quark". Now if there is some truth to such a translation there are important consequences for the usual diagonalisation procedures of the non-relativistic quark models, and this augments the surprise at their success. We defer further discussion of this until Sections 6 (neutron charge radius) and 7 (N-N force).

One major objection to the non-relativistic (or harmonic oscillator) quark model calculations is the tendency to ignore relativistic corrections. In computing μ_p and μ_n, for example, the up and down quark masses are chosen so that the corresponding Dirac moments $(e/2m_q)$ when added non-relativistically yield a good fit. Relativistic corrections are simply omitted despite the fact that typically $\langle p^2 \rangle/2m_q$ is bigger than m_q!

As Litchfield remarked (Isg 80, p.216), "despite the theoretical bricks thrown at Isgur and Karl's model the amazing thing is how well their formulae actually fit an extremely large and varied data set. This would seem to imply that there must be a basis of truth in the arithmetic and maybe more effort should go into seeing why their formula is so nearly correct". In the case of magnetic moments the bag model does just this. As we shall see in Section 3.2 there is no free parameter in computing the magnetic moment of a massless quark in a bag. The answer is, however, proportional to $(R/\omega_{n,\kappa})$ which we recognise as one over the energy of the appropriate level. This includes *all* relativistic effects. However, as we remarked earlier the bag model quark energy is essentially the constituent quark mass! Thus both models arrive at an answer proportional to m_q^{-1}, but the bag helps us to understand why there are no "relativistic corrections".

The non-relativistic models really have tremendous practical advantages in dealing with excited states. As DeGrand observes, "(although I hate to admit it) bag models are calculationally much more unwieldy than the non-relativistic quark models". The major problem is to deal with spurious centre of mass motion. It does not appear likely that this problem with bag models will improve in the near future.

Finally, as we shall re-emphasise in Section 4, the bag model can be formulated (at least in principle) as a relativistic local field theory. In particular it can readily be described by a Lagrangian and all the standard technology can be applied to it. This has been extremely important in discussions of symmetry properties (conserved currents) and was certainly an important factor in spurring the further development of chiral bags. It is dubious whether such considerations would ever have arisen out of the potential model calculations.

Figure Captions

Fig. 2.1. One gluon exchange contributions to the energy of the MIT bag.

Fig. 2.2. The mass spectrum of the low-lying hadrons calculated in the MIT bag model (DeG+ 75).

Fig. 2.3. Some low order gluon self-interactions.

Fig. 2.4. A typical form for the σ-potential energy, U(σ), in a soliton bag model.

Fig. 2.5. Numerical results from the soliton bag model calculations of Goldflam and Wilets (GW 81) showing: a) the σ-field for MIT-like solutions, and b) the quark density for MIT-like (g=15), SLAC-like (g=200) and intermediate bags.

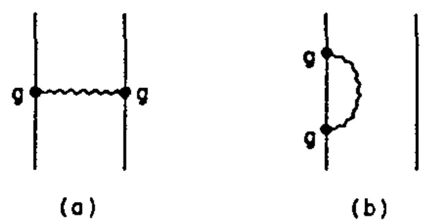

Fig. 2.1

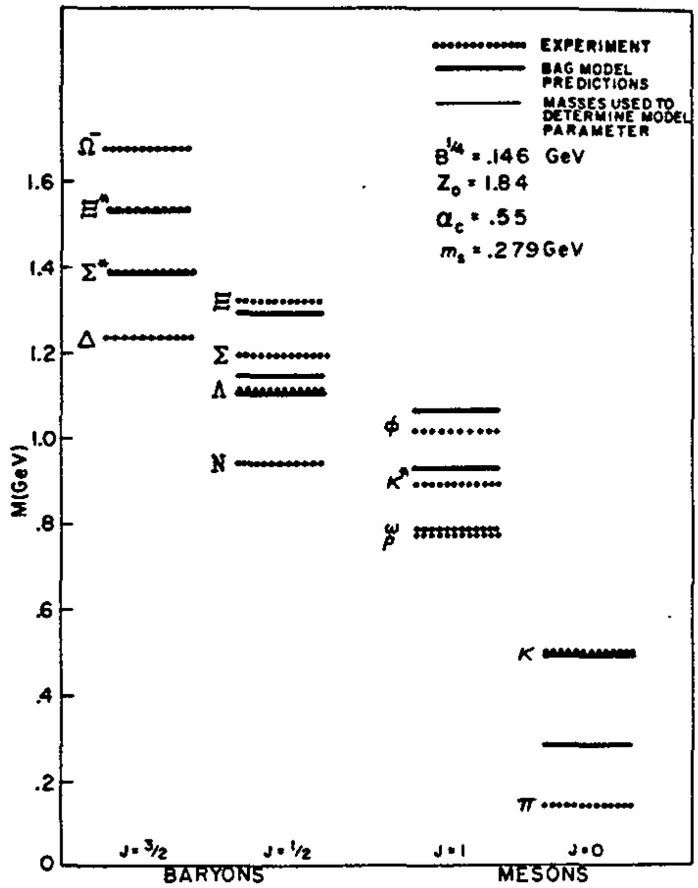

Fig. 2.2

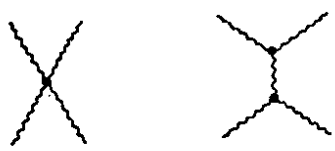

Fig. 2.3

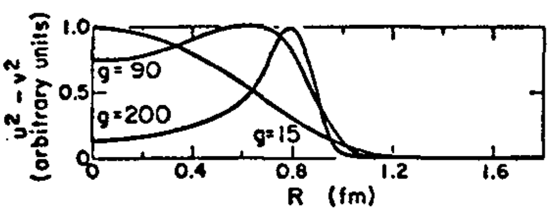

Fig. 2.5(a)

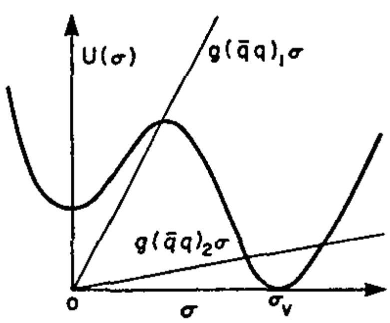

Fig. 2.4

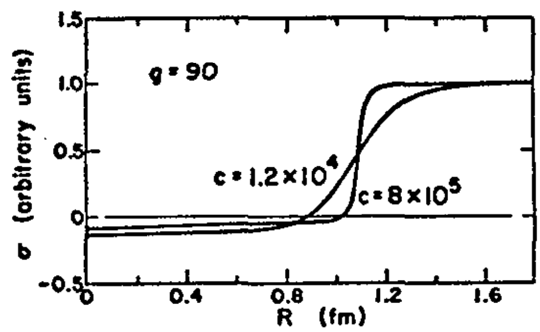

Fig. 2.5(b)

3. HADRONIC PROPERTIES IN THE MIT BAG MODEL

In the last section we gave some arguments to support the idea that QCD could lead to bag-like hadrons. We discussed the quark wave functions in the bag in great detail and showed that the fit to at least the low-lying baryon and meson masses was rather good. From the purist's point of view it is an attractive feature of the bag model that once the fit to the mass of a hadron has been made there are no further parameters to adjust. Either the calculated properties, r.m.s. charge radius, magnetic moment, axial coupling constant and so on agree with the data or they don't. In this section we shall show how these three basic properties are calculated. For the axial coupling constant (Section 3.3) the agreement with experiment is excellent—as realised originally by Bogolioubov. For the charge radii and magnetic moments (Sections 3.1 and 3.2 respectively) the model is not quite so successful, particularly when the question of c.m. corrections is raised again (Section 3.4).

3.1. Charge Radii

As we remarked in Section 2 the matter density for a particular quark, i, is given (as usual for a Dirac particle) by

$$j_i^0(\underline{r}) = \bar{q}_i(\underline{r})\,\gamma^0\,q_i(\underline{r})\theta_V = q_i^\dagger(\underline{r})\,q_i(\underline{r})\theta_V\ , \tag{3.1}$$

which is plotted in Fig. 3.1 for the $1s_{1/2}$ level. Clearly then the charge density for a given hadron is

$$\rho(\underline{r}) = \sum_i q_i^\dagger(\underline{r})\,Q_i\,q_i(\underline{r})\ , \tag{3.2}$$

where Q_i is the charge of the i^{th} quark. The r.m.s. charge radius is therefore

$$\langle r^2\rangle_{ch} = \sum_i Q_i \int_{Bag} d\underline{r}\ q_i^\dagger(\underline{r})\ r^2\ q_i(\underline{r})\ , \tag{3.3}$$

which is proportional to R^2. In fact for the proton it is easily shown, using Eq. (2.33), (2.34) and (3.3), that

$$\langle r^2\rangle^p_{ch} = \left\{\frac{\omega^3_{1-1}}{2(\omega_{1-1}-1)\sin^2\omega_{1-1}} \int_0^1 dx\, x^4\left(j_0^2(\omega_{1-1}x) + j_1^2(\omega_{1-1}x)\right)\right\} R^2. \quad (3.4)$$

With $\omega_{1-1} = 2.04$ this gives

$$\langle r^2\rangle^{p\,1/2}_{ch} = 0.73 \text{ fm} , \quad (3.5)$$

for R = 1 fm, as found in the fit of DeGrand *et al.* (DeG+ 75). This is to be compared with the experimental value of 0.82 fm.

Of more profound importance, as we shall discuss at length in Section 6, is the neutron charge distribution. Because each quark occupies the same spatial state, and the sum of the quark charges is zero, the mean square charge radius of the bag model neutron is zero in lowest order. Experimentally $\langle r^2\rangle^n_{ch}$ is known to be -0.116 fm^2, from very accurate experiments with thermal neutrons. Attempts have been made to obtain this negative tail of the charge distribution through the perturbation of the ground-state wave function by one-gluon exchange. In the neutron the dd pair necessarily has isospin one and hence spin one (because their colour wave function is anti-symmetric). From Eq. (2.83) we see that the dd interaction is therefore repulsive and will tend to push the dd pair out from the centre of the bag— hence a negative tail for the charge distribution. Quantitatively this idea fails for the bag. For example, Close and Horgan (CH 81) and Maxwell and Vento (MV 81) both find that this effect explains only about 6% of the observed ratio of $\langle r^2\rangle^n_{ch}/\langle r^2\rangle^p_{ch}$.*

*In fact Maxwell and Vento include sea quark contributions omitted by Close and Horgan. While the magnitude does not change, even the sign of $\langle r^2\rangle^n_{ch}$ is then in disagreement with experiment.

3.2. Magnetic Moments

In the bag model the quarks are structureless Dirac particles which therefore have no intrinsic moments. Indeed they yield a magnetic moment only because of the confinement. As for any current loop the magnetic moment is given by

$$\underline{\mu} = \frac{1}{2}\int (\underline{r} \times \underline{j}_{em})d\underline{r} \ . \tag{3.6}$$

Using the usual form for the Dirac electromagnetic current

$$= \frac{1}{2}\int_{Bag} d\underline{r}\ \underline{r} \times \left[\sum_i q_i^{\dagger}(\underline{r})\ \underline{\alpha}_i Q_i\ q_i(\underline{r})\right] , \tag{3.7}$$

and the $1s_{1/2}$ wave functions of Section 2 we find (for massless quarks)

$$\underline{\mu} = \frac{N^2}{2}\sum_i Q_i \int_0^R dr\ r^2\Big(j_o(\omega r/R),\ -i\underline{\sigma}_i\hat{\underline{r}}\ j_1(\omega r/R)\Big) \times$$
$$\times \begin{pmatrix} 0 & \underline{r}\times\underline{\sigma}_i \\ \underline{r}\times\underline{\sigma}_i & 0 \end{pmatrix} \begin{pmatrix} j_o(\omega r/R) \\ i\underline{\sigma}_i\cdot\hat{r}\ j_1(\omega r/R) \end{pmatrix} . \tag{3.8}$$

Finally after a little spin algebra Eq. (3.8) reduces to the form

$$\underline{\mu} = \mu_o \sum_i \underline{\sigma}_i\ Q_i \ , \tag{3.9}$$

where μ_o is directly proportional to the radius of the confinement region, that is

$$\mu_o = \frac{4\omega-3}{\omega(\omega-1)}\frac{R}{12}$$
$$= \frac{2.43}{12}(2m_N R)\mu_N \ . \tag{3.10}$$

Here μ_N is the usual nuclear magneton $(e/2m_N)$.

Therefore, just as in the non-relativistic constituent quark models we are left to evaluate a simple spin-isospin matrix element

$$\mu_p = \mu_o < p\!\uparrow\! | \sum_{i=1}^{3} \sigma_{iz} Q_i | p\!\uparrow\! > \ . \tag{3.11}$$

However, instead of μ_o being adjusted to fit experiment it is calculable in terms of the quark radial wave functions yielding Eq. (3.10). Using the standard mixed symmetry spin-flavour wave functions (Kok 69)

$$\psi' = 2^{-1/2}\ (\text{duu} - \text{udu}) \ ;\ \psi'' = -\left(\frac{2}{3}\right)^{1/2}\left[\text{uud} - \frac{1}{2}(\text{udu} + \text{duu})\right] , \qquad (3.12)$$

and similarly the mixed symmetry spin states χ' and χ'' [substitute $u \to \uparrow$ and $d \to \uparrow$ in Eq. (3.12)], we have

$$|p\uparrow\rangle = 2^{-1/2}\ (\psi'\ \chi' + \psi''\ \chi'') \ . \qquad (3.13)$$

It is a straightforward exercise to prove that

$$\langle p\uparrow| \sum_{i=1}^{3} Q_i \sigma_{iz} |p\uparrow\rangle = +1 \ , \qquad (3.14)$$

and hence

$$\mu_p = \mu_o \ . \qquad (3.15)$$

With the original radius R = 1.3 fm this gives $\mu_p = 2.6\ \mu_N$, but with the 'best fit' parameters of DeGrand *et al.* $\mu_p = 1.9\ \mu_N$ (DeG+ 75). As recognised by those authors, the failure to reproduce the magnitude of the proton magnetic moment was the most serious discrepancy of all the predictions of the model. Nevertheless, if one normalises all other moments to that of the proton it is a remarkable fact that the bag model is invariably an improvement over the naive SU(3) predictions — see Table 3.1.

3.3. The Axial Current

The accurate prediction of the axial coupling constant, g_A, is certainly one of the major successes of the MIT bag model. It is a direct consequence of the correct, relativistic treatment of the quarks. In view of the importance of the axial current in our later development of a chiral-symmetric model of hadronic structure we shall discuss the calculation of g_A in detail. First we briefly review the standard phenomenological treatment of weak interactions.

3.3.1. *A brief review of weak interactions*

The usual weak interaction Hamiltonian has a current-current form (Mar+ 69)

$$H_W = \frac{G}{2}\ J^{\mu}\ J_{\mu}^{+} \ , \qquad (3.16)$$

where the coupling constant is

$$G \simeq 10^{-5}\ m_p^{-2}\ . \qquad (3.17)$$

The current is a sum of hadronic and leptonic pieces

$$J^\mu = J_h^\mu + J_\ell^\mu\ , \qquad (3.18)$$

where (assuming V-A) ,

$$J_\ell^\mu = \overline{\nu}_e\ \gamma^\mu(1-\gamma_5)e + (e\rightarrow\mu\ ;\ \nu_e\rightarrow\nu_\mu)\ , \qquad (3.19)$$

and

$$J_h^\mu = V^\mu + A^\mu\ . \qquad (3.20)$$

The hadronic vector and axial vector components have both strangeness conserving ($\Delta S=0$, proportional to $\cos\theta_c$) and non-conserving ($\Delta S=1$, proportional to $\sin\theta_c$) pieces. For our purposes only the $\Delta S=0$ piece is relevant and we shall effectively set $\theta_c = 0$ for pedagogical purposes. Consider the semi-leptonic matrix element

$$i \rightarrow f + \text{leptons}\ , \qquad (3.21)$$

which would appear (for example) in β-decay. This matrix element is proportional to

$$\langle f,\ell|J_h^\mu\ J_\mu^{\ell\dagger}|i\rangle = \langle f|J_h^\mu|i\rangle\ \langle\ell|J_\mu^{\ell\dagger}|0\rangle\ , \qquad (3.22)$$

where the leptonic piece is known exactly. In the simple case of neutron β-decay

$$\langle f|J_h^\mu|i\rangle = \langle p|J_h^\mu|n\rangle$$

$$\sim e^{-i\underline{k}\cdot\underline{x}}\ \overline{u}_p[\gamma^\mu\ g_V(k^2) - \gamma^\mu\ \gamma_5\ g_A(k^2) + - - -]u_n\ , \qquad (3.23)$$

where the corrections are of order k^2, and therefore suppressed. In fact, in the limit of very small $\underline{k}$ Eq. (3.23) is simply

$$\langle p|J_h^\mu|n\rangle \simeq g_V(0)\ \delta_{\mu o} - g_A(0)\ \sigma_i\ \delta_{\mu i}\ , \qquad (3.24)$$

where

$$g_V \simeq 1 \text{ and } \frac{g_A}{g_V} = 1.24\ . \qquad (3.25)$$

The fact that $g_V = 1$ even in the presence of strong interactions is, of course, of great significance and is 'explained' by the CVC hypothesis of Feynman and Gell-Mann (BD 64). That is, the vector current is assumed to be directly porportional to the isospin current

$$V_\mu^a = 2g_V \, I_\mu^a \ , \tag{3.26}$$

where I_μ^a is the isospin current which is *conserved in strong interactions*.

The fact that g_A is so nearly one is also highly suggestive, as we shall discuss in the next Section. For the moment we ask only how to calculate this in the bag model. The isospin current in the MIT bag model is

$$\underset{\sim}{I}^\mu(\underline{x}) = \sum_i \bar{q}_i(\underline{x})\gamma^\mu \frac{\underset{\sim}{\tau}}{2} q_i(x)\theta_V \ , \tag{3.27}$$

and the axial current

$$\underset{\sim}{A}^\mu(\underline{x}) = \sum_i \bar{q}_i(\underline{x})\gamma^\mu \gamma_5 \frac{\underset{\sim}{\tau}}{2} q_i(\underline{x})\theta_V \ . \tag{3.28}$$

(By analogy with Eq. (3.26) we let $\underset{\sim}{A}^\mu = 2\underset{\sim}{A}^\mu$.)

Clearly then at $\underline{k} = 0$, the matrix element

$$\underset{\sim}{t}_{pn} = \int d^3r \langle p|\underset{\sim}{I}^0(r)|n\rangle\Big|_{\underline{k}=0} = \int d^3r \sum_i \langle p|q_i^\dagger(\underline{r}) \ \frac{\underset{\sim}{\tau}}{2} q_i(\underline{r})|n\rangle \ ,$$

and because the quark radial wave functions are normalised we find

$$\underset{\sim}{t}_{pn} = {}_{s-f}\langle p| \sum_i \frac{\underset{\sim}{\tau}_i}{2}|n\rangle_{s-f} \ , \tag{3.29}$$

where the subscript indicates a spin-flavour matrix element only. Using the wave function given earlier (and the analogous one for the neutron) this becomes

$$\underset{\sim}{t}_{pn} = \langle p|\frac{\underset{\sim}{\tau}}{2}|n\rangle \ . \tag{3.30}$$

On the other hand, again with $\underline{k} = 0$, we find

$$\int d^3x \langle p|\vec{\underset{\sim}{A}}(\underline{x})|n\rangle\Big|_{\underline{k}=0} = \int d^3x \sum_i \langle p|q_i^\dagger(\underline{x})\gamma^0 \ \vec{\gamma} \ \gamma_5 \frac{\underset{\sim}{\tau}}{2} q_i(\underline{x})|n\rangle \ . \tag{3.31}$$

However, with our conventions (Appendix I)

$$\gamma^0 \ \vec{\gamma} \ \gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & \underline{\sigma} \\ -\underline{\sigma} & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \underline{\sigma} & 0 \\ 0 & \underline{\sigma} \end{pmatrix} , \tag{3.32}$$

and hence

$$\int_{Bag} d\underline{x}\; q^+(\underline{x})\gamma^0\, \vec{\gamma}\, \gamma_5\, q(\underline{x}) = \int_{Bag} d\underline{x}\, \frac{N^2}{4\pi}(j_0,\, -i\underline{\sigma}\cdot\hat{x} j_1) \begin{pmatrix}\underline{\sigma} & 0\\ 0 & \underline{\sigma}\end{pmatrix}\begin{pmatrix} j_0 \\ i\underline{\sigma}\cdot\hat{x} j_1\end{pmatrix}$$

$$= \frac{N^2}{4\pi}\int_0^R dx\, x^2 \int d\hat{x}\left[j_0^2\, \underline{\sigma} + j_1^2\, \underline{\sigma}\cdot\hat{x}\, \underline{\sigma}\, \underline{\sigma}\cdot\hat{x}\right] . \tag{3.33}$$

However, after a little spin algebra we find

$$\int d\hat{x}\; \underline{\sigma}\cdot\hat{x}\; \underline{\sigma}\; \underline{\sigma}\cdot\hat{x} = \frac{-4\pi}{3}\,\underline{\sigma} , \tag{3.34}$$

and hence

$$\int d^3x \langle p|\vec{\underset{\sim}{A}}(\underline{x})\,|n\rangle|_{\underline{k}=0} = N^2 \int_0^R dx\; x^2\left[j_0^2\left(\frac{\omega x}{R}\right) - \frac{1}{3}\, j_1^2\left(\frac{\omega x}{R}\right)\right] \times$$

$$\times\; {}_{s-f}\langle p|\sum_{i=1}^{3}\vec{\sigma}_i\, \frac{\underset{\sim}{\tau}_i}{2}|n\rangle_{s-f} . \tag{3.35}$$

The first term can be evaluated analytically and for massless u and d quarks one finds (DeG+ 75)

$$N^2\int_0^R dx\; x^2\left(j_0^2 - \frac{1}{3}\, j_1^2\right) = 1 - \frac{1}{3}\left(\frac{2\omega-3}{\omega-1}\right) = 0.65 . \tag{3.36}$$

This is the crucial difference from non-relativistic (constituent) quark models.

In the usual non-relativistic quark model,

$$\vec{\underset{\sim}{A}}_{NR} = \sum_{i=1}^{3}\vec{\sigma}_i\, \frac{\underset{\sim}{\tau}_i}{2} , \tag{3.37}$$

and the spin-flavour matrix element is easily evaluated using the wave functions given earlier with the result

$${}_{s-f}\langle p|\sum_{i=1}^{3}\underline{\sigma}_i\, \frac{\underset{\sim}{\tau}_i}{2}|n\rangle_{s-f} = \frac{5}{3}\,\langle p|\underline{\sigma}\,\frac{\underset{\sim}{\tau}}{2}|n\rangle , \tag{3.38}$$

and hence $g_A^{N/R} = 5/3$. The bag model reduces this to 1.09 which is in much better agreement with the experimental result $g_A/g_V = 1.24$.

3.4. Centre of Mass Corrections

As it is usually presented, the bag model is effectively an independent particle shell model (IPSM) of hadron structure. In the nuclear context it is widely known that the IPSM is a terrible way to treat

^3He. For example, if one uses harmonic oscillator wave functions the removal of the spurious motion of the centre of mass reduces the value of the squared charge radius by a factor of 2/3. Thus one might expect that c.m. corrections should be extremely important for the bag model.

The procedures for removing spurious kinetic energy associated with the motion of the c.m. were discussed in Section 2.2.4. Here we are concerned with the effect on observables associated with the bag. In Section 3.4.1 we shall briefly describe what seems to be the most reasonable correction procedure, while in 3.4.2 we discuss the ambiguities which do not arise in the nuclear case.

3.4.1. *Centre of mass corrections in the independent particle approximation*

The recent discussions of c.m. corrections to observables began with the work of Donoghue and Johnson (DJ 80). These authors attempted to calculate the pion decay constant (f) in the bag model. Unfortunately their discussion contained an error which was recently corrected by Wong (Won 81). Finally, Carlson and Chachkhunashvili (CC 81) followed the same approach as Wong in order to derive corrections for hadronic properties—charge radii, magnetic moments and so on.

The technique used by Wong to remove the spurious c.m. motion is known as the Peierls-Yoccoz projection in nuclear physics (PY 57). The assumption is that the independent particle model wave function can be written as a superposition of *momentum eigenstates*—whose internal structure describes the true hadron. Suppose we have a bag fixed at the position $\underline{R}$, which we denote by $|B(\underline{R})\rangle$, then we have

$$|B(\underline{R})\rangle = \int \frac{d\underline{p}}{W(\underline{p})}\, e^{i\underline{p}\cdot\underline{R}}\, \phi(\underline{p})\, |b,\underline{p}\,\rangle, \qquad (3.39)$$

where $|b,\underline{p}\rangle$ is the momentum eigenstate representing particle b. This will be normalised as usual,

$$\langle b,\underline{p}'|b,\underline{p}\rangle = (2\pi)^3\ \delta(\underline{p}-\underline{p}')W(\underline{p})\ , \tag{3.40}$$

$$\begin{aligned} W(\underline{p}) &= 2\omega_p \quad \text{meson}\ , \\ &= (m_N{}^2+\underline{p}^2)^{1/2}/m_N \quad \text{baryon}\ . \end{aligned} \tag{3.41}$$

Finally $\phi(\underline{p})$ is the wave packet describing the momentum distribution in the bag. It can be obtained simply by inverting Eq. (3.39) to obtain

$$|b,\underline{p}\rangle = (2\pi)^{-3}\,\frac{W(\underline{p})}{\phi(\underline{p})}\int d\underline{R}\ e^{-i\underline{p}\cdot\underline{R}}|B(\underline{R})\rangle\ , \tag{3.42}$$

and substituting into Eq. (3.40) with the result

$$\phi^2(\underline{p}) = \frac{W(p)}{(2\pi)^6}\int d\underline{r}\ e^{-i\underline{p}\cdot\underline{r}}\langle B(-\underline{r}/2)|B(\underline{r}/2)\rangle\ . \tag{3.43}$$

Equation (3.43) will, of course, only receive contributions when r is less than twice the bag radius.

Having constructed an eigenstate of momentum we can now calculate any matrix element required. For example, the electric and magnetic form-factors of the nucleon are given by the matrix elements of j_0 and $\vec{j}$ respectively, in the Breit frame (CC 81, Bet 82),

$$G_E(Q^2) = \langle b,\underline{Q}/2|j^o(o)|b,-\underline{Q}/2\rangle\ , \tag{3.44}$$

and

$$G_M(Q^2)\chi^+_{\lambda'}\,\frac{i\underline{\sigma}\times\underline{Q}}{m}\,\chi_\lambda = \langle b_{\lambda'},\underline{Q}/2|\vec{j}(o)|b_\lambda,-\underline{Q}/2\rangle\ . \tag{3.45}$$

(In the second case we have explicitly shown spin labels, λ, for the hadronic state.) Carlson and Chachkhunashvili (CC 81) explicitly calculated the correction to the naive bag model predictions for the charge radius, magnetic moment and axial charge using this approach. For the r.m.s. charge radius they found about 20% reduction — in rough agreement with the factor 2/3 (for r^2) of the non-relativistic harmonic oscillator. In the case of the magnetic moment there was a 15%

reduction. The axial charge, g_A, increased by about 20%.

An important difficulty with the Peierls-Yoccoz procedure is that there is *no guarantee that the internal state of the momentum eigenstate*, $|b,\underline{p}\rangle$, will be independent of $\underline{p}$. Indeed the two complications of the bag model, namely its sharp boundary and the fact that its wave functions are highly relativistic, make it less likely that this technique will be reliable for the bag. One practical indication of this, suggested by Carlson and Chachkhunashvili, is to compute the correction for slightly altered wave functions. For example one might use the approximate Gaussian wave function of Duck (Duc 78),

$$\psi(\underline{r}) = \left[R_0^3\ \pi^{3/2}\left(1+\frac{3}{2}\ \beta^2\right)\right]^{-1/2}\ e^{-r^2/2R_0^2}\begin{pmatrix}1\\ i\beta\,\dfrac{\underline{\underline{\sigma\cdot r}}}{R_0}\end{pmatrix}\chi\ . \tag{3.46}$$

Whereas the results for the r.m.s. charge radius and g_A were not altered significantly by using Eq. (3.46) instead of Eq. (2.33), the magnetic moment increased by 8% for the former, in comparison with a 15% decrease noted above. Clearly the correction for the magnetic moment at least is untrustworthy. The ultimate c.m. correction which allows one to correct any spurious momentum dependence was developed by Peierls and Thouless (PT 62). This has never been applied to the bag model, mainly because of the complications introduced by relativity (Won 81).

3.4.2. *Ambiguities associated with the centre of mass correction*

In the previous Section the discussion was based on the nuclear physics analogy to the bag. However, as we discussed in Section 2.3 the bag itself *might* be expected to have some reality. Indeed, as discussed by Bardeen *et al.* (Bar+ 75) there is some momentum associated with the soliton bag. Thus, even though the MIT cavity does not carry momentum, in a better dynamical model one could conceive of the bag playing an important dynamical role.

With this in mind, Duck constructed a pion wave function in which the momentum of the bag balanced that of the two quarks (Duc 76). In that way the quarks were allowed to move independently of each other inside the hadron. While it was still necessary to construct momentum eigenstates, there was no centre of mass correction in that approach.

An excellent illustration of the dilemma to be faced if the bag does not carry momentum has been raised by Betz (Bet 82). Consider the physically unreasonable case of a single quark confined in the bag. In the IPSM approach all of its motion would be spurious c.m. motion which should be removed by the Peierls-Yoccoz procedure. On the other hand, if the soliton ideas are a reasonable representation of the physics the single quark could dig a hole in the vacuum, and there should be a form-factor associated with the internal structure of the system. We shall mention this again in connection with the cloudy bag model form-factor for pion-hadron coupling in Sections 5 and 6.

One other important aspect of this problem concerns the *n.l.b.e.* As we have discussed, the term $-Z_0/R$ is now thought to arise mainly as a centre of mass correction. Including this in the stability calculation [$\partial M/\partial R = 0$, see Eq. (2.98)] produces a bag radius that is smaller than that which would be obtained by first setting $\partial E/\partial R$ to zero and then correcting for c.m. motion. For the nucleon it is readily seen that this gives about a 10% reduction (for $Z_0 = 0.75$) in R.

In the absence of a truly covariant bag model it is not at all clear which of these choices of bag radii is most appropriate for computing hadronic properties. However, since both the r.m.s. radius and the magnetic moment are proportional to R, the answers depend crucially on the choice which is made. The parallel with nuclear physics is of no help because there is no analogue to the non-linear boundary condition. As a practical matter our choice has been to use the smaller radius but then omit further c.m. corrections. But to be honest any of the four possible options is equally acceptable and one has to accept an uncertainty of at least ±10% on bag model predictions of r.m.s. radii and magnetic moments. To end on a note of balance we might point out that this is still considerably better than the uncertainties associated with relativistic corrections in the non-relativistic quark models (see Section 2.4).

Table 3.1

Magnetic moments of the nucleon octet in units of the proton magnetic moment — from (DeG+ 75).

	MIT bag	Experiment	Naive SU(3)
μ_n	-2/3	-0.685	-2/3
μ_Λ	-0.26	-0.219	-1/3
μ_{Σ^-}	-0.36	-0.51	-1/3
μ_{Σ^+}	+0.97	+0.84	+1.0
μ_{Ξ^0}	-0.56	-0.45	-2/3
μ_{Ξ^-}	-0.23	-0.27	-1/3

Figure Caption

Fig. 3.1. Matter density in the $1s_{1/2}$ orbit for the MIT bag model.

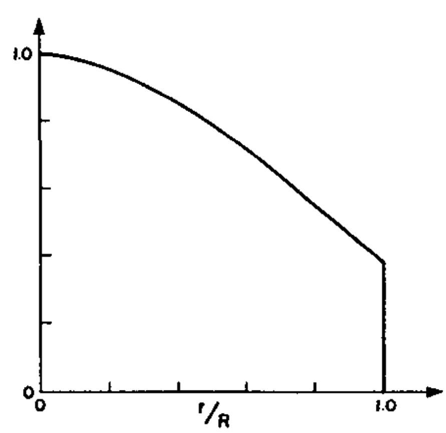

Fig. 3.1

4. CHIRAL SYMMETRY

In this section we first present the Lagrangian formulation of the MIT bag model. One of the most attractive features of this model as a basis for a pedagogical discussion is that it can be summarised in an extremely simple Lagrangian density. Using this we are able to formally derive conserved electromagnetic and isospin currents. However, in Section 4.3 we show that the axial current associated with the bag is not conserved. This is something of a disaster in view of the experimental successes of PCAC. Indeed, chiral SU(2) × SU(2) is known to be one of the best symmetries of the strong interaction (Pag 75). The classical representation of SU(2) × SU(2) is the so-called σ-model which we describe in Section 4.4. This discussion should also provide some background from which the later development of the cloudy bag model can be better appreciated.

4.1. Lagrangian Formulation of the MIT Bag Model

It is extremely convenient to have a concise mathematical summary of the MIT bag model as a Lagrangian density. In the limit of massless quarks, which we have seen to be a good starting point for dealing with non-strange hadrons, the following very simple expression gives the essential content for the fermions (CT 75, DeT 80, Jaf 79)

$$\mathcal{L}(x) = \left[\frac{i}{2}\,\bar{q}(x)\,\overleftrightarrow{\not\partial}\,q(x) - B\right]\theta(R-r) - \frac{1}{2}\,\bar{q}(x)\,q(x)\,\delta(r-R)\ . \qquad (4.1)$$

For pedagogical reasons we have specialised to the case of a static spherical bag of radius R. Of course the whole problem is usually formulated in a covariant fashion by replacing θ(R-r) by θ_V, which is one inside the bag and zero outside, and δ(r-R) by a general surface δ-function, Δ_S. As usual B denotes the phenomenological energy density of the bag. We also have

$$\overleftrightarrow{\not\partial} = \gamma^\mu(\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu) \ , \tag{4.2}$$

where the arrow indicates the direction in which the derivative acts. Lastly q(x) is the Dirac spinor describing the quarks. It actually has four Dirac components for each of two flavours (u and d) and three colours. (The extension to include strangeness, charm and so on requires no essential change, but one must then introduce a mass matrix.)

The last term in Eq. (4.1) may seem a little strange until we recall that the linear boundary condition, which ensured no current flow through the surface of the bag, amounted to the condition that $\bar{q}q$ should be zero on the surface [Eq. (2.41)]. This term is a Lagrange multiplier guaranteeing that $\bar{q}q$ is zero at the bag surface.

As usual the field equations are obtained by demanding that

$$S = \int d^4x \mathcal{L}(x) \ , \tag{4.3}$$

should be stationary under arbitrary changes in the fields

$$\begin{aligned} q_a &\to q_a + \delta q_a \ , \\ \bar{q}_a &\to \bar{q}_a + \delta\bar{q}_a \ , \end{aligned} \tag{4.4}$$

and in this case under changes in bag size (without change of shape). In the static spherical case this means

$$R \to R + \varepsilon \ . \tag{4.5}$$

In the general case such a variation leads to the following changes in θ_V and Δ_S

$$\delta\theta_V = \varepsilon \ \Delta_S \ , \tag{4.6}$$

$$\delta\Delta_S = -\varepsilon \ n\cdot\partial \ \Delta_S \ , \tag{4.7}$$

where n is the unit normal outward from the bag surface ($n^\mu = (0,\hat{r})$ for a static spherical bag). The coefficients of δq, $\delta q\Delta_S$ and ε in the expression for δS give the three bag model equations which were discussed at such length in Section 2, respectively

$$i\,\not\partial\, q(x) = 0, r \leqslant R\ , \tag{4.8}$$

$$i\ \gamma\cdot n q(x) = q(x), r = R\ , \tag{4.9}$$

$$B = -\frac{1}{2}\, n\cdot\partial[\overline{q}(x)\ q(x)], r = R\ . \tag{4.10}$$

4.2. Conserved Currents in Lagrangian Field Theory

In the previous section we referred to the mathematical convenience of a Lagrangian formulation. A prime example of this convenience is Noether's theorem which states that an invariance of the Lagrangian density is associated with a conserved quantity. Consider, for example, the Lagrangian density

$$\mathcal{L} = \mathcal{L}(\phi_i,\ \partial^\mu\phi_i)\ , \tag{4.11}$$

for which the equations of motion are determined by Hamilton's principle

$$\delta S = \delta\int d^4x\, \mathcal{L} = 0\ , \tag{4.12}$$

for arbitrary variations of the fields $\{\phi_i\}$, which vanish on the boundary (usually at infinity).

Suppose that we make a variation in these fields by an amount

$$\delta\ \phi_i(x) = f_i(\phi_j(x))\varepsilon\ , \tag{4.13}$$

where f_i is an arbitrary function of the fields $\{\phi_j\}$ at x, and ε is an infinitesimal constant. If $\mathcal{L}$ is invariant under the transformation (4.13), we have

$$\delta\mathcal{L} = \left[\frac{\partial\mathcal{L}}{\partial\phi_i}\ f_i + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\ \partial_\mu f_i\right]\varepsilon = 0\ , \tag{4.14}$$

where there is an implicit summation over repeated indices. If ε is no longer constant [$\varepsilon = \varepsilon(x)$], $\delta\mathcal{L}$ has an extra, non-vanishing term,

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\ f_i\ \partial_\mu\varepsilon(x)\ . \tag{4.15}$$

However, from Eq. (4.12) the integral of $\delta\mathcal{L}$ still vanishes, and integrating by parts we find

$$\int d^4x \; \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} f_i \right] \varepsilon(x) = 0 \; . \tag{4.16}$$

As this is true for arbitrary $\varepsilon(x)$, clearly we have constructed a *conserved current*. That is, if we define the current $j^\mu(x)$ as,

$$j^\mu(x) = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} f_i \; , \tag{4.17}$$

then it is locally conserved

$$\partial_\mu j^\mu(x) = 0 \; . \tag{4.18}$$

Finally we note that if $\mathcal{L}$ has two pieces, as is often the case in examples of physical interest, so that only $\mathcal{L}_o$ is invariant under the transformation while $\mathcal{L}_b$ is not, that is

$$\mathcal{L} = \mathcal{L}_o + \mathcal{L}_b \; , \tag{4.19}$$

then while j^μ is no longer conserved its divergence is easily written down,

$$\partial_\mu j^\mu = \frac{\partial \mathcal{L}_b}{\partial \phi_j} f_j \; . \tag{4.20}$$

4.2.1. *The usual charge current*

As the simplest possible example of a conserved current in the bag model, consider the simple gauge transformation:

$$q(x) \to q(x) + i\,\varepsilon\, q(x) \; , \tag{4.21}$$

$$(q^+(x) \to q^+ - i\,\varepsilon\, q^+)\gamma^o \; ,$$

$$\overline{q}(x) = \overline{q}(x) - i\,\varepsilon\,\overline{q}(x) \; . \tag{4.22}$$

Clearly $\mathcal{L}(x)$ in Eq. (4.1) is invariant under this transformation because it contains only the combinations $\overline{q}q(\to \overline{q}q - i\,\varepsilon\,\overline{q}q + \overline{q}\, i\,\varepsilon\, q = \overline{q}q)$. Therefore there is a conserved current which is easily found from Eq. (4.17)

$$j^\mu = \frac{i}{2}\,\overline{q}(x)\gamma^\mu(i\; q(x))\theta_V - \frac{i}{2}(-i\overline{q}(x))\gamma^\mu q(x)\theta_V \; , \tag{4.23}$$

or up to a minus sign

$$j^\mu = \overline{q}(x)\gamma^\mu q(x)\theta_V \; . \tag{4.24}$$

This has already been used in calculating the charge distribution (j^0) and magnetic moment ($\vec{j}$) of a bag model hadron in Section 3.

4.2.2. *Isospin conservation—invariance under SU(2)*

Now let us make an arbitrary, infinitesimal rotation in isospin,

$$q \to q + i\,\frac{\underset{\sim}{\tau}\cdot\underset{\sim}{\varepsilon}}{2}\,q\ ,$$

$$\bar{q} \to \bar{q} - i\,\bar{q}\,\frac{\underset{\sim}{\tau}\cdot\underset{\sim}{\varepsilon}}{2}\ , \tag{4.25}$$

with $\underset{\sim}{\varepsilon}$ constant. Once again $\mathcal{L}(x)$ is invariant, and hence $\underset{\sim}{I}^{\mu}$, given by

$$\underset{\sim}{I}^{\mu}(x) = \bar{q}(x)\gamma^{\mu}(\underset{\sim}{\tau}/2)\ q(x) \tag{4.26}$$

is a conserved current. Of course, the total isospin of the bag ($\underset{\sim}{t}$) is the integral of the isospin density,

$$\underset{\sim}{t} = \int d^3x\ \underset{\sim}{I}^{0}(x)\ , \tag{4.27}$$

and because $\partial_{\mu}\underset{\sim}{I}^{\mu}$ is zero, it is a constant of the motion.

4.3. The Axial Current

At last we have sufficient background material to begin consideration of the most recent developments in the bag model which are of direct relevance in nuclear physics. The natural starting point for this discussion is the axial current in the MIT bag model. As we shall see, unlike the charge and isospin currents which were decently conserved, the axial current is far from being conserved. Moreover this problem seems to be inescapably linked with the concept of confinement. For this reason we believe that the ideas presented here have a far more general validity than the MIT model on which the discussion is based.

4.3.1. *Non-conservation of the axial current*

Suppose that instead of just rotating in isospin-space, as in Eq. (4.25), we also operate with γ_5, thereby introducing a dependence on the quark's helicity

$$q \to q - i\,\frac{\underset{\sim}{\tau}\cdot\underset{\sim}{\varepsilon}}{2}\,\gamma_5\,q\ , \tag{4.28a}$$

$$q^+ \to q^+ + i\,q^+\,\gamma_5\,\frac{\underset{\sim}{\tau}\cdot\underset{\sim}{\varepsilon}}{2}\ ,$$

and therefore

$$\overline{q} \to \overline{q} - i\,\overline{q}\,\gamma_5\,\frac{\underset{\sim}{\tau}\cdot\underset{\sim}{\varepsilon}}{2}\ . \tag{4.28b}$$

Under this transformation we find

$$\mathcal{L} \to \mathcal{L} + \frac{1}{2}\,\overline{q}[\gamma_5\gamma^\mu + \gamma^\mu\gamma_5]\overleftrightarrow{\partial}_\mu\,\frac{\underset{\sim}{\tau}\cdot\underset{\sim}{\varepsilon}}{2}\,q\,\theta_V + \frac{i}{2}\,\overline{q}(x)\underset{\sim}{\tau}\cdot\underset{\sim}{\varepsilon}\,\gamma_5\,q(x)\Delta_S\ , \tag{4.29}$$

but whereas the second term vanishes because $\{\gamma^\mu,\gamma_5\} = 0$, the last is definitely non-zero. The jargon for this is that the surface term "$-1/2\ \overline{q}q\Delta_S$" is *"chirally odd"*. Figure 4.1 illustrates in a very simple way what this lack of invariance means physically. Confinement implies that any quark impinging on the bag surface must be reflected. However, there is no spin-flip associated with the reflection, and hence the chirality, or handedness, of the quark is changed. Formally this is known as a violation of chiral symmetry.

Because of the lack of invariance of the Lagrangian density under the transformation (4.28) we do not have a conserved current. In fact the axial current associated with Eq. (4.28) is

$$\underset{\sim}{A}^\mu(x) = \overline{q}(x)\gamma^\mu\gamma_5\,\underset{\sim}{\tau}/2\,q(x)\ \theta(R-r)\ , \tag{4.30}$$

and using Eq. (4.20) we find easily that its divergence is

$$\partial_\mu\,A^\mu(x) = -\frac{i}{2}\,\overline{q}(x)\gamma_5\,\underset{\sim}{\tau}\,q(x)\ \delta(r-R)\ . \tag{4.31}$$

This emphasizes once more that the essential problem is the confining wall at r = R. It also serves to remind us of Bogolioubov's relativistic potential model without the phenomenological energy density B. In that case [see Eq. (2.53) and (2.54)] the divergence of the energy momentum tensor was proportional to a surface δ-function times the

Dirac pressure exerted by the quarks. Indeed this was the observation that necessitated the introduction of B. In the same way we expect that something new will be required here. For guidance we recall the conventional description of the hadronic weak current.

4.3.2. *PCAC*

In Section 3.3 we reviewed the conventional theoretical description of the weak interaction. As an example we considered neutron β-decay which involves rather low momentum transfer. Accordingly Eq. (3.23) did not contain all the pieces of the vector and axial-vector currents. The most general expression for the hadronic axial current is

$$\langle p|A^\mu|n\rangle \sim \bar{u}_p[\gamma^\mu\gamma_5\, g_A(k^2) + k^\mu\gamma_5\, g_P(k^2)]u_n \ , \tag{4.32}$$

where u_n and u_p are Dirac spinors for the nucleons and the second term in brackets is the induced pseudoscalar term. If the momentum transfer, k^μ, is spacelike and small, we find the non-relativistic limits

$$\gamma^\mu\gamma_5 \rightarrow \underline{\sigma} \ ; \quad k^\mu\gamma_5 \rightarrow -\frac{\underline{\sigma}\cdot\underline{k}}{2m_N}\,\underline{k} \ , \tag{4.33}$$

and hence

$$\langle p|\underline{A}|n\rangle = \chi_p^+\left(g_A(k^2)\underline{\sigma} - \frac{g_P(k^2)}{2M}\,\underline{\sigma}\cdot\underline{k}\ \underline{k}\right)\chi_n \ , \tag{4.34}$$

where χ_n and χ_p are Pauli spinors.

Now the problem of concern to us in Section 4.3.1 was the non-conservation of the axial current in the bag model — or more specifically the fact that $\partial_\mu A^\mu$ was non-zero. In the present case $\partial_\mu A^\mu$ becomes simply $\underline{k}\cdot\underline{A}$, and we see from Eq. (4.34) that $\underline{k}\cdot\underline{A}$ is zero only if

$$\left(g_A(k^2) - \frac{g_P(k^2)k^2}{2M}\right)\underline{\sigma}\cdot\underline{k} = 0 \ , \tag{4.35}$$

which implies that g_P is related to g_A by

$$g_P(k^2) = \frac{2M\ g_A(k^2)}{\underline{k}^2} \ . \tag{4.36}$$

Since g_A is simply a constant as $\underline{k}^2 \to 0$ we see that *if the axial current is to be conserved the induced pseudoscalar term must have a pole corresponding to the propagation of a massless exchanged particle.* Furthermore, the quantum numbers of the exchanged particle are those of the pion.

If we then accept that the axial current may not be exactly conserved it seems very natural to replace Eq. (4.36) by

$$g_P(k^2) = \frac{2M\ g_A(k^2)}{\underline{k}^2 + m_\pi^2} . \tag{4.37}$$

Since m_π is unusually small on the scale of hadronic masses the axial current is said to be almost, or partially, conserved. In fact the correct statement of the PCAC (Partially Conserved Axial Current) hypothesis (Col 68) is that the extrapolation from zero pion mass to m_π should be smooth.

In the limited space available here we can not do justice to the depth of physics investigated using the PCAC hypothesis. At best we can refer to some excellent text-book presentations (GL 60, AD 68, Lee 68, Col 68, Zum 68, ER 72, Bro 79). In addition, we can get some physical insight into the structure of $\underset{\sim}{A}^\mu$ by referring to Fig. 4.2. As we see there are two essential contributions to it. The first is a direct term which reduces to $g_A\underline{\sigma}$, and is included in the bag model. Secondly, there is the possibility that the nucleon emits a pion which then decays via the axial current with amplitude $\sqrt{2}f\underline{k}$, where f = 93 MeV is the pion decay constant. If as suggested by Eq. (4.37) we equate these two terms when m_π^2 is zero we obtain

$$(g_A\ \underline{\sigma})\cdot\underline{k} = \sqrt{2}\left(\frac{f_{NN\pi}}{m_\pi}\right)\underline{\sigma}\cdot\underline{k}\ \frac{1}{\underline{k}^2}\left(\sqrt{2}f\underline{k}\right)\cdot\underline{k} , \tag{4.38}$$

and hence

$$\frac{g_A}{2f} = \frac{f_{NN\pi}}{m_\pi} . \tag{4.39}$$

Equation (4.39) provides a remarkable connection between weak and strong coupling constants and is known as the Goldberger-Trieman relationship. In conclusion let us re-emphasize that the massless pion pole term is essential if one wants $\partial_\mu A^\mu = 0$. In the real world where m_π is non-zero we have instead the relationship

$$\partial_\mu A^\mu = f\, m_\pi^2\, \underset{\sim}{\phi} , \tag{4.40}$$

where $\underset{\sim}{\phi}$ is the pion field (Col 68)

4.4. The σ-Model and Spontaneous Symmetry Breaking

4.4.1. *General discussion of SU(2) × SU(2)*

In the preceding sections we have discussed separately the vector and axial-vector currents in the bag model. However, the quantities of more general interest in particle physics are the combinations $(V \pm A)$. In the case of massless fermion fields these are the left- and right-handed currents. The original significance of these combinations lay in the current algebra hypothesis of Feynman and Gell-Mann (Gel 64, Fey+ 64, AD 68). This significance has only grown with the development of QCD over the past decade.

In particular, the underlying Lagrangian density for QCD contains a kinetic energy term for free massless quarks. As we have seen in Sections 4.2.2 and 4.3.1 such a Lagrangian density leads to conserved vector and axial-vector currents,

$$V_\mu^i = \bar{q}(x)\, \tau^i\, \gamma_\mu\, q(x) , \tag{4.41}$$

and

$$A_\mu^i = \bar{q}(x)\, \tau^i\, \gamma_\mu \gamma_5\, q(x) . \tag{4.42}$$

The combinations $V \pm A$ then describe the isospin structure of left- and right-handed quarks respectively,

$$L_\mu^i = \overline{q}(x)\ \tau^i\ \gamma_\mu(1-\gamma_5)q(x)\ ,$$

$$R_\mu^i = \overline{q}(x)\ \tau^i\ \gamma_\mu(1+\gamma_5)q(x)\ . \qquad (4.43)$$

Because of the commutation relations amongst the V and A currents, L and R form independent algebras under equal-time commutation (AD 68, Sak 69). That is, we have two independent representations of SU(2), one for left-handed particles and the other for right-handed particles. The invariance of the theory under separate transformations for left and right-handed particles is referred to as chiral SU(2) × SU(2) symmetry—or $SU(2)_L \times SU(2)_R$, in an obvious notation.

To restate this simply, the theory is chirally symmetric if no piece of the Lagrangian density mixes left- and right-handed particles. Figure 4.1 illustrated exactly why SU(2) × SU(2) is violated by the MIT bag model—or indeed any model where quarks are reflected by a boundary. Such a reflection changes helicity and thus mixes the left- and right-handed parts of the theory.

Thus the first argument that something is missing from the usual bag model is that it does not have a symmetry which is present in what is generally believed to be the correct theory of strong interactions—namely QCD. The second indication is rather more pragmatic. That is, there is an extremely successful phenomenology which has been built on the idea that chiral SU(2) × SU(2) is a good symmetry of strong interactions. An excellent discussion of the evidence can be found in the review by Pagels (Pag 75). Based on the comparison between theory and experiment for the Goldberger-Treiman relationship [Eq. (4.39)], the πN Σ-commutator and so on, Pagels concludes that "SU(2) × SU(2) is a good hadron symmetry to within 7%. *This makes chiral SU(2) × SU(2)* *the most accurate hadron symmetry after isotopic invariance*." (Pag 75, p.242).

We are therefore faced with a problem very similar to that encountered by Gell-Mann and Lévy in 1960. They had to reconcile the fact that the axial current for the nucleon was partially conserved, with the fact that the nucleon has a large mass. That is, the Lagrangian density for a free nucleon is

$$\mathcal{L}(x) = i\bar{\psi}\, \not{\partial}\, \psi - m_N\, \bar{\psi}\, \psi, \qquad (4.44)$$

where the mass term [as we saw in Eq. (4.29)] is "chirally odd". Their solution to the problem was the so-called σ-model, to which we turn in the following section. Although it is a very simple model it is of more than academic interest. It has been used as a method of incorporating the constraints of chiral symmetry in many applications in conventional nuclear theory, for example

a) exchange current corrections—e.g. for the axial charge density in nuclei (Gui+ 78, Ose 80);

b) the two-pion exchange N-N force (Bro 78, Bro 79);

c) many-body forces (MR 79);

d) exotic states of matter, such as Lee-Wick matter and pion condensation (LW 74, Cam 78, Bay 78, Mey 81).

For the present we simply observe that the cloudy bag model which will be described in Sections 5 and 6 is the natural generalisation of the σ-model to the case where the nucleon has structure. It invites application in each of the areas (a)-(d).

4.4.2. *The σ-model*

As we have remarked many times the essential problem in constructing a chiral symmetric theory containing fermions is the mass term proportional to $\bar{\psi}\psi$. The simplest way to avoid this problem is to introduce new fields $(\sigma, \underset{\sim}{\pi})$ — an isoscalar scalar field and an isovector pseudoscalar field — in addition to the nucleon, ψ. The generalisation of the infinitesimal transformation (4.28) [replace q(x) by $\psi(x)$] is

$$\psi \to e^{-i\underset{\sim}{\tau}\cdot\underset{\sim}{\alpha}\gamma_5/2}\,\psi\ , \tag{4.45a}$$

$$\bar{\psi} \to \bar{\psi}\, e^{-i\underset{\sim}{\tau}\cdot\underset{\sim}{\alpha}\gamma_5/2}\ , \tag{4.45b}$$

Then the idea is to replace $m_N\,\bar{\psi}\psi$ in Eq. (4.44) by $g\,\bar{\psi}(\sigma+i\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\gamma_5)\psi$, where σ and $\underset{\sim}{\pi}$ are defined to transform in exactly the right way to cancel the transformation (4.45). In particular, we demand that

$$(\sigma+i\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\gamma_5) \to e^{+i\underset{\sim}{\tau}\cdot\underset{\sim}{\alpha}\gamma_5/2}\,(\sigma+i\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\gamma_5)e^{+i\underset{\sim}{\tau}\cdot\underset{\sim}{\alpha}\gamma_5/2}\ . \tag{4.46}$$

If we now consider the case where $\underset{\sim}{\alpha}$ is infinitesimal, Eq. (4.46) implies that

$$\sigma \to \sigma - \underset{\sim}{\alpha}\cdot\underset{\sim}{\pi}\ , \tag{4.47a}$$

$$\underset{\sim}{\pi} \to \underset{\sim}{\pi} + \sigma\,\underset{\sim}{\alpha}\ . \tag{4.47b}$$

and of course

$$\psi \to \psi - i\,\frac{\underset{\sim}{\tau}\cdot\underset{\sim}{\alpha}}{2}\gamma_5\psi\ ;\quad \bar{\psi} \to \bar{\psi} - i\bar{\psi}\gamma_5\,\frac{\underset{\sim}{\tau}\cdot\underset{\sim}{\alpha}}{2} \tag{4.48}$$

It is a simple exercise to show that Eq. (4.47) implies that $(\sigma^2+\underset{\sim}{\pi}^2)$ is invariant under this chiral transformation. That is we are merely making a rotation in a four-dimensional (4D) space.

We mentioned above that under the familar SU(2) of isospin, σ is a scalar and $\underset{\sim}{\pi}$ a vector. Under an infinitesimal rotation in isospin-space

$$\sigma \to \sigma\ ;\quad \underset{\sim}{\pi} \to \underset{\sim}{\pi} - \underset{\sim}{\beta}\times\underset{\sim}{\pi}\ , \tag{4.49}$$

and [recall Eq. (4.25)]

$$\psi \to \psi + i\,\frac{\underset{\sim}{\tau}\cdot\underset{\sim}{\beta}}{2}\,\psi\ ;\quad \bar{\psi} \to \bar{\psi} - i\bar{\psi}\,\frac{\underset{\sim}{\tau}\cdot\underset{\sim}{\beta}}{2}\ . \tag{4.50}$$

Equation (4.49) also leaves $(\sigma^2+\underset{\sim}{\pi}^2)$ constant. Thus the most general transformation under SU(2) × SU(2) involves two parameters ($\underset{\sim}{\alpha},\underset{\sim}{\beta}$) and amounts to nothing more than a rotation in 4D space. Indeed, as discussed in detail by Lee (68) SU(2) × SU(2) is isomorphic to the rotation group in four dimensions, R(4). The basis of the regular (adjoint) representation of R(4) (Car 66) is in fact $(\sigma,\underset{\sim}{\pi})$.*

The most general renormalisable Lagrangian density involving nucleon, σ and $\underset{\sim}{\pi}$ fields which is consistent with chiral symmetry is therefore

$$\mathcal{L}(x) = i\bar{\psi}\,\partial\!\!\!/\,\psi + g\bar{\psi}(\sigma+i\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\gamma_5)\psi + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\pi)^2$$
$$- \frac{\lambda^2}{4}\left((\sigma^2+\underset{\sim}{\pi}^2) - \nu^2\right)^2 . \qquad (4.51)$$

In case it is not clear, we stress that the σ and $\underset{\sim}{\pi}$ kinetic energy terms are invariant under SU(2) × SU(2) because we are only discussing *global transformations* — that is $\underset{\sim}{\alpha}$ and $\underset{\sim}{\beta}$ constants, not functions of x.

Let us consider the potential energy term in Eq. (4.51) in more detail

$$V(\sigma,\underset{\sim}{\pi}) = \frac{\lambda^2}{4}(\sigma^2+\underset{\sim}{\pi}^2)^2 - \frac{\lambda^2\nu^2}{2}(\sigma^2+\underset{\sim}{\pi}^2) + \frac{\lambda^2}{4}\nu^4 . \qquad (4.52)$$

(There is a change of sign because the Hamiltonian goes as $-g^{00}\mathcal{L}$.) If the system is ever to be stable we obviously need $\lambda^2 > 0$. Then there are two possibilities. First it is possible that $\nu^2 \leqslant 0$, in which case the coefficient of the σ^2 and $\underset{\sim}{\pi}^2$ terms is positive and therefore an acceptable mass term. The σ and $\underset{\sim}{\pi}$ fields have the same mass, $(-\lambda^2\nu^2)^{1/2}$, and the potential $V(\sigma,\underset{\sim}{\pi}=0)$ has only one minimum—

*The regular representation is 4D because only four of the six operators $\underset{\sim}{\tau}_i$ and $\gamma_5\underset{\sim}{\tau}_i$ are independent. As we discussed in Section 4.4.1 the combinations $(1\pm\gamma_5)\underset{\sim}{\tau}_i$ are the operators for left- and right-handed SU(2).

at $\sigma = 0$. One could then deal with fluctuations of the σ and $\boldsymbol{\pi}$ fields in the normal vacuum.

A second possibility, which is far more interesting, is the case $\nu^2 > 0$. In this case the potential has the "Mexican hat" shape shown in Fig. 4.3. Remarkably, the point $\sigma = 0$ is no longer stable and it would be meaningless to talk about quantum fluctuations about that point! Instead there is a minimum on the surface $(\sigma^2 + \boldsymbol{\pi}^2) = \nu^2$. Since a non-zero classical expectation value for $\boldsymbol{\pi}$ would violate parity, it is natural to think of expanding about either of the equivalent minima of $V(\sigma, \boldsymbol{\pi} = 0)$ at $\sigma = \pm\nu$, for example

$$\sigma \to \sigma + \nu \ , \quad \boldsymbol{\pi} \to \boldsymbol{\pi} \ . \tag{4.53}$$

Once this transformation is made the symmetry of the original Lagrangian density (4.51) is hidden.

However, Goldstone's Theorem (Gol 61, Gol+ 62, Ber 74, Pag 75, Lee 81) tells us that when a continuous symmetry is hidden, a Goldstone boson or massless excitation of the system appears. Mathematically we find upon substituting Eq. (4.53) into Eq. (4.51)

$$\mathcal{L}(x) = \bar{\psi}(i\not{\partial} + g\nu)\psi + g\bar{\psi}(\sigma + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}\gamma_5)\psi + \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}(2\lambda^2\ \nu^2)\sigma^2 +$$

$$\frac{1}{2}(\partial_\mu\boldsymbol{\pi})^2 - \nu\lambda^2\ \sigma(\sigma^2+\boldsymbol{\pi}^2) - \frac{\lambda^2}{4}(\sigma^2+\boldsymbol{\pi}^2)^2 \ , \tag{4.54}$$

and the explicit SU(2) $\times$ SU(2) symmetry has certainly been lost. The nucleon, for example, now has a mass term with

$$m_N = -g\ \nu \ , \tag{4.55}$$

which arises because the vacuum state is now complicated and the nucleon always meets resistance. Similarly the σ now has a mass corresponding to the second derivative of $V(\sigma, \boldsymbol{\pi} = 0)$, at $\sigma = \pm\nu$, in the σ-direction

$$m_\sigma^2 = 2\lambda^2\nu^2 \ . \tag{4.56}$$

Furthermore, as advertised there is no mass term for the pion which is now a massless Goldstone boson corresponding to massless excitations around the rim of the "hat". We also observe that there are now σ-π-π and σ-σ-σ interaction vertices with strength proportional to the expectation value of the original σ-field.

Let us recall that the whole purpose of this exercise was to produce a chiral symmetric theory with a massive nucleon. Although it may not be obvious, we have succeeded, and the whole key is the spontaneous breaking of chiral symmetry associated with Eq. (4.53). Actually a much more appropriate term would be hidden chiral symmetry because Eq. (4.54) is invariant under the chiral transformation

$$\begin{aligned} \sigma &\to \sigma - \underset{\sim}{\alpha}\cdot\underset{\sim}{\pi} \\ \underset{\sim}{\pi} &\to \underset{\sim}{\pi} + (\sigma+\nu)\underset{\sim}{\alpha} \ , \end{aligned} \tag{4.57}$$

and hence there is still a conserved axial current

$$\underset{\sim}{A}^\mu(x) = \bar{\psi}\gamma^\mu\gamma_5\underset{\sim}{\tau}/2\psi - \underset{\sim}{\pi}\partial^\mu\sigma + \sigma\partial^\mu\underset{\sim}{\pi} \ . \tag{4.58}$$

Finally, the conserved vector current associated with Eq. (4.54) is

$$\underset{\sim}{V}^\mu(x) = \bar{\psi}\gamma^\mu\underset{\sim}{\tau}/2\psi + \underset{\sim}{\pi}\times\partial^\mu\underset{\sim}{\pi} \ . \tag{4.59}$$

4.4.3. *PCAC in the σ-model*

Having obtained a chiral symmetric theory with a nucleon mass, all we need to do to make contact with the real world is to introduce a mass for the pion. This is done by explicitly breaking the chiral symmetry of the original Lagrangian density (4.51) by "tipping" the Mexican hat

$$\mathcal{L} \to \mathcal{L} + c\sigma \ . \tag{4.60}$$

In this case there is a preferred direction in $(\sigma,\underset{\sim}{\pi})$ space and the minimum about which we expand is σ_0, where

$$\sigma_o(\sigma_o{}^2 - \nu^2) = c/\lambda^2 \ . \tag{4.61}$$

If we now let

$$\sigma \to \sigma + \sigma_o \tag{4.62}$$

in Eq. (4.60) it is a straightforward exercise to show that the nucleon, σ and $\underset{\sim}{\pi}$ all get masses, with

$$m_N = -g\ \sigma_o \ , \tag{4.63}$$

$$m_\sigma^2 = \lambda^2(3\sigma_o{}^2 - \nu^2) \ , \tag{4.64}$$

and

$$m_\pi^2 = \lambda^2(\sigma_o{}^2 - \nu^2) \ . \tag{4.65}$$

Because we broke the chiral SU(2) symmetry with the "$-c\sigma$" term the axial current is no longer conserved. Instead, from Eq. (4.20) we find

$$\partial_\mu \underset{\sim}{A}^\mu = -c\ \underset{\sim}{\pi}^\mu \ . \tag{4.66}$$

This is exactly the form given in Section 4.3.2 provided we identify

$$c = -fm_\pi^2 \ . \tag{4.67}$$

Using Eq. (4.67), (4.61) and (4.65) we find that the minimum about which we have expanded, σ_o, is equal to the pion decay constant

$$\sigma_o = -f \ . \tag{4.68}$$

Hence Eq. (4.63) becomes

$$m_N = g\ f \ , \tag{4.69}$$

which we recognize as the Goldberger-Treiman relation [see Eq. (4.39)] *with $g_A = 1$.* This is a defect of the σ-model which is usually overcome in practice by introducing $g_A = 1.24$ as a fudge-factor whenever needed!

In summary, we emphasize that the σ-model was presented not as the best one can do in imposing chiral symmetry, but in order to motivate what follows. In order to appreciate what is really new and advantageous about the CBM we need to understand what has been done in the past. Nevertheless, the σ-model is a beautiful case study, presenting as it does simple examples of chiral symmetry, spontaneous symmetry breaking, PCAC and the Goldberger-Treiman relation. The serious student should follow up our brief presentation by reading the appropriate sections of Lee 68, Bro 79, IZ 80 and Lee 81.

Figure Captions

Fig. 4.1. Violation of chiral symmetry at the bag surface.

Fig. 4.2. The direct and pion pole contributions to the nucleon axial current.

Fig. 4.3. The potential energy density $V(\sigma,\pi)$ with $\mu^2 < 0$, $\nu^2 > 0$, and $c_\pi = 0$.

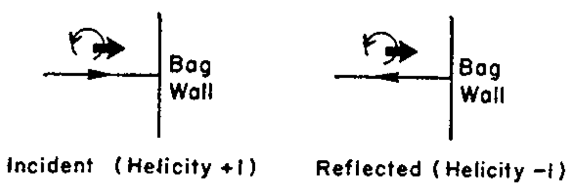

Fig. 4.1

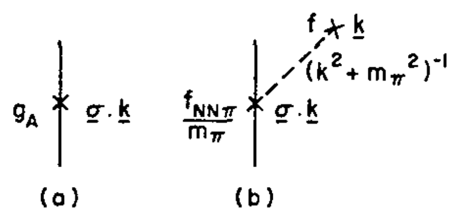

Fig. 4.2

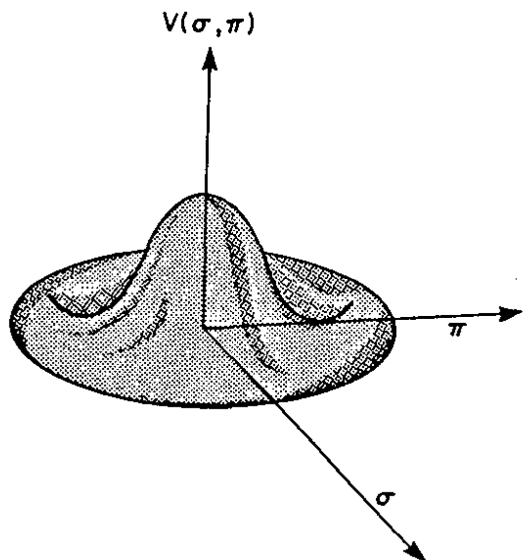

Fig. 4.3

5. BAG MODELS WITH CHIRAL SYMMETRY

In Sections 2 and 3 we took great care to explain the MIT bag model, its application to particle properties, and the attempts to derive it from a more fundamental theory. The concept of chiral symmetry was explained in Section 4. In particular we showed that the essential effect of confinement was to lead to the non-conservation of the axial current. We then examined the classical σ-model as an example of how chiral symmetry can be restored through the appearance of a Goldstone boson. The purpose of this section is to show how a number of groups have attempted to put these concepts together to create a hybrid model in which the experimental fact of PCAC is preserved. However, such a review would be incomplete without some discussion of the relationship of this phenomenology to QCD. It is the purpose of Section 5.1 to provide that background.

5.1. Motivation

Finding the solution of QCD, which is widely accepted as the correct theory of strong interactions, poses a very difficult problem (AL 73, MP 78). It is quite likely that some genuine physical insight will be required if we are ever to solve the QCD equations. Symmetry arguments may be of great importance in developing that insight. In the innocent days of 1968, when only three quark flavours were known, Gell-Mann, Oakes and Renner (GOR) proposed the following scheme (Gel+ 68). Beginning with three massless quarks, QCD (for the reasons reviewed in Section 4.4) would have an exact SU(3) $\times$ SU(3) symmetry. Because physical particles have definite parity the vacuum symmetry in this theory must be hidden — leading to an octet of massless Goldstone bosons (π, η, κ and $\bar{\kappa}$).

If the strange quark is then given a mass the symmetry group is broken to SU(2) × SU(2) — with only the pion still massless. Next one sets the masses of the u- and d-quarks to be non-zero ($m_u = m_d \neq 0$) leaving only SU(2) (isospin) and $m_\pi \neq 0$. Finally, in order to explain mass splittings in isospin multiplets one must set $m_u \neq m_d$ leaving U(1), or charge conservation as the only exact symmetry.

For the present we shall ignore chiral SU(3) × SU(3) because of the large mass of the kaon. (Nevertheless there may be a great deal to be learnt by extending the hybrid bag models to include strangeness (RT 82).) On the other hand, as we have stressed many times, SU(2) × SU(2) is found experimentally to be an excellent symmetry. It should therefore make a firm foundation for model building. As GOR observed on very general grounds the physical realization of chiral SU(2) × SU(2) must be the Goldstone mode. To see this, suppose

$$\partial_\mu \underset{\sim}{A}^\mu = 0 \ , \tag{5.1}$$

in all space. Therefore, if we integrate over all space

$$\int d^3x \ \partial_\mu \underset{\sim}{A}^\mu = 0 \ , \tag{5.2}$$

and use Gauss's theorem on the $\vec{\nabla} \cdot \vec{\underset{\sim}{A}}$ piece we find

$$\partial_o \ \underset{\sim}{Q}_5 = \partial_o \int d^3x \ \underset{\sim}{A}^o(\underline{x},t) = 0 \ . \tag{5.3}$$

Thus the axial charge is a constant of the motion, and therefore commutes with the Hamiltonian

$$[H, Q_5] = 0 \ . \tag{5.4}$$

If an eigenstate of H, namely $|N^+\rangle$, exists with mass m,

$$H|N^+\rangle = m|N^+\rangle \ , \tag{5.5}$$

then $|N^-\rangle$ defined as

$$|N^-\rangle = \underset{\sim}{Q}_5|N^+\rangle \ , \tag{5.6}$$

also has mass m, viz:

$$\underset{\sim}{Q}_5 \, H|N^+\rangle = H \, \underset{\sim}{Q}_5|N^+\rangle = H|N^-\rangle$$
$$= m|N^-\rangle \, . \qquad (5.7)$$

Since $|N^-\rangle$ necessarily has opposite parity from $|N^+\rangle$ there is an unobserved, opposite-parity partner for each hadron!

The only way around this theorem is the Goldstone representation of chiral symmetry in which $\underset{\sim}{Q}_5$ does not annihilate the vacuum (Pag 75, Gol+ 62), i.e.

$$\underset{\sim}{Q}_5 \, |0\rangle \neq 0 \, . \qquad (5.8)$$

In that case, rather than being a parity partner of $|N^+\rangle$, the state $|N^-\rangle$ contains an arbitrary number of massless, pseudoscalar, Goldstone bosons. (Recall Section 4.4.2 where we showed explicitly how such bosons can appear as a result of spontaneous symmetry breaking—SSB.) Thus on very general grounds *the pion must be present as a Goldstone boson in this ideal chiral-symmetric world (with $m_\pi = 0$).*

While the σ-model was pedagogically very useful for introducing the ideas of SSB and chiral symmetry, it is physically very unsatisfactory. The nucleon is point-like and there is no way to relate it to QCD. Thus it certainly does not help to resolve the problem of $\partial_\mu A^\mu \neq 0$ which we found in the bag model. We recall that the essential difficulty there was the confining surface of the bag, and this has led to speculation of a phase change at the bag surface. Briefly the idea is that chiral symmetry would be realized in the Wigner-Weyl mode inside the bag (massless quarks, no pions) and in the Goldstone mode outside (Cal+ 78, Cal+ 79, BR 79). In such a picture the pion field outside the bag could (but need not) play an essential role in the confinement process—even contributing significantly to the bag pressure.

Very recently Goldman and Haymaker have taken some steps which may provide the link between QCD and the appearance of the Goldstone mode (GH 81, HG 81). Their considerations were based upon an effective Lagrangian of the Nambu-Jona-Lasinio type

$$\mathcal{L}_{eff} = i\,\bar{q}\,\not{\partial}\,q - g\{(\bar{q}q)^2 + (i\bar{q}\,\gamma_5\underset{\sim}{\tau}q)^2\}\,. \tag{5.9}$$

Actually they used a rather more general form than this with the δ-function 4-quark interaction replaced by exchange of a massive vector particle, but the idea is the same. Moreover there have been indications that such an effective Lagrangian density could come out of QCD after transforming away the gluons (Cal+ 79). The properties of (5.9) have been well studied (NJ 61, GN 74, GH 81), indeed it provides the classic example of a dynamically broken symmetry. Beyond a certain critical value of the coupling constant, g_c, one finds that the quarks become massive and the pion appears as a massless, composite Goldstone boson. The breaking of chiral symmetry as a result of the dynamics of the system is (not surprisingly) referred to as dynamical symmetry breaking (DSB).

Put very briefly the essential idea of Goldman and Haymaker is the following. The one gluon exchange is very strongly attractive in the state with pion quantum numbers [see Eq. (2.83)]. It is quite conceivable that the one-gluon-exchange ladder graphs alone could bind a $q\bar{q}$ pair in that channel. Then the large distance, non-perturbative aspects of QCD responsible for confinement need not alter the properties of the pion very much. Chiral symmetry could be dynamically broken, with the appearance of a Goldstone pion, independently of the usual mechanism of confinement. Naturally this leads to a rather small pion, with a hydrogen-like relative $\bar{q}q$ wave function. At present the only experimental problem this presents would be the measured r.m.s. charge radius of 0.56 ± 0.04 fm (Dal+ 81). However, theoretical corrections to the charge distribution from processes like $\pi \to 3\pi$ have not been estimated.

Whatever the nature of the pion, there is strong theoretical justification for treating it as a Goldstone boson arising from some DSB mechanism. In addition, it is unique amongst hadrons in having a size (less than or equal to its r.m.s. charge radius) considerably less than its Compton wavelength. Thus in first approximation it should be reasonable to construct a theory in which chiral symmetry is retained in the Goldstone mode but the internal structure of the pion is neglected. This would be essentially a long wavelength approximation.

5.2. Chodos and Thorn

The lack of chiral symmetry in the MIT bag model was recognised immediately by the MIT group. One attempt was made to deal with this problem as early as 1975 by Chodos and Thorn (CT 75) — see also Inoue and Maskawa (IW 75). Their proposal was a simple generalisation of the σ-model which we described in Section 4. That is the surface term in the MIT Lagrangian density (4.1), $\bar{q}q\delta_s$, is replaced by the chiral invariant form $\bar{q}(\sigma+i\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\gamma_5)q\delta_s$. The new Lagrangian density involving the extra, elementary fields σ and $\underset{\sim}{\pi}$ is therefore

$$\mathcal{L}_{CT}(x) = (i\ \bar{q}\ \overleftrightarrow{\partial\!\!\!/}\ q - B)\theta_V - \frac{\lambda}{2}\bar{q}(\sigma+i\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\gamma_5)q\delta_s + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\underset{\sim}{\pi})^2\ , \qquad (5.10)$$

where λ is a Lagrange multiplier which turns out to be simply $(\sigma^2+\underset{\sim}{\pi}^2)^{-1/2}$. By construction Eq. (5.10) is invariant under the chiral transformations (4.47) and (4.48) ($\psi \to q$), and the conserved axial current analogous to Eq. (4.58) is

$$\underset{\sim}{A}^{\mu} = \frac{1}{2}\,\bar{q}\,\gamma^{\mu}\gamma_5\underset{\sim}{\tau}/2\;q\;\theta_V - \underset{\sim}{\pi}\,\partial^{\mu}\sigma + \sigma\partial^{\mu}\underset{\sim}{\pi}\;. \tag{5.11}$$

Having written down the classical field equations corresponding to (5.10) for the case of a static, spherical bag, namely

$$i\,\partial\!\!\!/\,q = 0\;,\quad r < R\;, \tag{5.12}$$

$$i\,\underline{\gamma}\cdot\hat{r}\;q = -\frac{1}{(\sigma^2+\underset{\sim}{\pi}^2)^{1/2}}\,(\sigma+i\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\gamma_5)q, r = R\;, \tag{5.13}$$

$$\nabla^2\sigma = \frac{1}{2}\,\frac{1}{(\sigma^2+\underset{\sim}{\pi}^2)^{1/2}}\,\bar{q}q\;\delta(r-R)\;, \tag{5.14}$$

$$\nabla^2\underset{\sim}{\pi} = \frac{1}{2}\,\frac{1}{(\sigma^2+\underset{\sim}{\pi}^2)^{1/2}}\;i\;\bar{q}\;\gamma_5\underset{\sim}{\tau}\;q\;\delta(r-R)\;, \tag{5.15}$$

Chodos and Thorn attempted to find an exact classical solution. The only case for which this was feasible was a highly idealised baryon called the "hedgehog". If we define a spin-flavour wave function ν as (u and d are up and down, and the arrows describe spin direction)

$$|\nu\rangle = (|u\!\downarrow\rangle - |d\!\uparrow\rangle)/\sqrt{2}\;, \tag{5.16}$$

that is a mixed spin-flavour singlet, then the hedgehog has the spin-flavour wave function

$$|h\rangle_{s-f} = |\nu\rangle_1\;|\nu\rangle_2\;|\nu\rangle_3\;. \tag{5.17}$$

Such an animal clearly has no place in the real world, as it is an eigenstate of neither isospin nor angular momentum. In fact, with three quarks in $1s_{1/2}$ orbitals it is a linear superposition of N and Δ states of all charges. However, the choice of h leads to a very simple form for the source term in Eq. (5.15). In fact one can easily show that $\bar{q}\;\gamma_5\underset{\sim}{\tau}\;q$, a vector in isospin space, always points in the radial direction $\hat{r}$! That is, for a quark in a $1s_{1/2}$ hedgehog orbit (q_h),

$$\bar{q}_h\;\underset{\sim}{\tau}\gamma_5\;q_h = -2ij_0j_1\;\nu^{+}\nu\;\hat{r}\;. \tag{5.18}$$

It is then obvious that the set of equations (5.12)-(5.15) allow a solution of the form

$$q(\underline{r}) = \begin{pmatrix} j_0(\omega r/R) \\ i\underline{\sigma}\cdot\hat{r} \quad j_1(\omega r/R) \end{pmatrix} \nu \, e^{-i(\omega/R)t} \tag{5.19}$$

$$\underset{\sim}{\pi}(\underline{r}) = \hat{r}\, g(r) \ , \tag{5.20}$$

$$\sigma(\underline{r}) = f(r) \ . \tag{5.21}$$

Although no explanation for the hedgehog was given by Chodos and Thorn, the form (5.20) is identical to the monopole solutions which were under investigation at about the same time. Solving explicitly for the pion field they found,

$$g(r) = -\beta\left(\theta(R-r)r + \theta(r-R)\,\frac{R^3}{r^2}\right) , \tag{5.22}$$

where β measures the strength of coupling at the bag surface. The quark frequency ω is obtained by solving a transcendental equation.

If it was not obvious from Eqs. (5.14) and (5.15), it is obvious from the explicit solution (5.22) that the $\underset{\sim}{\pi}$ field has a discontinuous derivative at the bag surface. (Although we do not show it there is a similar discontinuity in the derivative of $\sigma(r)$.) Such a discontinuity is actually essential if the axial current is to be conserved and simply serves to balance the source of axial current arising from quark reflection at the surface. We mention it here because in the non-linear boundary condition

$$\frac{\partial}{\partial r}[\bar{q}(\sigma + i\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\gamma_5)q] = -2(\sigma^2+\underset{\sim}{\pi}^2)^{1/2}\, B, \ r = R \ , \tag{5.23}$$

consistency with energy momentum conservation requires that one use the average of the $\underset{\sim}{\pi}$- and σ-field derivatives inside and outside the bag surface. The solutions for several values of the bag radius are displayed in Fig. 5.1.

As we have hinted, although the existence of hedgehog-like solutions is fascinating, they are not of much physical significance because of the lack of rotational invariance in space and isospin. An alternative approach suggested by Chodos and Thorn, which was not inconsistent with the model results for the hedgehog, was to make a perturbative expansion about the MIT solution, with a constant classical σ-field and zero classical pion field. Since the same approach was used by Jaffe, whose work is discussed in Section 5.3.3 below, we shall defer discussion of the perturbative approach.

5.3. Further Developments

5.3.1. *General Considerations*

Of course the form of the classical σ-field obtained by Chodos and Thorn is rather different from what we obtained in a soliton bag model in Section 2.3. That discussion suggested that the bag should correspond to a region where $\langle\sigma\rangle$ was zero. From the phenomenological point of view it is possible to impose this simply by multiplying the kinetic energy for the σ-field, $(\partial_\mu\sigma)^2$, in Eq. (5.10) by $\theta_{\overline{V}}$ (which is zero for $r < R$ and unity elsewhere). Indeed, if one identifies σ_V, the expectation value of the σ-field in free space with f [the pion decay constant, see Eq. (4.68)] it is easily seen that one gets volume energy contribution $-B\theta_V$ with B about 20 MeV/fm^3, or about one half of the phenomenological MIT value (Ros 81).

It seems to us extremely worthwhile to extend the soliton bag model of Section 2.3 by making it at least approximately chirally symmetric. For example, one might start with the symmetric form,

$$\mathcal{L}_s(x) = i\,\bar{q}\,\not{\partial}\,q + g\,\bar{q}(\sigma+i\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\gamma_5)q + \frac{1}{2}(\partial_\mu\underset{\sim}{\pi})^2 + \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{\lambda^2}{4}(\sigma^2+\underset{\sim}{\pi}^2 - \nu^2)^2 - p\ , \qquad (5.24)$$

and then explicitly break the symmetry with, say

$$\mathcal{L}_b(x) = C\,\sigma \quad \text{or} \quad C\,\sigma^3 \,. \tag{5.25}$$

In either case the sum $(\mathcal{L}_s + \mathcal{L}_b)$ is equivalent to the form used by Lee, or Goldflam and Wilets, when $\underset{\sim}{\pi}$ is set to zero [see Eq. (2.105)]. Unfortunately none of the parameters actually used by Goldflam and Wilets gives the right pion decay constant, but their study covered a very limited range of parameters.

Another persistent problem in any version of the σ-model is that the mass of its quantum fluctuations must be large. In fact the lightest isoscalar two-pion resonances are the narrow $S^*(980)$ which is possibly exotic, and the $\varepsilon(1300)$ which does at least have a large width. It is not at all clear that either of these should be identified with the fluctuations in the σ-field. Such problems led even the earliest investigators (GL 60) to consider eliminating the σ-field altogether. In that case one is forced to deal with *non-linear representations* of SU(2) × SU(2).

One example of such a non-linear representation is obtained by the Cayley transformation, in which σ and $\underset{\sim}{\pi}$ are replaced by a new pion field $\underset{\sim}{\xi}$ (Zum 68). That is,

$$\sigma + i\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\gamma_5 \rightarrow \frac{1 - i\xi\gamma_5}{1 + i\xi\gamma_5} = \Xi \quad , \tag{5.26}$$

where

$$\xi \equiv \underset{\sim}{\tau}\cdot\underset{\sim}{\xi} \,. \tag{5.27}$$

Just as we discussed in Section 4.4.2 the transformation properties of $\underset{\sim}{\xi}$ must be such as to keep $\bar{q}\,\Xi\,q$ invariant under a chiral transformation [Eq. (4.28)]. It is an easy algebraic exercise to show that this implies

$$\xi \rightarrow \xi - \delta\xi \ ; \quad \delta\xi = \varepsilon + \xi\,\varepsilon\,\xi \ , \tag{5.28}$$

which is clearly non-linear, involving ξ^2 on the right (By analogy with (5.27) $\underset{\sim}{\tau}\cdot\underset{\sim}{\varepsilon}$ is denoted ε.)

An alternate approach introduced by Gell-Mann and Lévy was to use the fact that chiral transformations leave $\sigma^2+\underset{\sim}{\pi}^2$ constant to eliminate σ^2. In fact, we saw that in the σ-model $(\sigma^2+\underset{\sim}{\pi}^2)$ was equal to f^2 (Section 4.4). Substituting the relation

$$\sigma^2 = f^2 - \underset{\sim}{\pi}^2 , \tag{5.59}$$

in the σ-model Lagrangian density (4.51) with $\nu \equiv f$, we obtain the non-linear sigma model. Clearly there are many other possibilities which use Eq. (5.59). For a general discussion of non-linear representations of SU(2) × SU(2) we refer to the work of Weinberg (Wei 67, Wei 68) and the lectures of Zumino (Zum 68).

5.3.2. *The little brown bag*

For a period of about four years the work of Chodos and Thorn was more or less forgotten. (A notable exception was the calculation of $B \to B\pi$ matrix elements in the MIT bag model by LeRoy (LeR 78).) Then in early 1979 several groups returned to this problem of imposing chiral symmetry (Bar+ 79, BR 79). Undoubtedly the largest shock wave was associated with the Stony Brook group. Brown and Rho proposed that one should take seriously the idea of a two phase picture of physical hadrons. The interior of the static MIT bag was to contain asymptotically free, massless quarks while the exterior would contain pions—the Goldstone bosons of SU(2) × SU(2).

Most notably from the point of view of this review Brown and Rho addressed the problem which we raised in Section 1—namely the compatibility of the bag model of the nucleon (with its large radius) with classical nuclear physics. Their proposal was that the pion coupling should have a dramatic effect on the bag, compressing it to a radius of say 3/10 fm! In that way the nucleon structure would be irrelevant at normal nuclear matter density.

In order to avoid the σ-meson the Stony Brook group worked with a non-linear version of the σ-model (Ven+ 80, Ven 80). In fact, their Lagrangian density can be obtained from that of Chodos and Thorn [Eq.. (5.10)] by multiplying $\theta_{\overline{V}}$ (1 outside, 0 inside the bag volume V) into the $\underset{\sim}{\pi}$ and σ kinetic energy terms

$$\mathcal{L}_{SB} = (i\, \bar{q}\, \not{\partial}\, q - B)\theta_V - \frac{1}{2f}\, \bar{q}(\sigma + i\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\gamma_5)q\delta_S$$

$$+ \frac{1}{2}(\partial_\mu\sigma)^2\ \theta_{\overline{V}} + \frac{1}{2}(\partial_\mu\underset{\sim}{\pi})^2\ \theta_{\overline{V}} \tag{5.60}$$

and eliminating $(\sigma,\underset{\sim}{\pi})$ in favour of a new pion field, $\underset{\sim}{\xi}$, defined by

$$\underset{\sim}{\pi} = \underset{\sim}{\xi}(1 + \xi^2/f^2)^{-1/2}$$

$$\sigma = f(1 + \xi^2/f^2)^{-1/2}\ . \tag{5.61}$$

Equation (5.61) is just one of the many non-linear transformations consistent with Eq. (4.59).

If the pion field is to drastically alter the equilibrium radius it is clear that a non-perturbative treatment must be used. There's the rub! The only case for which a non-perturbative treatment is feasible is once again the hedgehog. Even then the solution is no longer algebraic, instead Vento *et al.* obtained an ordinary second order differential equation for the classical pion field [i.e. for G(r), where $\underset{\sim}{\xi}(r) \equiv \hat{r}\, G(r)$].

We display in Fig. 5.2 some typical results from the Stony Brook group. For all these curves the πNN coupling constant, as measured by the asymptotic strength of the pion field, has been fixed at $f^2_{\pi NN}$ = 0.081 by varying f. There are clearly two rather different regions. For large values of R the graph of mass versus R is very flat and the result would be much like the usual MIT solution. Alternatively the hedgehog tends to collapse as R goes below about 0.6 fm. Indeed, within the crude model proposed there is nothing to stabilise it, and the mass goes to zero at $R \lesssim 0.3$ fm! It has been suggested that this problem can be overcome by coupling the ω-meson to the bag, in which case there is a stable minimum at about 0.5 fm (Ven 81).

As a model system the hedgehog is great fun to play with. For systems of six quarks it has provided some insight into the physics of the short distance repulsion in N-N scattering (Section 7). However, there are too many inconsistencies in this approach for it be considered realistic. In particular, all of the successes of the MIT bag model, which motivated the whole discussion of chiral symmetry breaking are lost in the small-R, non-perturbative limit. Moreover, as we shall argue in more detail in presenting the cloudy bag model (Section 5.4), when multi-pion effects are important the long-wavelength approximation breaks down and one can no longer justify neglecting the internal structure of the pion itself.

5.3.3. *Classical perturbation theory*

The revival of interest in "hybrid" bag models (pions and quarks) continued through 1979. In his lectures at the Erice school, Jaffe continued the work of Brown and collaborators in a different direction (Jaf 79). He too worked with a classical pion field, but (not surprisingly!) took the view that the MIT bag should not be drastically altered by its pion couplings. (A similar approach was taken by Musakhanov (Mus 80).) Thus he developed a systematic expansion of the pionic corrections in terms of a small parameter ε,

$$\varepsilon = \frac{9A}{8\pi f^2 R^2} , \qquad (5.62)$$

which essentially measures the strength of the classical pion field at the bag surface.

Formally Jaffe's work is almost identical to that of Vento *et al.* (Ven+ 80). Instead of the non-linear transformation (5.61) Jaffe chose to define a new pion field, $\underset{\sim}{\phi}$, using the relations

$$\underset{\sim}{\pi} = f\ \hat{\phi}\ \sin(\phi/f)\ ,$$
$$\sigma = f\ \cos(\phi/f)\ , \tag{5.63}$$

which obviously respects the condition $\sigma^2+\underset{\sim}{\pi}^2 = f^2$. In Eq. (5.63) ϕ is the magnitude of the three component vector $\underset{\sim}{\phi}$, and $\hat{\phi}$ is the unit vector giving its direction in isospin space,

$$\phi = (\underset{\sim}{\phi}\cdot\underset{\sim}{\phi})^{1/2}\ ;\quad \hat{\phi} = \underset{\sim}{\phi}/\phi\ . \tag{5.64}$$

It will be useful to have the following simple identities,

$$\partial_\mu\phi = \hat{\phi}\cdot\partial_\mu\underset{\sim}{\phi}\ , \tag{5.65}$$

$$\partial_\mu\hat{\phi} = [\hat{\phi}\times(\partial_\mu\underset{\sim}{\phi}\times\hat{\phi})]/\phi\ , \tag{5.66}$$

if the reader intends to follow the original papers in detail.

With the transformation (5.63) the surface coupling term becomes

$$-\frac{1}{2f}\,\bar{q}(\sigma+i\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\gamma_5)q\delta_s \rightarrow -\frac{1}{2}\,\bar{q}\ e^{i\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}\gamma_5/f}\ q\delta_s\ . \tag{5.67}$$

To prove this, simply make a power series expansion of the r.h.s. of Eq. (5.67) and use the identities

$$\gamma_5^2 = (\underset{\sim}{\tau}\cdot\underset{\sim}{\hat{\phi}})^2 = +1\ . \tag{5.68}$$

The kinetic energy pieces of the usual σ-model can be written in the form

$$\frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\underset{\sim}{\pi})^2 = \frac{1}{2}\left\{(\partial_\mu\phi)^2 + f^2\,\sin^2(\phi/f)(\partial_\mu\hat{\phi})^2\right\}\ . \tag{5.69}$$

However, if we define a "covariant derivative" as

$$D_\mu\underset{\sim}{\phi} \equiv (\partial_\mu\phi)\hat{\phi} + f\,\sin(\phi/f)\partial_\mu\hat{\phi}\ , \tag{5.70}$$

it is easy to see from the orthogonality of $\hat{\phi}$ and $\partial_\mu\hat{\phi}$ [see Eq. (5.66)] that

$$\frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\underset{\sim}{\pi})^2 \rightarrow \frac{1}{2}(D_\mu\underset{\sim}{\phi})^2\ . \tag{5.71}$$

Finally, if we exclude the pion field from the interior of the bag—in line with the simple minded, two-phase picture — the Lagrangian density is [use Eqs. (5.71) and (5.67) in Eq. (5.60)],

$$\mathcal{L}(x) = (i\,\bar{q}\,\not{\partial}\,q - B)\theta_V - \frac{1}{2}\bar{q}\,e^{i\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}\gamma_5/f}\,q\delta_S + \frac{1}{2}(D_\mu\underset{\sim}{\phi})^2\theta_{\bar{V}}\ . \tag{5.72}$$

By construction we know that Eq. (5.72) must be invariant under a non-linear chiral transformation. We leave it as an exercise for the reader to show that the appropriate transformation is

$$q \rightarrow q - i\,\frac{\underset{\sim}{\tau}\cdot\underset{\sim}{\varepsilon}}{2}\gamma_5\,q\ ,$$

$$\underset{\sim}{\phi} \rightarrow \underset{\sim}{\phi} + \underset{\sim}{\varepsilon}f + f(\underset{\sim}{\varepsilon}\times\hat{\phi})\times\hat{\phi}[1-(\phi/f)\cot(\phi/f)]\ , \tag{5.73}$$

and the corresponding, conserved axial current has the form

$$\underset{\sim}{A}^\mu = \bar{q}\,\gamma^\mu\gamma_5\,\underset{\sim}{\tau}/2\,q\,\theta_V + \left\{f\hat{\phi}\partial^\mu\phi + \frac{f^2}{2}\,\partial^\mu\hat{\phi}\,\sin(2\phi/f)\right\}\theta_{\bar{V}}\ . \tag{5.74}$$

(The latter is easily obtained by direct substitution for $\underset{\sim}{\pi}$ and σ in terms of $\underset{\sim}{\phi}$ in Eq. (5.11).) For completeness we also give the expression for the conserved vector current which, by analogy with the discussion of the σ-model in Section 4.4.2 [Eqs. (4.49) and (4.50)], arises from the invariance of Eq. (5.72) under the transformation

$$q \rightarrow q + i\,\frac{\underset{\sim}{\tau}\cdot\underset{\sim}{\beta}}{2}\,q$$

$$\underset{\sim}{\phi} \rightarrow \underset{\sim}{\phi} - \underset{\sim}{\beta}\times\underset{\sim}{\phi}\ . \tag{5.75}$$

It is

$$\underset{\sim}{V}^\mu = \bar{q}\,\gamma^\mu\underset{\sim}{\tau}/2\,q\,\theta_V + j_0^2(\phi/f)(\underset{\sim}{\phi}\times\partial^\mu\underset{\sim}{\phi})\theta_{\bar{V}}\ , \tag{5.76}$$

with $j_0(\phi/f)$ the s-wave spherical bessel function ($j_0(x) = \sin x/x$).

The corresponding non-linear field equations were written down by Jaffe "in their full non-linear ugliness", to emphasise that "if no sensible approximation scheme exists the situation is hopeless". We do not repeat those equations here, but simply summarise the results of the perturbative solution of the classical problem. The small parameter in the expansion is taken to be $(\underset{\sim}{\phi}/f)$. Then to zero'th order only the quark fields are non-zero and we have the MIT solution (q_o). To next order we have to solve the free Klein-Gordon equation for a massless pion field outside the bag

$$\nabla^2 \underset{\sim}{\phi}_1(\underline{r}) = 0 \ , \quad r \geq R \ , \tag{5.77}$$

subject to the boundary condition

$$\frac{\partial}{\partial r} \phi_1(\underline{r}) = \frac{i}{2f} \overline{q}_o(\underline{r})\gamma_5 \underset{\sim}{\tau} \, q_o(\underline{r}) \ , \quad r = R \tag{5.78}$$

This is exactly the phenomenon we observed earlier, that the discontinuity in the derivative of the pion field compensates for the source of axial charge due to the quarks at the surface of the bag.

Equations (5.77) and (5.78) are easily solved, and we obtain

$$\frac{\underset{\sim}{\phi}_1(\underline{r})}{f} = \varepsilon \left(\frac{R}{r}\right)^2 \underline{\sigma}\cdot\hat{r}\underset{\sim}{\tau} \ , \quad r \geq R \ . \tag{5.79}$$

Thus, as we advertised below Eq. (5.62), ε measures the strength of the pion field at the bag surface. Using $f = 93$ MeV and $g_A = 1.24$ we see that for the typical MIT bag radius, $R \approx 1$ fm, ε is about 0.2, and one would expect this perturbation expansion to work very well. However, for a "little bag" ($R \simeq 0.3$ fm) ε would be about 2, and perturbation theory useless.

In order to be consistent in the classical scheme one must calculate the first order correction to the quark fields as well,

$$q_o \to q_o + \varepsilon \, q_1 + O(\varepsilon^2) \ . \tag{5.80}$$

The correction q_1 must be calculated from the equations

$$\begin{aligned} \not{\partial} \, q_1 &= 0 \ , \quad r \leq R \ , \\ (i\gamma\cdot\hat{r} - 1)q_1 &= 3i \, \underline{\sigma}\cdot\hat{r} \, \gamma_5 \, q_o \ , \quad r = R \ , \end{aligned} \tag{5.81}$$

in order to consistently obtain the lowest order corrections to the energy, axial coupling constant and so on ...

$$E = E_o + \varepsilon\, E_1 + O(\varepsilon^2)\ ,$$
$$g_A = g_A^{(o)} + \varepsilon\, g_A^{(1)} + O(\varepsilon^2)\ . \tag{5.82}$$

For example, Jaffe obtained the result

$$\varepsilon E_1 = \frac{-3 g_A \varepsilon}{50R} \sum_{i,j} \underline{\sigma}_i \cdot \underline{\sigma}_j\ \underset{\sim}{\tau}_i \cdot \underset{\sim}{\tau}_j\ , \tag{5.83}$$

where for an N-quark bag of total spin S, and isospin I (with all quarks in the same spatial state),

$$\langle \sum_{i,j} \underline{\sigma}_i \cdot \underline{\sigma}_j\ \underset{\sim}{\tau}_i \cdot \underset{\sim}{\tau}_j \rangle = 3N^2 + 12N - 4S(S+1) - 4I(I+1)\ . \tag{5.84a}$$

(Actually Eq. (5.84a) is only correct for N=3, when the colour wave function is totally anti-symmetric. For example, when N=6 one finds instead (Mul+ 82)

$$\langle \sum_{i,j} \underline{\sigma}_i \cdot \underline{\sigma}_j\ \underset{\sim}{\tau}_i \cdot \underset{\sim}{\tau}_j \rangle = 20N - N^2 - 4S(S+1) - 4I(I+1)\ , \tag{5.84b}$$

which implies somewhat smaller pionic corrections.)

There are several satisfying features of this classical treatment. None of the major features of the MIT bag model are altered much. For example, Eqs. (5.83) and (5.84) give changes in the N and Δ masses by -100 MeV and -65 MeV respectively (for $R \approx 1$ fm) (Jaf 79). In addition the pion current also contributes to the magnetic moment of the hadron (BH 80). Indeed, Myhrer and collaborators have recently shown (using the classical approach still) that not only are the proton and neutron moments improved by the addition of pionic corrections, but that the Λ magnetic moment also comes out rather well (Myh+ 81). We shall not discuss the calculation of magnetic moments further here as the most extensive investigations have been carried out in the cloudy bag model, which will be discussed in Section 6.

On the other hand the model proposed by Jaffe does raise some problems. We recall from Section 3.3 that the correct prediction of the axial charge of the nucleon, g_A, was a major triumph of the MIT bag.

Once the pion field is included, however, there is a contribution to $\vec{A}(\underset{\sim}{x})$ from the gradient of the pion field—see Eq. (5.74) which to first order in ϕ/f is simply

$$\vec{A}(\underset{\sim}{x}) = \bar{q}(x)\, \vec{\gamma}\gamma_5\, \tau/2\; q(x)\theta_V + f\, \vec{\nabla}\phi(\underset{\sim}{x})\theta_{\bar{V}}\ . \qquad (5.85)$$

Now if the pion field were not excluded from the bag (by $\theta_{\bar{V}}$) the integral over $\vec{\nabla}\phi$ could be converted to a vanishing surface integral.* In the presence of $\theta_{\bar{V}}$ there remains a non-vanishing contribution from the integral of the pion field over the bag surface. As verified by a number of groups this surface contribution from the pion field increases the overall value of g_A by a factor of approximately 3/2 (Jaf 79, BH 80, Ven+ 80). Thus the hybrid bag model gives a value of g_A very close to the 5/3 of the "good old (non-relativistic) quark model"—a retrograde step to be sure. Further investigation of higher order corrections only makes the situation worse, with g_A rising above 2 (Hul+ 81).

Quite apart from the disaster for g_A one might expect to find some contribution to hadronic charge densities from the pion field. Unfortunately the charge density involves the time derivative of the pion field which vanishes in the classical limit.

Finally, classical models of the type considered by Jaffe offer little connection with nuclear physics. Indeed Jaffe seemed to feel that the hybrid bag models, although an entertaining sidelight to serious physics, were rather sterile. To quote directly, "it should be clear to the reader that hybrid chiral models are of limited theoretical

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*The astute reader may have observed that this is not actually true in the case of massless pions because $\phi(\underset{\sim}{r}) \propto r^{-2}$ and hence there is a constant contribution from the surface at infinity. However, for any finite pion mass (no matter how small) this will vanish.

interest. They are entirely ad hoc ... and restricted to the low energy regime."

We have taken a rather different and far more optimistic point of view. It seems to us that understanding the transition to the Goldstone realization of chiral symmetry will be an essential step in the solution of the QCD equations. Moreover, as we shall demonstrate, one particular hybrid model, the cloudy bag model (CBM), overcomes all of the objections raised above, while retaining the positive features. Most significantly for the present review it goes further, offering a basis for optimism in low and medium-energy nuclear physics which has simply not been conceivable before. The CBM will be introduced in Section 5.4 and its applications for hadronic properties described in Section 6. First, however, we summarise some of the other attempts to deal with pion-bag interactions.

5.3.4. *Other bag model calculations*

As we have already remarked, the first in what we may regard as modern investigations of hybrid bags after Chodos and Thorn was the work of LeRoy (LeR 78). He used the Chodos-Thorn surface coupling to estimate the strength of various $B'B\pi$ couplings. This was then compared with the more conventional Melosh analysis (Mel 74). In spite of the simplicity of this first analysis of a wide range of decays in the bag model, rather good qualitative agreement with experiment was obtained. For the specific examples of the nucleon and delta we shall see in Section 6 some of the corrections which would need to be incorporated in a more detailed investigation.

The first studies of the effect of pion coupling, dictated by chiral symmetry, on hadronic properties were those of Brown and Rho (BR 79) and Barnhill *et al.* (Bar+ 79). As Jaffe demonstrated at length (Jaf 79) neither of these works gave a fully consistent set of field equations for the coupled quark-pion system. In particular Barnhill *et al.* omitted the first order correction to the quark field [q_1 in Eq. (5.80)] caused by the non-zero pion field. This actually leads to the wrong sign for the first-order correction to the energy [E_1 in Eq. (5.82)]. We have already shown above that the later Stony Brook work on the hedgehog was based on a suitable hybrid extension of the non-linear σ-model.

Several other groups have used essentially the linearised version of Jaffe's equations in redoing the MIT spectroscopy for low-lying states (Cot+ 80, McM 81, Myh+ 81, Thé 82). Although the details of these fits vary a little, the overall conclusion is that there is no difficulty refitting the mass spectrum with pionic corrections. If anything, there is some improvement.

In concluding this section we note that there have been a number of other attempts to deal with pion coupling to the MIT bag which have *not* been motivated by considerations of chiral symmetry. In the sense that it is a non-perturbative treatment the work of Weber (Web 80, Web 81) is probably the most closely related to our present discussion. This will be treated in more detail in Section 7 (on the N-N force). Both Duck (Duc 76) and Weise (Wei 81) attempted to calculate the pion emission perturbatively. It is interesting that Weise also finds the pions to be predominantly created in the surface region of the bag. What is perhaps remarkable is that the coupling even has the right order of magnitude. We look with great interest for future work which might indicate why low order perturbation theory should yield sensible numerical results in the region where confinement (quark reflection) is occurring, and the effective qq-gluon coupling α_c is varying rapidly.

In comparison with these rather ambitious calculations the hybrid bag models *in general, and the CBM in particular are more phenomenological.* On the other hand, chiral symmetry is imposed as a crucial guide in constructing the theory, and one is in that sense not compelled to rely on perturbation theory in the bag surface which is not an asymptotically free region.

5.4. The Cloudy Bag Model

The starting point for the development of a model of hadron structure of relevance to nuclear physics is the model of Jaffe. As we saw in Section 5.3.3, even in its linearised form that model had some unfortunate features. The cloudy bag model (CBM) (Thé+ 80, Tho+ 81, Tho 81, Mil+ 81) also relies on a perturbative approach. However, it overcomes all of the problems encountered in Jaffe's model by: (a) dealing with a quantised pion field, and (b) not explicitly excluding the pion from the static bag volume. Some compelling, but nevertheless qualitative arguments will be given to suggest that not only does this approach yield good results, but that it may also be the best approximation to the underlying physics.

5.4.1. *The non-linear equations*

We have already explained in great detail how to obtain a chiral invariant Lagrangian density involving only quark and pion fields by making the substitution (5.63) in the Chodos-Thorn Lagrangian density. In the case where the pion is not excluded from the interior of the static bag volume this yields [c.f. Eq. (5.72)]

$$\mathcal{L}(x) = (i\,\bar{q}\,\not{\partial}\,q - B)\theta_V - \frac{1}{2}\,\bar{q}\,e^{i\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}\gamma_5/f}\,q\delta_S + \frac{1}{2}(D_\mu\underset{\sim}{\phi})^2 \;. \qquad (5.86)$$

All of the formal results of Section 5.3.3 hold and need not be repeated here. (The covariant derivative, $D_\mu\underset{\sim}{\phi}$, was given in Eq. (5.70).)

The only change is that wherever $\theta_{\overline{V}}$ appeared in Section 5.3.3 it should be replaced by 1. If an explicit symmetry breaking term, $-1/2\ m_\pi^2\ \underset{\sim}{\phi}^2$, were introduced in Eq. (5.86) the axial current of the model [c.f. Eq. (5.74)]

$$\underset{\sim}{A}^\mu = \overline{q}\ \gamma^\mu\gamma_5\ \underset{\sim}{\tau}/2\ q\ \theta_V + \left[f\ \hat{\phi}\ \partial^\mu\phi + \frac{f^2}{2}\ \partial^\mu\hat{\phi}\ \sin(2\phi/f)\right] \qquad (5.87)$$

would satisfy the PCAC condition

$$\partial_\mu \underset{\sim}{A}^\mu = -f\ m_\pi^2\ \underset{\sim}{\phi} + O(\underset{\sim}{\phi}^2)\ . \qquad (5.88)$$

5.4.2. *Pions inside the bag?*

None of the hybrid bag models which have been developed so far have really constituted a dynamical description of the process of pion emission. It is difficult enough to believe that the static MIT bag model itself, with its rigid, spherical boundary is more than a mathematically convenient idealisation of a real hadron. However it is impossible to believe that the boundary remains static and unperturbed by the creation of a pion with several hundred MeV/c momentum. Thus the very concept of interior and exterior, which was taken to be sacrosanct in the models discussed in Section 5.3, is by no means clear cut.

A useful model to consider at this stage is the soliton bag model discussed in Section 2.3.2. There we saw that with a suitable interaction between an effective σ-field and a fermion field it is possible for the fermions to dig themselves a "hole" (or bag). Within the hole the vacuum would be simple, with the expectation value of the σ-field very near zero. Outside the bag, where $\overline{q}q$ is zero, the σ-field has a non-vanishing expectation value. The transition region between these two extremes is the bag surface. It has been shown that results very similar to those of the MIT bag model can be obtained for a variety of parameters and surface thicknesses (GW 82).

Suppose that a $\bar{q}q$ pair is produced by some perturbative interaction in the surface of such a bag. This pair could also start to dig a hole and eventually move into the vacuum as a new particle, as illustrated schematically in Fig. 5.3. It is clear that creation of such a pair could occur anywhere inside the bag, although our ideas of asymptotic freedom suggest it would be most likely in the surface where the effective value of α_s is growing rapidly.

Of course this sort of pair creation process in a cavity has been studied other ways (MV 81, CH 81, DG 77) and such pairs are referred to as "sea quarks". Usually such pairs are treated like exchange current corrections in nuclear physics with the quarks being put in cavity eigenstates, rather than exhibiting any coherence. DeTar (DeT 81) suggested, without much conviction, that one might be able to treat pairs with pion quantum numbers as though they were coherent — in that way deriving a model identical to the CBM (Thé+ 80). However, the essential justification for such a procedure can come only from dynamical symmetry breaking (DSB) — in particular a model such as that proposed by Goldman and Haymaker (GH 81, HG 81). If their idea (see Section 5.1) that short-distance one-gluon-exchange suffices to bind a $\bar{q}q$ pair with pion quantum numbers (thereby producing DSB and a Goldstone boson) is correct, then it would be essential to treat such pairs coherently — even inside another bag!

Thus it should be clear for a number of reasons that the insistence on excluding pions from the interior of a static, spherical MIT bag is not only an unreasonable simplification, it may be wrong. On the other hand, it is clearly an approximation to treat the pion as a free particle through all space, as we assumed in writing Eq. (5.86). A more sophisticated treatment would perhaps involve the expansion of the pion field in eigenfunctions of some effective potential. Nevertheless, incorporating exact chiral symmetry and the concept of DSB, the CBM seems, a priori, to be a good place to begin.

5.4.3. *Linearisation of the CBM equations*

If the discussion towards the end of Section 5.1 did not make it clear let us stress again that it will only make sense to write down a hybrid model if the problem to be examined is one where the internal structure of the pion can reasonably be ignored. In this sense we are making a long wavelength approximation from the beginning. Therefore we must agree with Jaffe that either perturbation theory about the usual MIT solution is adequate or we should attack the problem in a different way.

As it stands, the Lagrangian density in Eq. (5.86) is probably not renormalisable. However, if it could be generalised to include the internal structure of the pion there would be a natural mechanism for cutting off higher order terms. This is a challenging problem for the future. For now, bearing all of these arguments in mind we have chosen (like Jaffe) to deal with small fluctuations in the pion field about the point $\underset{\sim}{\phi} = 0$. In that case we find the simplifications

$$\frac{1}{2}(D_\mu \underset{\sim}{\phi})^2 \rightarrow \frac{1}{2}(\partial_\mu \underset{\sim}{\phi})^2 \tag{5.89}$$

$$-\frac{1}{2}\,\overline{q}\, e^{i\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}\gamma_5/f}\, q\delta_s \rightarrow -\frac{1}{2}\,\overline{q}\, q\delta_s - \frac{i}{2f}\,\overline{q}\,\gamma_5\,\underset{\sim}{\tau}\, q\cdot\underset{\sim}{\phi}\,\delta_s\ , \tag{5.90}$$

in Eq. (5.86). The resulting Lagrangian density (Thé+ 80, DeT 81)

$$\mathcal{L}_{CBM}(x) = (i\,\overline{q}\,\not{\partial}\,q - B)\theta_V - \frac{1}{2}\,\overline{q}\,q\delta_s + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}\,m_\pi^2\,\underset{\sim}{\phi}^2$$

$$- \frac{i}{2f}\,\overline{q}\,\gamma_5\,\underset{\sim}{\tau}\,q\cdot\underset{\sim}{\phi}\,\delta_s\ , \tag{5.91}$$

will be treated in great detail in Section 6.

It will be an essential part of the discussion in Section 6 to show that the hadronic states resulting from Eq. (5.91) do not contain large multi-pion components. As long as perhaps one or two pions dominate, the large Compton wavelength of the pion ensures that the internal structure of the pion can be neglected. If on the other hand we find that there is an appreciable probability of finding, say, four or five pions, the distance scale of 5 $m_\pi^{-1} \sim (0.1\text{-}0.2)$ fm would simply make nonsense of our long wavelength approximation. This is also the reason why we oppose the inclusion of vector mesons as an explicit component of the hadronic wave function—such heavy $\bar{q}q$ pairs are best treated as sea quarks. (This will be discussed further in Sections 6 through 8 because it impacts severely on the conventional description of nuclear physics!)

Fortunately, we shall find that over a wide range of bag sizes a perturbative expansion in the number of pions converges extremely rapidly and the linearisation and long-wavelength approximation do produce a consistent solution! Indeed we shall show that Eq. (5.91) constitutes a renormalisable theory of bare bags coupled to a pion field within which the renormalisations are not only finite but small. For example, the bare NNπ coupling constant is within 10% of the renormalised value for any bag radius greater than 0.8 fm.

In motivating the present model rather than those considered in Section 5.3 we noted that the CBM would overcome all of the problems connected with the classical model of Jaffe. Hopefully it is obvious that as there is no exclusion of the pion from the bag interior there is no surface contribution to g_A from the pion field. Thus in lowest order the good bag model result that g_A is 1.09 is retained. Of course we have a Goldberger-Treiman relation and g_A will be renormalised in exactly the same way as the NNπ coupling constant. However, as we remarked above, such renormalisations are small in the CBM. Incidentally it is interesting to contrast this beautifully simple picture of PCAC and the fact that g_A is near one with the classical version described in Section 4. In the CBM g_A is near one because that's what three confined, relativistic, massless quarks give. The renormalisation is small because the cavity containing the quarks is large and low order perturbation theory in the pion field makes sense!

5.4.4. *An alternative formulation*

The implications of Eq. (5.91) for pion-nucleon scattering, particularly in the P_{33} channel, will be discussed in detail in the next Section. However, it is worth noting at this stage that the one disappointing feature of the CBM Lagrangian density is that there is no obvious prediction for low energy pion-baryon scattering. One of the triumphs of the soft-pion ideas of the late 60's was the Weinberg-Tomozawa relationship (Wei 66, AD 68). That is the prediction that in a chirally symmetric world the scattering length for a pion on *any* target of isospin T_t, with total isospin T, is exactly

$$a_T = (g/2m)^2\left(\frac{g_V}{g_A}\right)^2 \frac{m_\pi}{2\pi} (1 - m_\pi/m_t)^{-1}[T(T+1) - T_t(T_t+1) - 2] \ , \qquad (5.92)$$

where (g/2m) is the pseudoscalar NNπ coupling constant and m_t the target mass. Thus, the scattering length is purely isovector in the soft-pion limit. Much of the popularity of the non-linear sigma model in fact followed from Weinberg's proof (Wei 67) that it provided a convenient dynamical framework which incorporated Eq. (5.92) explicitly in an effective Lagrangian.

It is possible to make a unitary transformation on the original, non-linear Lagrangian density (5.86) in such a way that the Weinberg-

Tomozawa result appears explicitly (Tho 81(b)). However, the price is a redefinition of the quark fields which essentially get dressed by the pions. Only one of these two quark fields can be canonical and one must make a choice.

To be specific, consider the new quark field q_W, defined by the transformation

$$q \rightarrow q_W = S\, q \;, \tag{5.93}$$

$$\bar{q} \rightarrow \bar{q}_W = \bar{q}\, S \;, \tag{5.94}$$

with

$$S = \exp(i\, \underset{\sim}{\tau}\cdot\underset{\sim}{\phi}\, \gamma_5/2f) \;. \tag{5.95}$$

Then $\mathcal{L}(x)$ becomes

$$\mathcal{L}(x) = (i\, \bar{q}_W\, S^+ \not\partial\, S^+\, q_W - B)\theta_V - \frac{1}{2}\, \bar{q}_W q_W \delta_S + \frac{1}{2}(D_\mu \underset{\sim}{\phi})^2 \;. \tag{5.96}$$

(As usual the explicit, symmetry-breaking pion mass is omitted, but it can of course be put in with no change in our argument.) The $\not\partial$ in Eq. (5.96) acts both on S^+ and q_W, so it is convenient to separate the two pieces with the result

$$\mathcal{L}(x) = (i\, \bar{q}_W \not\partial\, q_W - B)\theta_V - \frac{1}{2}\, \bar{q}_W q_W \delta_S + \frac{1}{2}(D_\mu \underset{\sim}{\phi})^2$$

$$+ \bar{q}_W\, \gamma^\mu\, i(S\, \partial_\mu\, S^+) q_W \theta_V \;. \tag{5.97}$$

(We have used $\{\gamma^\mu, \gamma_5\} = 0$ to change $S^+\gamma^\mu$ to $\gamma^\mu S$.)

At this stage there is an extremely useful identity which appears in a paper by Au and Baym (AB 74):

$$S\, \partial_\mu\, S^+ = \int_0^1 d\lambda\, S^\lambda\, \partial_\mu (\ln S^+)(S^+)^\lambda \;. \tag{5.98}$$

The essential feature of Eq. (5.98) is that the logarithm reduces $\exp(i\, \underset{\sim}{\tau}\cdot\underset{\sim}{\phi}\, \gamma_5/2f)$ to a form linear in $\underset{\sim}{\phi}$. We leave it as a fairly straightforward algebraic exercise using Eq. (5.98) and

$$S = \cos(\phi/2f) + i\, \underset{\sim}{\tau}\cdot\hat{\phi}\, \gamma_5 \sin(\phi/2f) \quad , \tag{5.99}$$

to prove that

$$i\, S\, \partial_\mu\, S^+ = \frac{\gamma_5}{2f}\, \underset{\sim}{\tau}\cdot D_\mu \underset{\sim}{\phi} + \left[\frac{\cos(\phi/f)-1}{2}\right] \underset{\sim}{\tau}\cdot(\hat{\phi}\times\partial_\mu\hat{\phi}) \quad . \tag{5.100}$$

Thus, if we define the covariant derivative on the quark fields as

$$D_\mu\, q_W = \partial_\mu\, q_W - i\left[\frac{\cos(\phi/f)-1}{2}\right] \underset{\sim}{\tau}\cdot(\hat{\phi}\times\partial_\mu\hat{\phi})q_W \quad , \tag{5.101}$$

the transformed Lagrangian density takes the form

$$\mathcal{L}'_{CBM}(x) = (i\, \bar{q}_W\, \not{\partial}\, q_W - B)\theta_V - \frac{1}{2}\bar{q}_W q_W \delta_S + \frac{1}{2}(D_\mu \underset{\sim}{\phi})^2$$

$$+ \frac{1}{2f}\, \bar{q}_W\, \gamma^\mu\gamma_5\, \underset{\sim}{\tau}\, q_W \cdot (D_\mu\underset{\sim}{\phi})\theta_V \quad . \tag{5.102}$$

Clearly the surface coupling of the pion has been transformed into volume pseudovector coupling. This is exactly what one expects from current algebra considerations (AD 68). At $k = 0$ the strength of the coupling is simply related to the axial charge of the bag state

$$\sqrt{4\pi}\, f_{NN\pi}/m_\pi = g_A/2f \quad . \tag{5.103}$$

The Goldberger-Treiman relation is thereby made explicit. It has been proven by Betz (Bet 82) that the form-factor associated with this $NN\pi$ vertex is identical to that in the first version of the CBM, namely $3j_1(kR)/kR$—see Section 6.1 for details of the $NN\pi$ form-factor. Thus both versions are identical in all predictions associated with single pion emission and absorption.

To illustrate the consequences for s-wave pion scattering from a bag let us consider the zero energy limit and as suggested in Section 5.4.3 work to lowest *non-trival order in* $\underset{\sim}{\phi}$. Then the covariant derivative on the quark fields [Eq. (5.101)] leads to an interaction term quadratic in the pion field,

$$\mathcal{L}_s(x) = -\frac{1}{2f^2}\, [\bar{q}_W\, \gamma^0 \underset{\sim}{\tau}/2\, q_W](\underset{\sim}{\phi}\times\partial_0\underset{\sim}{\phi})\theta_V \quad . \tag{5.104}$$

But the term in square brackets is just the isospin density of the bag target [see Eq. (4.26)] and $(\underset{\sim}{\phi}\times\partial_0\underset{\sim}{\phi})$ the usual pion isospin density. For pions of zero three-momentum, $\underset{\sim}{\phi}\times\partial_0\underset{\sim}{\phi}$ is independent of $\underline{x}$, and integrating Eq. (5.104) to give the matrix element of the Hamiltonian between pion states of zero momentum

$$H_S = -\int d^3x\, \mathcal{L}_S(x)$$
$$= \underset{\sim}{T}_t \cdot \underset{\sim}{t}_\pi / 2f^2 , \qquad (5.105)$$

in an obvious notation. Thus, to lowest order we obtain a general relationship for *pion scattering from any* hadronic bag (except another pion!) which is identical to the Weinberg-Tomozawa result, Eq. (5.92). (To see this, use the Goldberger-Treiman relationship (5.103) and the familiar equivalence of pseudoscalar and pseudovector coupling constants $g/2m = \sqrt{4\pi}\; f_{NN\pi}/m_\pi$.) This result has been obtained independently by Szymacha and Tatur (ST 81).

Thus the alternate form of CBM Lagrangian density (Tho 81(b))

$$\mathcal{L}'_{CBM}(x) = (i\,\bar{q}_W \not\partial\, q_W - B)\theta_V - \frac{1}{2}\bar{q}_W q_W \delta_S - \frac{\theta_V}{4f^2}\bar{q}_W\, \gamma^\mu \underset{\sim}{\tau}\, q_W \cdot (\underset{\sim}{\phi}\times\partial_\mu\underset{\sim}{\phi})$$
$$+ \frac{\theta_V}{2f}\bar{q}_W\, \gamma^\mu\gamma_5\, \underset{\sim}{\tau}\, q_W \cdot \partial_\mu\underset{\sim}{\phi} + \frac{1}{2}(\partial_\mu\underset{\sim}{\phi})^2 - \frac{1}{2}m_\pi^2\, \underset{\sim}{\phi}^2 , \qquad (5.106)$$

incorporates both major results of the current algebra for low energy pion scattering and generalises the Weinberg Lagrangian (which applied to the NNπ system only) to any hadron describable by the MIT bag model. Furthermore, with the cautions given in the next section, rather than being used simply as an effective Lagrangian it defines a renormalisable theory of strong *interactions—thereby permitting* the systematic calculation of higher order corrections.

Figure Captions

Fig. 5.1. Behaviour of the quark density and the σ- and π-fields for various choices of parameters in the hedgehog model of Chodos and Thorn (CT 75).

Fig. 5.2. Rest energy of the hedgehog bag versus radius—with f^2=0.081 (Ven+ 80).

Fig. 5.3. Schematic illustration of the soliton bag: a) before, b) during, and c) after emission of a $q\bar{q}$ pair.

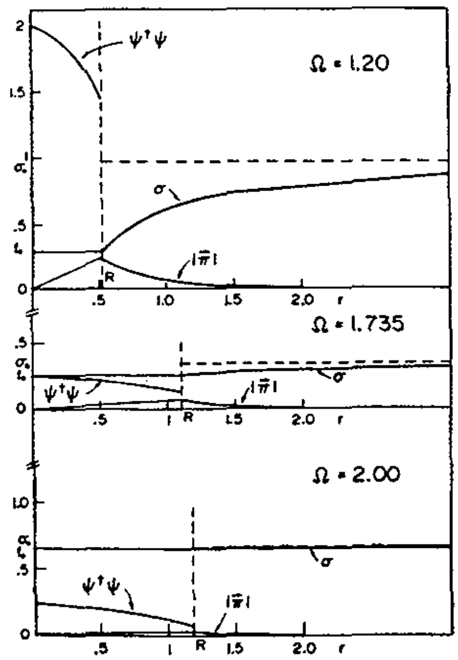

Fig. 5.1

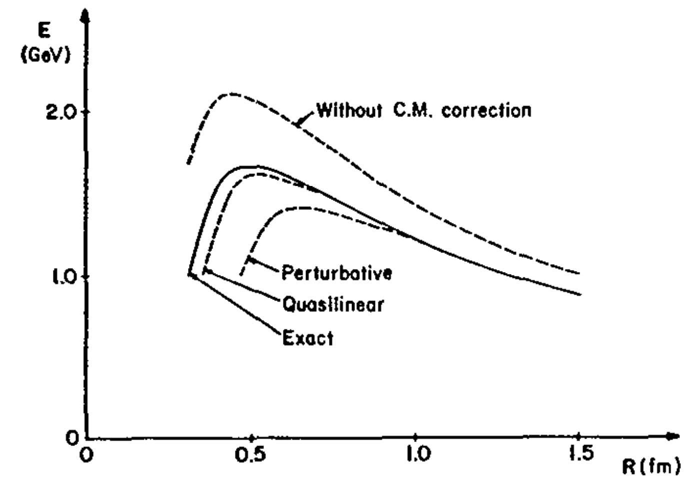

Fig. 5.2

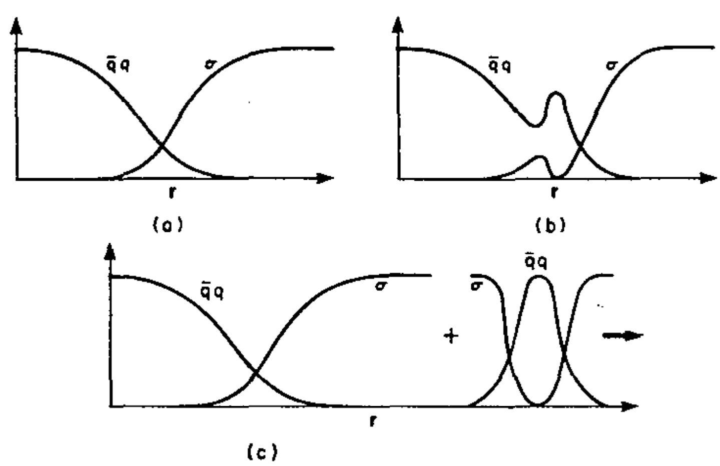

Fig. 5.3

6. APPLICATIONS OF THE CLOUDY BAG MODEL

At last we have established the chiral-bag formalism, and can begin to apply it to cases of physical interest. Our starting point will be the linearised version of the cloudy bag model given in Eq. (5.91). Essentially all of the applications so far have relied on the linear coupling of the form $B'B\pi$, and as we have remarked the alternate form of the Lagrangian density, Eq. (5.106), would give identical results. On the other hand, if one is concerned with s-wave π-π scattering, or reactions like $(\pi,2\pi)$ it would be necessary to retain terms of higher order in $\underset{\sim}{\phi}$. In that case, as we have already seen in obtaining the Weinberg-Tomozawa relationship, it is most fruitful to simply go to higher order in the expansion of the alternate Lagrangian density (5.102). For example, to $O(\phi^3)$ there will be an explicit term describing $\pi + B \rightarrow \pi + \pi + B'$, which arises from the pseudovector coupling to the axial current (KE 81, Tho 81b).

The natural first step in making practical calculations is to obtain a Hamiltonian from the underlying Lagrangian density. This Hamiltonian can be written entirely in terms of bags with the quantum numbers of observed particles, rather than in terms of quarks. At that stage the theory will look very much like the starting point for many calculations in medium energy physics. To first order, what we have gained is a microscopic understanding of the high-momentum cut-off in the theory, and relationships between the relevant coupling constants. Looked at in more detail, we shall see that the model is conceptually quite different, and the difference should have important consequences for our understanding of nuclear physics—particularly at high density.

6.1. A Hamiltonain for Low- and Medium-Energy Physics

The linearised CBM Lagrangian density, Eq. (5.91), breaks very nicely into three separate pieces

$$\mathcal{L}_{CBM}(x) = \mathcal{L}_{MIT}(x) + \mathcal{L}_{\pi}(x) + \mathcal{L}_{int}(x) \ , \tag{6.1}$$

where $\mathcal{L}_{MIT}$ was given in Eq. (4.1), $\mathcal{L}_{\pi}$ describes a free pion field

$$\mathcal{L}_{\pi} = \frac{1}{2}(\partial_{\mu}\underset{\sim}{\phi})^2 - \frac{1}{2} m_{\pi}^2 \underset{\sim}{\phi}^2 \ , \tag{6.2}$$

and

$$\mathcal{L}_{int} = -\frac{i}{2f} \bar{q} \gamma_5 \underset{\sim}{\tau} q \cdot \underset{\sim}{\phi} \ \delta_s \ . \tag{6.3}$$

Without $\mathcal{L}_{int}$, which was dictated by chiral symmetry, the theory would describe stable MIT bag states, and free pions.

Once gluon degrees of freedom are included in $\mathcal{L}_{MIT}$ only colourless states have finite energy.* Our emphasis in this review will be on baryon structure and interactions, although similar ideas could be applied to the heavy mesons. Thus we are naturally led to consider first colourless bag states with baryon number one. These will contain 3q (three quarks), $4q\text{-}\bar{q}$, $5q\text{-}2\bar{q}$ and so on. In view of the success of the bag model in describing the low-lying baryons without exotic components, it is reasonable to divide the space of baryon number one hadrons into two pieces (P+Q)

$$P = \sum_{\substack{\alpha=\text{non-exotic}\\ \text{baryons}}} |\alpha\rangle\langle\alpha| \ , \tag{6.4}$$

$$Q = \underset{\sim}{1} - P \ . \tag{6.5}$$

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*For pedagogic simplicity our discussion has ignored the role of gluons in the MIT bag model, except where they are absolutely essential—as in Section 3 for hyperfine splitting of hadronic levels. Nevertheless any realistic calculation must include the gluons. Since they play no role with respect to chiral symmetry they will only appear in $\mathcal{L}_{MIT}$.

That is, P is a projection operator onto non-exotic bag states such as N, Δ, R (the roper resonance) etc. The wave functions for these states are simply the usual bag-model SU(6) wave functions—see for example Eq. (3.13) for the nucleon. The unit operator $\underset{\sim}{1}$ refers to the space of B=1 bag states, and Q is a projection operator onto exotic states.

Formally, the inclusion of corrections arising from coupling to the Q-space is equivalent to evaluating the lowest order sea quark corrections. Such corrections have been shown numerically to be rather small (MV 81, CH 81, DG 77), so for the present purposes we shall neglect off-diagonal terms connecting P and Q. In that case the Hamiltonian obtained from $\mathcal{L}_{MIT}$ in the canonical way is simply (Thé+ 80)

$$H_{MIT} = \int d^3x \; T^{oo}_{MIT}(x) \quad , \tag{6.6}$$

where the energy-momentum tensor ($T^{\mu\nu}$) is

$$T^{\mu\nu}_{MIT}(x) = \frac{\partial \mathcal{L}_{MIT}(x)}{\partial(\partial_\mu q)} (\partial^\nu q) - g^{\mu\nu} \mathcal{L}_{MIT}(x) \; . \tag{6.7}$$

Explicitly this gives a bag model Hamiltonian

$$H_{MIT} = \int d^3x \left[\overline{q}(-i\underline{\gamma}\cdot\underline{\nabla})q + B + \frac{1}{2}\sum_{a=1}^{8}(E_a^2 - B_a^2)\right]\sigma_V \; . \tag{6.8}$$

However, the states α are eigenstates of Eq. (6.8) with masses $m_\alpha^{(b)}$, where the superscript means "bag". Thus we obtain

$$H_{MIT} \simeq P\, H_{MIT}\, P$$

$$= \sum_\alpha |\alpha> m_\alpha^{(b)} <\alpha| \; . \tag{6.9}$$

In terms of more conventional second quantisation this can be written

$$H_{MIT} = \sum_\alpha \alpha^+ \alpha \; m_\alpha^{(b)} \quad , \tag{6.10}$$

where α creates a three-quark bag state with the quantum numbers of N, Δ, R, etc.

(There is one rather innocent assumption implicit in the last step, namely that two different bag states α and β, with different masses, are orthogonal. Unfortunately this is not completely correct in the naive bag model, because the radii of those two bag states will not be exactly equal—as a result of the non-linear boundary condition. Nevertheless one expects on physical grounds that the orthogonality must hold in a more sophisticated formulation, such as the soliton bag model, and we simply impose it here.)

In the canonical way we also obtain the Hamiltonian for a free pion field corresponding to Eq. (6.2) namely,

$$H_\pi = \frac{1}{2}\int d^3x\left((\partial_0\underset{\sim}{\phi})^2 + (\underline{\nabla}\underset{\sim}{\phi})^2 + m_\pi^2\ \underset{\sim}{\phi}^2\right) , \qquad (6.11)$$

with $\underset{\sim}{\phi}$ the quantised pion field

$$\phi_i(\underline{x}) = (2\pi)^{-3/2}\int\frac{d\underline{k}}{(2w_k)^{1/2}}(a_{\underline{k}i}\ e^{i\underline{k}\cdot\underline{x}} + a^+_{\underline{k}i}\ e^{-i\underline{k}\cdot\underline{x}}) . \qquad (6.12)$$

The creation and destruction operators obey the usual commutation relations,

$$[a_{\underline{k}i}, a_{\underline{k}'j}] = [a^+_{\underline{k}i}, a^+_{\underline{k}'j}] = 0 ,$$

$$[a_{ki}, a^+_{k'j}] = \delta_{ij}\ \delta(\underline{k}-\underline{k}') . \qquad (6.13)$$

After normal ordering, Eq. (6.11) takes the rather simple and familiar form

$$H_\pi = \sum_i \int d\underline{k}\ w_{\underline{k}}\ a^+_{\underline{k}i}\ a_{\underline{k}i} . \qquad (6.14)$$

Finally, and of course this was the whole point of the exercise, there is an interaction term

$$P\ H_{int}\ P = \frac{i}{2f}\sum_{\alpha,\beta}\int d^3x \langle\beta|\bar{q}(x)\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}(\underline{x})\gamma_5\ q(x)|\alpha\rangle\delta_s\ \beta^+\alpha . \qquad (6.15)$$

Using the expansion (6.12) for the pion field, and assuming static, spherical bags of equal radii $[\delta_s \equiv \delta(x-R)]$, Eq. (6.15) becomes

$$P\,H_{int}\,P = (2\pi)^{-3/2} \sum_{\alpha,\beta,i} \int d\underline{k}\,(v^{\beta\alpha}_{\underline{k}i}\,\beta^{+}\alpha\,a_{\underline{k}i} + \text{h.c.})\ , \qquad (6.16)$$

where h.c. denotes hermitian conjugate and

$$v^{\beta\alpha}_{\underline{k}i} = \frac{i}{2f}\,\frac{1}{(2w_{\underline{k}})^{1/2}} \int d^3x\; e^{i\underline{k}\cdot\underline{x}}\,\delta(x-R)\,\langle\beta|\bar{q}(\underline{x})\tau_i\gamma_5\,q(\underline{x})|\alpha\rangle\ . \qquad (6.17)$$

Thus, as promised, all $B'B\pi$ couplings can be calculated in terms of the pion decay constant, $f = 93$ MeV.

6.1.1. *The* NNπ *vertex*

To see what is involved in Eq. (6.17) let us consider the NNπ vertex in this theory. In that case the spatial orbits of all quarks in the initial and final hadrons are the same, namely $1s_{1/2}$. The spatial portion of Eq. (6.17) is therefore [from Eqs. (2.33) and (2.34)]

$$\bar{q}_{1,-1}(\underline{x})\;\gamma_5\;q_{1,-1}(\underline{x})\Big|_{x=R} = \frac{N^2_{1,-1}}{4\pi}\,2i\,j_0(\omega)\,j_1(\omega)\,\underline{\sigma}\cdot\hat{r}\ ,$$

$$= \frac{\omega}{(\omega-1)}\,\frac{i}{4\pi R^3}\,\underline{\sigma}\cdot\hat{r}\ , \qquad (6.18)$$

and we have used the fact that the surface δ-function restricts the integral to x=R. Using Eq. (6.18) to perform the integral over coordinates in Eq. (6.17) we find that

$$v^{NN'}_{\underline{k}i} = (2w_{\underline{k}})^{-1/2}\,\frac{i}{2f}\,\frac{\omega}{(\omega-1)}\,\frac{j_1(kR)}{kR}\;{}_{s-f}\langle N|\sum_{a=1}^{3}\tau_{ai}\,\underline{\sigma}_a\cdot\underline{k}|N'\rangle_{s-f}\ , \qquad (6.19)$$

where the sum over a runs over the three quarks, and the subscript s-f denotes the spin-flavour part of the nucleon wave function. Now we recognise that the combination $\sum_{a=1}^{3} \underset{\sim}{\tau}_a \underline{\sigma}_a$ appeared in our discussion of the axial current. Indeed, from Eq. (3.38) we know that

$${}_{s-f}\langle N|\sum_{a=1}^{3}\tau_{ai}\,\underline{\sigma}_a\cdot\underline{k}|N'\rangle_{s-f} = \frac{5}{3}\,\langle N|\tau_i\,\underline{\sigma}\cdot\underline{k}|N'\rangle\ . \qquad (6.20)$$

Let us now define a form-factor u(k) which goes to one as $k\to 0$, namely

$$u(k) = 3j_1(kR)/kR\ , \qquad (6.21)$$

and recognise [from Eqs. (3.36) and (3.38)] that

$$g_A^{BAG} = \frac{5}{9}\frac{\omega}{(\omega-1)} . \tag{6.22}$$

Putting all of this together we find a very natural expression for the operator at the NNπ vertex,

$$v_{\underline{k}i}^{NN} = i(2w_{\underline{k}})^{-1/2}(g_A^{BAG}/2f)u(k)\ \tau_i\ \underline{\sigma}\cdot\underline{k} . \tag{6.23}$$

This should be compared with the usual static interaction (Wic 55, HT 62, Che 54, CL 55)

$$v_{i\underline{k}} = i(4\pi)^{1/2}(2w_k)^{-1/2}(f_{NN\pi}^{(o)}/m_\pi)v(k)\ \tau_i\ \underline{\sigma}\cdot\underline{k} , \tag{6.24}$$

where $f_{NN\pi}^{(o)}$ is the bare, pseudovector NNπ coupling constant whose renormalized value is $f_{NN\pi}^2 = 0.081$ — if the phenomenological cut-off function, v(k), is defined to be one at $k = im_\pi$.

If for the present we ignore questions of renormalisation and so forth, it is clear that the CBM makes a remarkably accurate prediction for $f_{NN\pi}$. Using $g_A = 1.09$ gives a value of 0.23 in comparison with the observed value of 0.28. However, including the c.m. correction discussed in Section 3.4.1 (about 20% increase in g_A), we find that theory and experiment agree within a few percent! We shall see in Section 6.2.1 that renormalisation will not significantly alter this success.

In addition to predicting the NNπ coupling constant we see that the CBM provides a very beautiful explanation for what was previously an ad hoc high momentum cut-off. The form-factor u(k), which is plotted in Fig. 6.1, simply reflects the fact that the violation of chiral symmetry, and therefore pion coupling to the bag, is associated with its surface. Since the bag is far from being point-like there is a natural cut-off in the theory with a range related to the radius of the source, R. Far from being specific to the CBM we expect such a cut-off to be a general feature of any model which treats the quark structure of the hadrons explicitly.

6.1.2. *The general* $B'B\pi$ *vertex*

Let us return to the general pion absorption vertex (6.17). If the hadrons α and β have the same radii it is well defined. But, as we have already remarked this will not usually be the case because of the non-linear boundary condition. Nevertheless, the radii of the members of the lowest baryon octet and decuplet do not vary by more than about 10% from the mean value. Thus in computing ratios of coupling constants we have assumed that these radii are all equal. (A more satisfying procedure would be to use the pseudovector volume coupling described in Section 5.4.3 (Tho(b) 81).)

A very basic example of an interaction which is extremely important in medium energy physics is the $\Delta N\pi$ vertex. In the CBM the pion induces this transition by flipping the spin and isospin of a quark at the bag surface (I=1/2, J=1/2 $\rightarrow$ I=3/2, J=3/2). Figure 6.2 illustrates some of these fundamental vertices. The form-factor at all such vertices will be the same function u(k) derived above. In the general case the vertex function associated with the $B'B\pi$ process is [from Eq. (6.17)],

$$v_{\underset{\sim}{k}i}^{B'B} = \frac{i}{2f}(2w_k)^{-1/2}\int d^3x\ e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}}\ \delta(x-R)\langle B'|\bar{q}(x)\ \times$$

$$\times\ \tau_i\gamma_5\ q(\underset{\sim}{x})\,|B\rangle\ , \tag{6.25}$$

which can always be summarised as

$$v_{\underset{\sim}{k}i}^{B'B} = i\left(\frac{4\pi}{2w_k}\right)^{1/2}(f^{(o)}_{B'B\pi}/m_\pi)u(k)\ \underset{\sim}{S}^{B'B}\cdot\underset{\sim}{k}\ T_i^{B'B}\ . \tag{6.26}$$

In general $\underset{\sim}{S}$ and $\underset{\sim}{T}$ are transition spin and isospin operators defined by

$$\underset{\sim}{S} = \sum_{m=-1}^{+1} S_m \, \hat{s}_m^* \, ,$$

$$\underset{\sim}{T} = \sum_{m=-1}^{+1} T_m \, \hat{t}_m^* \, , \tag{6.27}$$

with $\hat{s}_m$ and $\hat{t}_m$ unit vectors in a spherical basis (Edm 60)

$$\varepsilon_{\pm 1} = \mp(\hat{x} \pm i\hat{y})/\sqrt{2} \, , \quad \varepsilon_0 = \varepsilon_z \, . \tag{6.28}$$

The transition spins are given in terms of their reduced matrix elements, for example,

$$\langle S_{B'} \, s_{B'} | S_m | S_B \, s_B \rangle = C^{s_B \; m \; s_{B'}}_{S_B \; 1 \; S_{B'}} \tag{6.29}$$

and similarly for T_m. (For a more symmetric definition, which is not so widely used, see Dod+ 81.)

The coupling constants appropriate to transitions between all members of the nucleon octet have been summarised in the paper of Théberge and Thomas (TT(b) 82) — see also Thé 82. In the specific case that is of most interest to us after the nucleon, namely the Δ, the appropriate vertex functions are

$$v^{\Delta N}_{\underset{\sim}{k} i} = i\left(\frac{4\pi}{2w_k}\right)^{1/2}\left(\frac{f^{(o)}_{\Delta N\pi}}{m_\pi}\right) u(k) \; \underset{\sim}{S}\cdot\underset{\sim}{k} \; T_i \, , \tag{6.30}$$

$$v^{\Delta\Delta}_{\underset{\sim}{k} i} = i\left(\frac{4\pi}{2w_k}\right)^{1/2}\left(\frac{f^{(o)}_{\Delta\Delta\pi}}{m_\pi}\right) u(k) \; \underset{\sim}{\Sigma}\cdot\underset{\sim}{k} \; \mathcal{T}_i \, , \tag{6.31}$$

where $\underset{\sim}{S}$ and $\underset{\sim}{T}$ are the transition spins and isospins of Brown and Weise ($S_B = 1/2$, $S_{B'} = 3/2$) in Eq. (6.29), $\underset{\sim}{\Sigma}$ and $\underset{\sim}{\mathcal{T}}$ are the usual spin-3/2 spin and isospin operators, and the bare coupling constants are in the SU(6) ratios

$$f^{(o)}_{NN\pi} : f^{(o)}_{\Delta N\pi} : f^{(o)}_{\Delta\Delta\pi} = 1 : \sqrt{\frac{72}{25}} : \frac{4}{5} \, . \tag{6.32}$$

6.2. The Nucleon

We have seen that the practical effect of imposing chiral symmetry on the bag model is to dictate the pion coupling term in the Hamiltonian. Thus the physical hadrons will be dressed by a pion cloud. As we discuss in the next Section the Δ becomes unstable once the interaction with the pion field is turned on and is no longer strictly an eigenstate of the Hamiltonian. The nucleon must, of course, remain as a discrete eigenstate. Denoting the dressed nucleon as $|\tilde{N}\rangle$ (and the bare three quark nucleon as $|N\rangle$) we have

$$H|\tilde{N}\rangle = m_N|\tilde{N}\rangle \; , \tag{6.33}$$

where from the discussion in Section 6.1

$$H = H_{MIT} + H_\pi + H_{int} \tag{6.34}$$

[see Eqs. (6.10), (6.14) and (6.16)].

In recognition of the central importance of the nucleon in nuclear physics we shall discuss its properties in great detail. We shall see that unlike older static meson theories (HT 62), the convergence properties of the CBM are excellent. Whereas in the Chew-Low model the ratio of bare to renormalised coupling constants squared was about three, in the CBM this ratio is within 20% of unity (Thé+ 82)! Moreover the average number of pions in the "cloud" has been rigorously proven to be small. The average number of pions of any charge or momentum ($\langle n\rangle$) is less than or equal to a parameter, Λ, which is order 0.9 for a bag radius bigger than 0.8 fm—Section 6.2.1. A low order perturbative calculation actually yields $\langle n\rangle \simeq 0.5$. Thus the pion "cloud" about the nucleon is rather sparse!

Given these excellent convergence properties the calculation of electromagnetic properties of dressed nucleons (and other members of the nucleon octet) is straightforward. One is justified in making a perturbative expansion of the state $|\tilde{N}\rangle$ as *

$$|\tilde{N}\rangle \cong Z^{1/2}|N\rangle + c|N\pi\rangle + c'|\Delta\pi\rangle \; . \tag{6.35}$$

*See also the recent discussion of Bolsterli (Bol 81, Bol 82) who describes the use of "coherent meson pair states" to simplify the calculations when first order perturbation theory is not adequate.

Perhaps the most significant observation concerning nucleon electromagnetic structure in this model is the charge form-factor of the neutron, G_{En}. It is discussed at length in Section 6.2.2, where we stress the significance of a good experimental determination. In the CBM it is inescapable that the measurement of the zero in the neutron charge distribution measures the bag size—modulo surface thickness corrections.

Finally in Section 6.2.5 we note that the CBM has obvious implications for calculations of nucleon decay—as suggested by grand unification. In particular, considerations of chiral symmetry suggest a rather strong enhancement of the $p \to e^+\pi^0$ decay mode.

6.2.1. *Convergence properties of the CBM*

In this section we briefly indicate how the bounds on the pion content of the nucleon were obtained. Then we look at the pionic self-energy contribution for the nucleon. Finally we discuss the renormalisation of the bare $NN\pi$ coupling constant and show that it is small for two reasons—first because of the rather strong cut-off provided by the vertex function $u(k)$, and second because of the explicit appearance of the Δ.

Following the discussion of Dodd, Thomas and Alvarez-Estrada (Dod+ 81) we write the most general solution of Eq. (6.33) as

$$|\tilde{N}s\tau\rangle = Z^{1/2}|Ns\tau\rangle + \sum_{r=1}^{\infty}\sum_{\alpha}\sum_{k_1\ldots k_n} c_n(\alpha;k_1\ldots k_n;\tilde{N}s\tau) \times$$
$$\times\ (n!)^{-1/2}\, a^+_{k_1}\ldots a^+_{k_n}|\alpha\rangle\ , \tag{6.36}$$

where $|\alpha\rangle$ represents a colourless, three-quark bag state, s and τ are spin and isospin labels for the nucleon, and the $\{c_n\}$ are expansion coefficients. For notational convenience we have also followed the common practice (Wic 55) of replacing the sum over pion isospin and integral over momenta by a formal sum,

$$\sum_k \equiv \sum_{\substack{\text{isospin} \\ \text{labels}}} \int \frac{dk}{(2\pi)^3} . \tag{6.37}$$

Clearly the coefficients c_n are given by

$$c_n = (n!)^{-1/2} \langle\alpha|a_{k_n} a_{k_{n-1}} \cdots a_1|\tilde{N}st\rangle , \tag{6.38}$$

and we can see that it may be useful to define a state $|\phi_n\rangle$ by removing n pions with specific isospin and momentum from a physical nucleon

$$|\phi_n\rangle = (n!)^{-1/2} a_{k_1} \cdots a_{k_n}|\tilde{N}st\rangle , \tag{6.39}$$

so that

$$c_n = \langle\alpha|\phi_n\rangle . \tag{6.40}$$

Since the physical nucleon state must be normalised we find

$$\langle\tilde{N}st|\tilde{N}st\rangle = Z + \sum_{n=1}^{\infty} P_n = 1 , \tag{6.41}$$

where P_n, the probability of finding n pions in the physical nucleon, is

$$P_n = \sum_\alpha \sum_{k_1 \ldots k_n} |\langle\alpha|\phi_n\rangle|^2 . \tag{6.42}$$

Now, from the completeness of the states $|\alpha\rangle$ in the single baryon subspace, Eq. (6.42) implies

$$P_n \leqslant \sum_{k_1 \ldots k_n} \langle\phi_n|\phi_n\rangle = \sum_{k_1 \ldots k_n} \||\phi_n\||^2 . \tag{6.43}$$

One can now use the defining equation for $|\tilde{N}\rangle$ [Eq. (6.33)], and the commutation relations of the pion creation and destruction operators to manipulate the expression for ϕ_n. For example, in the case n=1, using Eq. (6.33) and the relation

$$[H_\pi, a_k] = -w_k a_k , \tag{6.44}$$

we find

$$\phi_1 = a_{k_1}|\tilde{N}\rangle = (\tilde{m}_N - w_{k_1} - H)^{-1}[a_{k_1}, H_{int}]|\tilde{N}\rangle . \tag{6.45}$$

However, the commutator in Eq. (6.45) is readily found from Eq. (6.16),

$$C_1 \equiv [a_{k_1}, H_{int}] = \sum_{\alpha\beta} (v_{k_1}^{\beta\alpha})^+ \beta^+ \alpha . \tag{6.46}$$

In addition, the spectrum of the full Hamiltonian H begins at $\tilde{m}_N$, so that $(H-\tilde{m}_N+w_{k_1})$ is greater than or equal to w_{k_1}. Thus we have a bound on P_1,

$$P_1 \leqslant \sum_{k_1} ||\phi_1||^2 \leqslant \sum_{k_1} \frac{||C_1||^2}{w_{k_1}^2} , \tag{6.47}$$

and in general one finds (Dod+ 81)

$$P_n < (n!)^{-1} \Lambda^n , \tag{6.48}$$

$$\Lambda = \sum_{k_1} \frac{||C_1||^2}{w_{k_1}^2} . \tag{6.49}$$

It is also rather easy to obtain a bound on the average number of pions. Consider the normalised expectation value of the number operator,

$$\langle n\rangle = || \, |\tilde{N}\rangle ||^{-2} \langle\tilde{N}| \sum_k a_k^+ a_k |\tilde{N}\rangle , \tag{6.50}$$

$$= \sum_{k_1} ||\phi_1||^2 \leqslant \Lambda . \tag{6.51}$$

All that remains is to evaluate the norm of the commutator, which is simply the maximum value of the vector $C_{k_1}|\psi\rangle$, with $|\psi\rangle$ any normalised linear combination of baryon-number one bags. As shown by Dodd *et al.*, if we include only N and Δ states (which we expect to dominate because of their closeness in mass and radius — see Section 6.1.2) Λ has the form

$$\Lambda = \frac{57}{25} 4\pi \left(\frac{f_{NN\pi}^{(o)}}{m_\pi}\right)^2 \frac{3}{(2\pi)^2} \int_0^\infty \frac{k^4 u^2(k)}{w_k^3} dk , \tag{6.52}$$

where $f_{NN\pi}^{(o)}$ is the bare coupling constant, and the CBM form factor u(k) was given in Eq. (6.21). In Table 6.1 we give the value of Λ and the corresponding bounds on P_n for a bag radius of 0.82, and $f_{NN\pi}^{(o)2} =$ 0.078, as found by Thomas *et al.* (Tho+ 81) from pion-nucleon scattering. For comparison we show the results for R=1 fm (the MIT bag radius), and also a bound for the old static Chew-Wick meson theory. Clearly the convergence properties of the model are remarkable. Indeed, this bound is probably still a little loose, for the calculated values of P_1, P_2 and <n> in case (a) are 0.35, ≤0.05 and ~0.5 respectively.

The coupling of the pion field to the bag will of course shift its energy, as we have already discussed in Section 5.3.3. In the present quantised description of the problem, the lowest order self-energy corrections are the single loop contributions shown in Fig. 6.3—for N and Δ. For the nucleon this mass shift is

$$\delta m_N^{(2)} = -\sum_k \left(\frac{v_k^{NN}\, v_k^{NN*}}{w_k} + \frac{v_k^{N\Delta}\, v_k^{\Delta N*}}{m_\Delta - m_N + w_k} \right). \tag{6.53}$$

This behaves roughly as $R^{-3.5}$ (Thé 82), and therefore grows rapidly as the bag radius decreases. With R=1 fm the pionic contribution to the self-energy of the nucleon is about 200 MeV, which is comparable to the one-gluon exchange, volume and centre-of-mass corrections. As we mentioned, a number of groups have shown that quite respectable fits can be obtained for the masses of the low-lying baryons when this correction is included—e.g. Myh+ 81. We shall discuss the mass splitting of the nucleon and delta further in Section 6.3.

To conclude this Section we consider the renormalisation of the NNπ coupling constant. In a theory without anti-nucleons (and therefore with no renormalisation of the pion propagator) the renormalised coupling constant is given by

$$f_{NN\pi}^{(r)} = Z\, f^{(o)}/Z_1 \; . \tag{6.54}$$

The factor Z (usually Z_2) measures [see Eq. (6.41)] the probability that the physical nucleon contains a bare nucleon—it therefore reduces $f^{(r)}$ from the bare value. The dressing of the vertex, which tends to increase the coupling strength, is described by Z_1. Figure 6.4

shows the first order dressing of the bare NNπ vertex. We see that the Δ-bag enters very naturally in this model, unlike the earlier static theories where only Fig. 6.4(a) would appear (HT 62).

Complete expressions for Z_1 can be found in the article of Théberge *et al.* (Thé+ 82). In Fig. 6.5 we show the corresponding bare coupling constant squared necessary to reproduce the observed renormalised NNπ coupling constant squared (0.081), as a function of bag radius. It is remarkable that for any radius greater than about 0.8 fm, $f^{(o)}$ is within 10% of $f^{(r)}$! This should be compared with the old static meson theories (HT 62) where, as shown in column (c) of Table 6.1, $f^{(o)2}$: $f^{(r)2}$ was about 3 : 1. There are two reasons for this dramatic improvement. First, the nucleon is now a rather large object, and the form factor u(k) cuts off the integrals describing Z. (Iterative solution of Eq. (6.33) implies

$$|N\rangle \simeq Z^{1/2}|N\rangle - Z^{1/2}\sum_k\left(\frac{v_k^{NN*}}{w_k}\Big|N,k\rangle + \frac{v_k^{\Delta N*}}{m_\Delta - m_N + w_k}\Big|\Delta,k\rangle\right) , \qquad (6.55)$$

so that

$$Z^{-1} \simeq 1 + \sum_k\left[\frac{v_k^{NN}\ v_k^{NN*}}{w_k^2} + \frac{v_k^{N\Delta}\ v_k^{\Delta N*}}{(m_\Delta + w_k - m_N)^2}\right], \qquad (6.56)$$

and we see that the summation term is just the derivative with respect to energy of the self-energy term $\delta m^{(2)}$ — Eq. (6.53) — as it must be in general.) Thus Z is typically greater than about 2/3 for $R \geqslant 0.8$ fm, compared with 1/3 for the Chew-Wick case.

The second reason why $f^{(r)}$ is so close to $f^{(o)}$ is the occurrence of the Δ in the vertex renormalisation — Fig. 6.4. To see this consider first Fig. 6.4(a) which goes like

$$\lambda_{NN} \sim \sum_k \underline{\sigma}\cdot\hat{k}\tau_k\ \underline{\sigma}\cdot\underline{q}\tau_q\ \underline{\sigma}\cdot\hat{k}\tau_k = \frac{1}{9}\,\underline{\sigma}\cdot\underline{q}\tau_q\left[\sum_k(\underline{\sigma}\cdot\hat{k})^2\tau_k^2\right] , \qquad (6.57)$$

where

$$Z_1^{-1} = 1 + \lambda_{NN} + \lambda_{N\Delta} + \lambda_{\Delta N} + \lambda_{\Delta\Delta} \tag{6.58}$$

and we have used the commutation relations of $\underline{\sigma}$ and $\underline{\tau}$. The factor of 1/9 essentially kills any compensation for the small value of Z in the Chew-Wick theory. This does not happen for those terms involving an explicit Δ. Indeed, if the N and Δ were degenerate, the ratio of the four terms in Fig. 6.4 would be respectively (Thé+ 82)

$$a : b : c : d \equiv \lambda_{NN} : \lambda_{N\Delta} : \lambda_{\Delta N} : \lambda_{\Delta\Delta}$$

$$= 1 : 16\left(\frac{f^{\Delta N}}{f^{NN}}\right)^2 : 16\left(\frac{f^{\Delta N}}{f^{NN}}\right)^2 : 20\left(\frac{f^{\Delta N}}{f^{NN}}\right)^2 . \tag{6.59}$$

To summarise, the CBM with bag radii greater than about 0.8 fm is remarkably convergent. This convergence arises because of the rapid cut-off of high momentum components and the explicit treatment of the Δ. In fact, we shall see in Section 6.3 that these factors are related; it is only because of the presence of the explicit Δ that one can understand pion-nucleon scattering with a strong high-momentum cut-off. In this light the pessimism of Henley and Thirring (HT 62): " For a long time it has been one of the main goals of meson theory to analyse the physical nucleon in terms of the bare nucleon and its meson cloud. This led to a dead end road ... The reason is that the ... resonant state of the nucleon is not important for the ground state", should rather be regarded as a clue for future development!

In case the point has not been made clear let us repeat it briefly. We have been led to the remarkable conclusion that if QCD results in large composite baryons with a structure like the MIT idealisation, the usual world of so-called "strong" interactions is amenable to solution by low order perturbation theory!

6.2.2. *The neutron charge distribution*

Let us briefly recall the discussion of hadronic charge distributions in the MIT bag model given in Section 3.1. We noted that since in lowest order the neutron bag has three quarks, whose charges sum to zero, in identical spatial orbits, it has no charge distribution. There are a number of higher order effects which tend to mix other configurations into the ground state (CH 81, MV 81) but none of these give even the right order of magnitude for $\langle r^2\rangle^n_{ch}$ in the bag model.

On the other hand, if we truncate the perturbation expansion of the physical neutron wave function in the CBM at one pion we find

$$|\tilde{n}\rangle = Z^{1/2}|n\rangle + c_{N\pi}\left(\sqrt{\frac{2}{3}}|p\pi^-\rangle - \sqrt{\frac{1}{3}}|n\pi^0\rangle\right) , \qquad (6.60)$$

where $|c_{N\pi}|^2$ is the probability for finding the nucleon to consist of a nucleon bag and a pion (of order 20% depending on R—Tho+ 81, Thé+ 81). As indicated in Eq. (6.35) there is also a $|\Delta\pi\rangle$ component which is included in all calculations. However it is much less important for the charge distribution because the $\Delta^-\pi^+$ piece tends to cancel against $\Delta^+\pi^-$, and the 300 MeV excitation energy of the Δ also makes the range of the pion field much smaller. Equation (6.60) shows quite explicitly that the charge distribution of the neutron in the CBM is a *first order effect* of the pion coupling—arising directly from the $|p\pi^-\rangle$ component.

This was first observed by Théberge *et al.* (Thé+ 80, Mil+ 81). Earlier calculations in classical models missed this because time derivatives vanish in the classical limit, and the pion charge current is

$$j^0_\pi(x) = -ie\left(\phi(x)\partial_0\,\phi^*(x) - \phi^*(x)\partial_0\,\phi(x)\right) ,$$

$$\phi(x) = \left(\phi_1(x) - i\,\phi_2(x)\right)/\sqrt{2} . \qquad (6.61)$$

Thus one really needs an explicit treatment of the quantum fluctuations of the pion field—as in the CBM—in order to see the effect. In terms of the creation and annihilation operators for pions of specific momentum and isospin Eq. (6.61) becomes (Tho+ 81)

$$j_\pi^o(\underline{x}) = \frac{-ie}{2} \sum_{i,j=1}^{2} \frac{\varepsilon_{ij3}}{(2\pi)^3} \int d\underline{k}\, d\underline{k}' \left(\frac{w_{k'}}{w_k}\right)^{1/2} e^{i(\underline{k}-\underline{k}')\cdot\underline{x}} \times$$

$$\times\ (-a_{i,-\underline{k}'} + a^+_{i,\underline{k}'})(a_{j,\underline{k}} + a^+_{j,-\underline{k}}) \ . \qquad (6.62)$$

The calculation of the pion contribution to the charge distribution then amounts to evaluating the expectation value of the operator in Eq. (6.62) in the state (6.55)—i.e. essentially (6.60). The quark contribution was already explained in Section 3.1.

Since the charge of the proton bag in Eq. (6.60) is confined inside the bag volume (i.e. radii less than R), and the pion field has its source at the bag surface and extends outside, the model obviously predicts a positive core and a negative tail. The details are illustrated in Fig. 6.6. It is clearly an inescapable conclusion of the CBM that the zero in the neutron charge distribution necessarily occurs at the bag radius. An accurate experimental determination of G_{En} would thus provide us with a direct measure of the size of the confinement volume! (Note that there is certainly no physical significance to the discontinuity of $\rho^n_{ch}(r)$ at r=R, it is a consequence of the oversimplification of the description of the bag surface as a rigid sphere. It is unlikely that any more realistic treatment would do more than smooth out the charge density in the surface region without altering our conclusion.) The r.m.s. radius of the neutron is not strongly dependent on R, varying from -0.391 fm at 0.8 fm to -0.327 at 1.1 fm (Thé+ 82)—in excellent agreement with the experimental value of -0.342 fm (obtained by dropping thermal neutrons on an electron target— Eri 78). Similar results have since been obtained by DeTar (DeT 81) and Myhrer (Myh 82).

Of course the idea of associating the negative tail of the neutron charge distribution with the process $n \to p\pi^-$ is very old—dating back to the late 50's and static meson theory (HT 62). However, that approach had two very important problems. First the properties of the core of the nucleon were unknown. Second, the interpretation of G_{En} was complicated by the presence of the Darwin-Foldy term, whereby a Dirac particle with an anomalous magnetic moment appears to have a charge distribution—because of zitterbewegung. Indeed the observed neutron magnetic moment is sufficient to explain all of $\langle r^2\rangle^n_{ch}$ (Fol 58, Eri 78).

In the quark model there is no Darwin-Foldy term. The photon interacts with three confined quarks and the pion. Thus there is no ambiguity in the interpretation of G_{En} in the CBM and the agreement with the data both for $\langle r^2\rangle^n_{ch}$ and G_{En} is highly significant! In conclusion, let us stress once more the importance of a better measurement of G_{En} in determining the size of the confinement region.

6.2.3. *Further nucleon electromagnetic properties*

It is of course of great interest to calculate the other nucleon electromagnetic properties, such as the proton charge radius ($\langle r^2\rangle^p_{ch}$), and proton and neutron magnetic moments (μ_p and μ_n), even though the pionic contribution is not the leading term there. The calculation of the proton charge radius proceeds exactly as we described above for the neutron, except that the bare bag makes a major contribution. Théberge *et al.* found a proton r.m.s. charge radius between 0.73 and 0.91 fm for R between 0.8 and 1.1 fm (Thé+ 82). However, the c.m. correction to the bag contribution is somewhat controversial as we described in Section 3.4.1. Without any c.m. correction the results of Théberge *et al.* lay between 0.71 and 0.87 fm. This is still in rather good agreement with the experimental value of 0.836 fm (Nag+ 79). Finally we note that very similar results have been obtained by DeTar (DeT 81) and Myhrer (Myh 82).

The pionic contribution to the magnetic moments involves the spatial component of the pion current

$$\vec{j}_\pi(\underline{x}) = ie\Big(\phi(\underline{x})\vec{\nabla}\,\phi^*(\underline{x}) - \phi^*(\underline{x})\vec{\nabla}\,\phi(\underline{x})\Big)\ , \tag{6.63}$$

which eventually can be written as (Sal 57, HT 62),

$$\vec{j}_\pi(\underline{x}) = \frac{-ie}{2}\sum_{i,j=1}^{2}\frac{\varepsilon_{ij3}}{(2\pi)^3}\int\frac{d^3k\ d^3k'}{(w_k w_{k'})^{1/2}}\,\vec{k}(a^{+}_{i\vec{k}'} + a_{i-\underline{k}'})\ \times$$

$$\times\ (a_{j,\vec{k}} + a^{\dagger}_{j-\underline{k}})\ e^{i(\underline{k}-\underline{k}')\cdot\underline{x}}\ . \tag{6.64}$$

Once again we need to evaluate this operator between nucleon wave functions of the form given in Eq. (6.35) — that is, including both nucleon and delta intermediate states.

The bag contribution itself (while the pion is "in the air") is also rather interesting. It is possible for the quark magnetic moment operator (unlike the charge operator) to induce an N-Δ transition. Thus one must compute all of the processes shown in Fig. 6.7. Unlike the direct interaction with the pion cloud, the core interactions will have both an isoscalar and an isovector piece. Thus it is not true, as one can find in the literature, that the pionic contribution is purely isovector.

Once again the comparison of calculational results with experiment is somewhat clouded by the uncertainty over c.m. corrections. Nevertheless this uncertainty is smaller than for the charge radii. Including the Donoghue-Johnson correction (DJ 80) μ_p and μ_n range between (2.43, 2.78) and (-1.97, -2.07) nuclear magnetons respectively for

R (0.8, 1.1) fm—Thé+ 82). With no c.m. corrections the corresponding values are (2.20-2.43) and (-1.80, -1.82) μ_N. Recalling that the MIT results with and without c.m. corrections were (2.24, -1.49 μ_N) and (1.9 μ_N, -1.26 μ_N) respectively (DeG+ 75), we see that *the inclusion of pionic corrections has made a tremendous quantitative improvement in the agreement with data.* In particular, the residual discrepancy of (5-10)% is well within the uncertainties of the calculation—e.g. from sea quarks, configuration mixing and so on.

In conclusion we make a couple of qualitative remarks about the role of the intermediate Δ in these calculations. For the charge distribution the Δ_π contribution tends to reduce that associated with $N\pi$. For example, the proton goes predominantly to π^+n and $\pi^-\Delta^{++}$. However, for magnetic moments the spin of the Δ is very important. The π^+ cloud around the n-core obviously gives a positive contribution to the magnetic moment. But when the proton with spin-up goes to Δ^{++} the Δ tends to have spin +3/2 so that the π^- orbits in the opposite direction to the π^+. Therefore, we get a positive contribution from the pion cloud—see Fig. 6.8. Again we see that the explicit presence of the Δ-bag is rather important for the quantitative success of the model.

6.2.4. *Weak interactions*

In view of the long development of the ideas of chiral symmetry and PCAC in Section 4.3 it should be clear that the chiral bag models necessarily produce an acceptable description of the axial current. The presence of an explicit pion field means that there is an induced pseudoscalar term in $A^\mu(\underline{x})$. Furthermore, the imposition of chiral symmetry implies that the relative strengths of the axial and induced pseudoscalar terms is consistent with the Goldberger-Treiman relation.

In both these respects the chiral bag models are, by construction, superior to the original MIT bag model.

A somewhat deeper feature of the cloudy bag model (CBM) is the interpretation of PCAC implicit in it. From Section 4.3 we recall that the correct statement of PCAC is that the dependence of physical quantities on the pion mass should be smooth. Both this and the nearness of g_A to one are directly related to the remarkable convergence properties of the model. We demonstrated in Section 5.3.3 that there is no direct pionic contribution to g_A in the CBM—as opposed to those models where the pion is excluded from the interior of the static bag. In addition, the renormalisation of g_A is identical to that of the $NN\pi$ coupling constant. But we showed in Section 6.2.1 that the large size of the nucleon bag, plus the presence of the Δ, mean that this renormalisation is 10% or less for a bag radius of 0.8 fm or larger—consistent with nucleon electromagnetic properties. In summary, the successful prediction of g_A in the MIT bag model (see Section 3.3) is preserved by the CBM.

6.2.5. *Proton decay*

There has been a great deal of excitement in the last couple of years since it was realized that the beautiful ideas of grand unification (PS 73, GG 74, Bur+ 78) may actually lead to the decay of the proton at an observable rate (Wei 79, WZ 79, Lan 81). From the practical point of view of our experimental colleagues the interesting question is what are the dominant decay modes—i.e. to what should a detector be sensitive. For someone with a classical nuclear physics background the idea of $p \to e^+\pi^0$, $e^+\rho^0$, $e^+\omega$ being the dominant processes (KK 80, Gav+ 81, Lan 81) seems absurd! For example, I had always believed that the point-like nucleon of nuclear text books must carry a large number of virtual pions with it—just as predicted by the Chew-Wick static meson theory mentioned earlier (see Table 6.1). If that were the case, even in the "unlikely" event that the small core contains three quarks which convert to $e^+\pi^0$ [Fig. 6.9(a)] all those pions in the cloud would be observed too. Thus because of phase space the only decay mode would be e^+ with many pions.

Because of its remarkable convergence properties the CBM provides a rather beautiful resolution of this difficulty. Most of the time the proton consists of a three-quark bag for which the usual calculations apply. However, there is also a chance of about one in three that the physical nucleon consists of a pion in the air with a three-quark core. The latter, being off-shell, can decay directly to e^+ as shown in Fig. 6.9(b). The probability of finding more than one pion in the cloud is negligibly small as we showed above. Calculations of the process in Fig. 6.9(b) have been made on the basis of current algebra (Tom 81) and chiral SU(3) × SU(3) (Wis+ 81, Cla+ 81). However, the question of proton structure was not addressed in either of these approaches.

McKellar and Thomas recently carried out a calculation motivated by the CBM (MT 82a). The pole graph [Fig. 6.9(b)] enhances the matrix element by a model-dependent factor of 3 to 6, and hence decreases the proton lifetime by at least an order of magnitude. [Both Tomozawa and Claudson *et al.* found a model-independent enhancement like $(1+g_A)$.] Thus within the SU(5) model of grand unification, considerations of chiral symmetry seem to imply both that $e^+\pi^0$ should be the dominant mode of decay, and that for a unification mass of order 4×10^{14} GeV the proton lifetime is about 3×10^{29} years (MT 82). The deep mine physicists can live in hope of seeing daylight soon!

6.3. Pion Nucleon Scattering

6.3.1. *The P_{33} resonance*

We recall from Section 3 that the Δ played a role as important as the nucleon in fixing the parameters of the MIT bag model. Indeed the colour coupling constant α_c was essentially determined from the hyperfine splitting of N and Δ. Once the constraint of chiral symmetry is imposed on the bag model, leading to the Hamiltonian given in Eq. (6.34), there is a qualitative change in the interpretation of the Δ. Whereas N, Δ, R and so on are eigenstates of H_{MIT}, once the pionic coupling is turned on only N (actually $\tilde{N}$ in our earlier notation) remains as an eigenstate of the full H. (Of course the other members of the nucleon octet should also remain stable under strong interactions.) The Δ is sufficiently high in mass that it can decay into Nπ, and can therefore at best be regarded as an approximate eigenstate of the full Hamiltonian with complex eigenvalue (FP 58, GK 57). In this case it seems most appropriate to discuss directly the predictions of the CBM for πN scattering in the P_{33} channel.

When the first crude calculation of pion nucleon scattering was made in the original Brown-Rho bag model (Mil+ 80) there was considerable concern in the medium energy community about double counting. That is, the old Chew-Wick meson theory, which involves just an NNπ vertex function can generate a resonance in the P_{33} channel (Che 54, Wic 55). The reason is that the crossed Born graph (u-channel nucleon pole) shown in Fig. 6.10(a), produces a strongly attractive, effective potential in the (3,3) channel

$$v_C(\underline{k}',\underline{k};w) = 4\pi P_{33}\left(-\frac{4}{3}\,\frac{f^2_{NN\pi}}{m^2_\pi}\,\frac{k'k\;v(k')v(k)}{(2w_{k'}2w_k)^{1/2}}\,\frac{1}{w_{k'}+w_k-w}\right) \qquad (6.65)$$

where P_{33} is the usual projection operator onto the isospin -3/2, spin -3/2 πN channel (Sch 64), and v(k) provides the high momentum cut-off. When iterated (as in Fig. 6.10(b) and so on), this potential produces a good description of the P_{33} scattering phase shifts up to 300 MeV—with a suitable choice of cut-off—e.g. $v(k) \approx \theta(m_N-k)$. Such a model of the P_{33} resonance is still widely used in the medium energy physics literature (typified by Physical Review C—e.g. Ban 79, Mil 79, EJ 80) and is often (somewhat incorrectly) referred to as the Chew-Low model.

The apparent problem with the CBM is that it naturally incorporates *both* this crossed graph and a direct coupling to the delta bag [Fig. 6.10(c)]—because both $NN\pi$ and $\Delta N\pi$ couplings occur on the same footing. One might ask whether there is not some double counting, or perhaps even two Δ resonances! The answer is simply that there is no double counting and the pion nucleon t-matrix defined by the CBM satisfies the Low equation (Low 55) as it should (Thé+ 80). Both the Chew-Wick and direct-Δ mechanisms contribute to πN scattering in the (3,3) channel (and interfere with each other) with a relative strength dictated directly by the CBM Hamiltonian, as illustrated in Fig. 6.10. One is no longer free to arbitrarily adjust the $NN\pi$ vertex function so that the Chew-Wick mechanism produces a resonance by itself, because the same vertex function occurs at the $\Delta N\pi$ vertex (see Section 6.1.2).

To summarise, far from raising problems of double counting, the CBM provides an explicit and physically well motivated example of an alternate solution to the (non-linear) Low equation, as discussed by Castillejo, Dalitz and Dyson (Cas+ 56). Moreover, it provides a precise answer to the rather confused question I asked Gerry Miller at the Houston meeting some three years ago (Mil 79):

"While the Chew-Low model is a useful model of the P_{33} resonance, it is very dated. Since then we have discovered ... quarks etc. In that model there is unambiguously an elementary $\Delta \equiv (qqq)$ state. ... Is it not possible that the truth about the πN interaction is that the elementary Δ contributes a short-range piece, while the πN rescattering ... results in a relatively long range piece of the interaction? On a more philosophical level, why must physics be split into two non-overlapping camps ...?"

The treatment of πN scattering in the CBM therefore involves solving the scattering equation

$$t = (v_C + v_\Delta) + (v_C + v_\Delta) G_0 \, t \; . \tag{6.66}$$

Here v_Δ is given by Fig. 6.10(c),

$$v_\Delta(\underline{k}',\underline{k};w) = 4\pi P_{33}\left(\frac{f^{(o)2}_{\Delta N\pi}}{3m_\pi^2} \frac{k'k\; u(k')u(k)}{(2w_{k'}2w_k)^{1/2}} \, S_\Delta^{(o)}(w)\right) , \tag{6.67}$$

where

$$S_\Delta^{(o)}(w) = \left(w - (m_\Delta^{bag} - m_N) - \Sigma_\Delta^{h.o.}(w)\right)^{-1} , \tag{6.68}$$

and $\Sigma_\Delta^{h.o.}$ is the sum of all the irreducible pionic self-energy contributions for the Δ, which do not involve an intermediate $N\pi$ state. The Chew-Wick driving term v_C is identical to that given in Eq. (6.64), except that the CBM form-factor $u(k)$ replaces $v(k)$. Considerable numerical simplification is obtained by approximating the propagator of the crossed Born graph as

$$(w - w_k - w_{k'})^{-1} \simeq -\frac{w}{w_k w_{k'}} , \tag{6.69}$$

which has been shown by Miller and Henley to be good to ~15% in the usual Chew-Wick theory (MH 80). In that case we find

$$v_C(\underline{k}',\underline{k};w) \simeq 4\pi P_{33}\left(-\frac{4}{3} \frac{f^{(o)2}_{NN\pi}}{m_\pi^2} \frac{k'k\; u(k')u(k)}{(2w_{k'}2w_k)^{1/2}} \frac{w}{w_{k'}w_k}\right) \tag{6.70}$$

and both v_C and v_Δ are separable. Then the solution to Eq. (6.66) can be written down in closed form (Thé+ 80).

Another advantage of the analytic form for the pion-nucleon t-matrix is that one can very easily see what is involved in the renormalisation process. In fact one can explicitly show that the bare coupling constants in Eqs. (6.67) and (6.70) are replaced by their renormalised values and the bare nucleon and delta masses get dressed by pionic interactions—as illustrated (in lowest order at least) in Fig. 6.3. The one free parameter of the model is the bag radius which can be adjusted to fit the P_{33} scattering data. While the best fit is obtained with R = 0.82 fm (Tho+ 81), any bag radius between 0.7 and 1.1 fm provides a fairly good description (Thé 82).

Of course the model we have described is fairly crude. The $B'B\pi$ vertices have all been calculated for a static bag. Nucleon kinetic energies have been neglected in all propagators and so on. It would certainly be worthwhile to repeat this work using (say) the Blankenbecler-Sugar equation, with improved vertex functions. In that case one might be able to pin down the bag radius somewhat more reliably. However, the essential physics, which is the participation of a relatively large composite Δ on the same footing as the nucleon will not be altered.

From the point of view of the bag model it is very interesting to ask whether the pionic self-energy corrections affect the Δ-N mass splitting. To lowest order in the pion coupling (which should be a rather good approximation for large bag radii*) the self-energy loops shown in Fig. 6.3 give rise to the following corrections

$$\Sigma_N(E) = \frac{3f^2_{NN\pi}}{\pi m^2_\pi}\int_0^\infty \frac{k^4\ u^2(k)\ dk}{w_k(E-w_k-m_N)} + \frac{4}{3}\,\frac{f^2_{\Delta N\pi}}{\pi m^2_\pi}\int_0^\infty \frac{k^4\ u^2(k)\ dk}{w_k(E-w_k-m_\Delta)}\ , \tag{6.71}$$

(which was called $\delta m_N^{(2)}$ earlier) and

$$\Sigma_\Delta(E) = \frac{f^2_{\Delta N\pi}}{3\pi m^2_\pi}\int_0^\infty \frac{k^4\ u^2(k)\ dk}{w_k(E-w_k-m_N)} + \frac{75}{16}\,\frac{f^2_{\Delta\Delta\pi}}{\pi m^2_\pi}\int_0^\infty \frac{k^4\ u^2(k)\ dk}{w_k(E-w_k-m_\Delta)}\ , \tag{6.72}$$

*See, however, the recent discussion of Hoodbhoy (Hoo 82).

where the "physical" N and Δ masses are defined by

$$m_N = m_N^{bag} + \Sigma_N(m_N) \ ,$$

$$m_\Delta = m_\Delta^{bag} + \mathrm{Re}\Sigma_\Delta(m_\Delta) \ . \tag{6.73}$$

Whenever the energy denominators in Eqs. (6.71) and (6.72) can vanish the self-energy becomes complex (corresponding to the width of the Δ for example), and the real part is given by the principal value prescription. The difference $(m_\Delta - m_N)$ was used as a fitting parameter in the CBM work (because the interference with Chew-Wick terms could shift the resonance position), but in fact the best fit value of 280 MeV is very close to the value one would naively extract from the particle data book (1231-940 = 291 MeV).

Recalling the CBM relationships between coupling constants from Eq. (6.32) we see that the first term in Eq. (6.71) at $E = m_N$ equals the second term in Eq. (6.72) at $E = m_\Delta$. On the other hand the $N\pi$ contribution to the Δ self-energy and the $\Delta\pi$ effect on N can only be compared numerically because of the principal value in the former. For the parameters of Thomas *et al.* (i.e. R = 0.82 fm, $m_\Delta - m_N$ = 280 MeV—Tho+ 81) $\Sigma_\Delta(m_\Delta)$ is actually 80 MeV less attractive than $\Sigma_N(m_N)$. Consequently the QCD splitting of the N and Δ bag masses is only 200 MeV. Since the hyperfine splitting due to one gluon exchange goes as 1/R [Eq. (2.83)] this means one does not need anywhere near as large a value of α_c as in the original MIT work. Indeed α_c of order 0.3 to 0.4 (rather than 0.55—as in DeG+ 75) is sufficient (Thé+ 82). This is much more consistent with the idea of treating gluon exchange in the bag in low order perturbation theory.

Very similar conclusions regarding the N-Δ mass splitting have been reached by Lichtenberg and Wills on the basis of a non-relativistic quark model (LW 81). They also treated the strong coupling of the ρ-meson to two pions in a coupled channels formalism. Once again the effect of the channel coupling was to reduce the splitting between π and ρ required from one gluon exchange. If, as we strongly suspect, the same result were to hold in a bag model description this would also be consistent with a smaller value of α_c.

In concluding this discussion we note that there is a considerable amount of loose discussion about the delta. For example, it is often claimed that the quark model ΔNπ coupling constant (i.e. $f_{\Delta N\pi} = (72/25)^{1/2} f_{NN\pi}$) is not sufficient to explain the width of the Δ. That is, the δ-function piece of Eq. (6.72) contributes only about 80 rather than 110 MeV to the width of the P_{33} resonance. However, it should be clear from our discussion of the CBM that this is not the only contribution to the width. For example, the intermediate pion in Fig. 6.10(b) or 6.10(e) can also be on-shell. Niskanen has given a rather nice summary of this recently (Nis 81). It is quite possible that the solution to the problem of the difference between predicted (Tho+ 81) and extracted (Arn+ 79) values of the ΔΔπ coupling constant raised recently by Duck and Umland (DU 82), may also be related to the subtlety of the structure of the P_{33} resonance. But in any case this problem deserves more work.

It may also be a source of confusion to some readers that processes such as Fig. 6.10(e), (f) etc., which appear naturally when Eq. (6.66) is iterated, are not simply incorporated into a renormalised ΔNπ coupling constant. The answer is unitarity! That is, above the Nπ threshold such terms contribute an imaginary part to the πN scattering amplitude. Any theory which seriously expects to explain the width of the Δ must include them explicitly! A similar observation must also be made about the magnetic moment of the Δ. The photon can couple to any of the intermediate pion legs in Fig. 6.10, just as we explained for the nucleon in Section 6.2.3. For the reasons we have just outlined the effective magnetic moment of an on-shell Δ will necessarily be complex. It is absolutely pointless to expect to test so-called "quark models" of the Δ magnetic moment without incorporating pionic effects (e.g. see the rather simple model of Moniz (Mon 82), which could easily be extended along CBM lines).

We might also make some brief remarks concerning the behaviour of the Δ in dense nuclear matter. For example, it is commonly believed (BP 75, CL 78, BP 79) that the Δ^- should be an important component of nuclear matter at the core of a neutron star. It is very easy to see that imbedding a Δ in nuclear matter would severely inhibit the self-energy contribution involving an intermediate Nπ state (Saw 72, Tho+ 80). Since this term is of order 160 MeV for R = 0.8 fm this can obviously be a large effect! Of course, the tendency to raise the mass of the Δ may be counteracted by the interaction with other nucleons in the medium. It is not even clear that one can simply Pauli-block the intermediate nucleon once its quark structure is being considered and the density is high. In short we shall have to develop a many-body theory of confined quarks and pions—at least! This will be discussed a little more in Section 7. For the present we merely note that the internal structure of the isobar (and the nucleon) may significantly modify our predictions for dense nuclear matter (Tho+ 80, Dre+ 82).

6.3.2. *Other partial waves*

One of the attractive features of the Chew-Low model was that it not only explained the resonant behaviour of the P_{33} interaction but that it also explained (qualitatively at least) the behaviour of the other P-wave πN phase shifts at low energy. It is therefore not unreasonable to ask that any theory which purports to replace Chew-Low should do as well. For the small repulsive P_{13} and P_{31} phase shifts this has been established by Israilov and Musakhanov (IM 81).

The P_{11} is rather more interesting for a number of reasons. This channel contains the nucleon pole, as a result of which the low energy phase shifts are negative. However, at about 150 MeV the phase shift changes sign and rises rapidly through 90° at the highly inelastic Roper resonance (at 520 MeV). Within the MIT bag model we expect that the Roper (R) should be predominantly a $(1s^2,2s)$ configuration— although as mentioned in Section 2.4 the MIT bag model is not overwhelmingly successful for excited hadrons. Just like the Δ, the R is stable in the absence of pion coupling. Once the full Hamiltonian is used R will of course move into the complex plane, obtaining its width predominantly from the coupling to $N\pi$ and $\Delta\pi$. Although the Roper necessarily involves higher energies, which means that the neglect of recoil corrections (and the difference in R and N bag radii) will be more drastic than for the Δ, Rinat has shown that the CBM can provide quite a good qualitative description of the P_{11} data (Rin 81). As we have stressed several times the development of the CBM description of this channel will be crucial in the rather ambitious attempts to develop a microscopic understanding of the prototype π-nucleus system, namely the pion deuteron system including absorption (Bet+ 82, Tho 82).

These results combined with the excellent fits to the P_{33} phase shifts and the derivation of the Weinberg-Tomozawa relationship in s-wave mean that the overall description of low energy πN scattering is in rather good shape.

6.4. Magnetic Moments of the Nucleon Octet

Looked at objectively there is not a great deal of data at our disposal for testing models of hadron structure. One important data set which has seen a dramatic improvement in quality recently, as a result of improved hyperon beams, are the magnetic moments of the stable hyperons (Ove 81, Lip 81). In view of the success of the CBM with the nucleon magnetic moments described above it is reasonable to ask what its predictions might be for the strange partners of the nucleon. This is even more critical in view of the findings of Brown and co-workers that the Σ^- moment was in the range $(-0.54, -0.64)\mu_N$ (Bro+ 80), in comparison with the experimental value of $-1.41 \pm 0.25\ \mu_N$ (Rob+ 79, Han+ 78)—see also the discussion of Franklin (Fra 80) and Lipkin (Lip 81).

It is a rather beautiful feature of the CBM Hamiltonian that there is very little freedom in the calculation of these magnetic moments. Equation (6.17) can be used to relate all of the $B'B\pi$ coupling constants to that for $NN\pi$. The results are summarised in the paper of Théberge and Thomas (TT 82). Furthermore, once the strange quark mass is chosen—see Section 2.2.3—the photon coupling to the bag is determined (Section 3.2). The calculation involves exactly the same diagrams as that for the nucleon except that the intermediate bag states (while the pion is in the air—see Fig. 6.7) must have the correct strangeness—e.g. for the Σ^- we can have intermediate Λ, Σ, Σ^*, (Λ,Σ), (Λ,Σ^*) and (Σ,Σ^*)

baryons. (Such terms were first discussed by Pilkuhn and Eeg from a different point of view, with quite different numerical results — EP 78.)

The results of a calculation using the same bag radii and strange quark mass as the original MIT work are shown in Table 6.2 (TT 82a). Clearly the overall agreement of the CBM with data is excellent. A more detailed study of the dependence on bag radius and strange quark mass has confirmed that this is no accident (TT 82b).

In view of the theoretical uncertainties associated with configuration mixing (IG 80), sea quarks (DG 77, MV 81) and centre of mass corrections it appears unlikely that a more accurate description of the data is likely in the near future.* Nevertheless it does seem that the inclusion of the lowest order pionic corrections does result in a good overall description. Clearly a definitive experimental result for both the Σ^- and Ξ^- would be most welcome.

6.5. Summary

Our considerations of chiral symmetry and the MIT bag model have led us to a remarkably optimistic new theory of strong interactions. There is hope that, once the non-perturbative region of QCD is understood and quarks are confined in bag-like objects, the conventional strong interactions may converge in low order perturbation theory.

To illustrate this we discussed the renormalisation properties of the CBM Hamiltonian in detail. It is a remarkable fact that in every

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*One might also consider generalizing the CBM to $SU(3)_L \times SU(3)_R$ and including a kaon-cloud. We chose not to do so because the large mass of the kaon means there is no longer such a clean separation between the phenomenology of the bag and the mesonic corrections — see the introduction to Section 7.

case where the CBM has been applied, it has either led to better agreement with data than the original MIT model, or in other cases provided new insight to old problems. The results that we have described strongly support our belief that the CBM is an excellent model on which to begin to build a new, unified description of nuclear, medium-energy and high-energy physics.

Table 6.1

Bounds on the pion content of the dressed nucleon (from Dod+ 81).

	CBM[a)]	CBM[b)]	Chew-Wick[c)]
$f^{(o)\,2}_{NN\pi}$	0.078	0.096	0.22
R	0.82	1.0	0.28
Λ	0.9	0.68	2.16
$P_1 \leqslant$	0.9	0.68	2.16
$P_2 \leqslant$	0.40	0.23	2.33
$P_3 \leqslant$	0.12	0.05	1.67
$P_4 \leqslant$	0.03	0.009	0.90
$\langle n \rangle \leqslant$	0.90	0.68	2.16

a) Using parameters of Tho+ 81.

b) Using MIT radius and value of $f^{(o)}_{NN\pi}$ necessary to reproduce the observed renormalised coupling constant $f^2_{NN\pi} = 0.081$ — from Thé+ 82.

c) Bare coupling constant and sharp cut-off from HT 62.

Table 6.2

Comparison of the predictions of the CBM for the magnetic moments (in nuclear magnetons) of the nucleon octet—from TT 82a.

	CBM	Experiment
P	2.60	2.79
n	-2.01	-1.91
Λ	-0.58	-0.61
Σ^-	-1.08	-1.41 ± 0.25 -0.89 ± 0.14[a]
Σ^+	2.34	2.33 ± 0.13
Ξ^-	-0.51	-0.69 ± 0.04
Ξ^0	-1.27	-1.25

[a]Preliminary result from T. Devlins, private communication (Dec, 1981).

Figure Captions

Fig. 6.1. The form factor in the CBM compared with a best-fit Gaussian—from TT 82.

Fig. 6.2. Pion-baryon couplings which appear naturally in the CBM Hamiltonian.

Fig. 6.3. Lowest order nucleon and delta self-energy corrections.

Fig. 6.4. Lowest order contributions to the vertex renormalisation of the $NN\pi$ coupling constant in the CBM.

Fig. 6.5. Radius dependence of the bare $NN\pi$ coupling constant necessary to reproduce the observed, renormalised coupling constant $f^2=0.081$.

Fig. 6.6. The neutron charge distribution $4\pi r^2 j_n^0(r)$ versus the radial distance r (shaded area). Also shown are the quark (Q) and the pion (π) charge distribution inside the neutron. The neutron charge radius is set at one fermi (Thé 82).

Fig. 6.7. Contribution to the magnetic moment of the nucleon from a) the quark current, b) and c) the pion current with an intermediate nucleon or delta.

Fig. 6.8. Illustration of the pionic contribution to the proton magnetic moment with an intermediate nucleon or delta.

Fig. 6.9. a) The conventional mechanism for proton decay to $e^+\pi^0$; b) the pion pole term which dominates in the CBM.

Fig. 6.10. Some low order contributions to πN scattering in the CBM—from Thé+ 80.

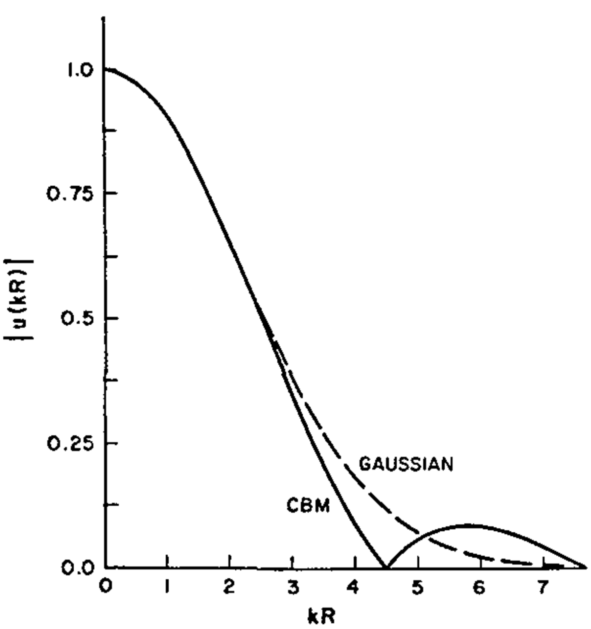


Fig. 6.1

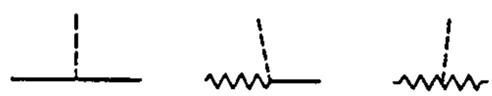


Fig. 6.2

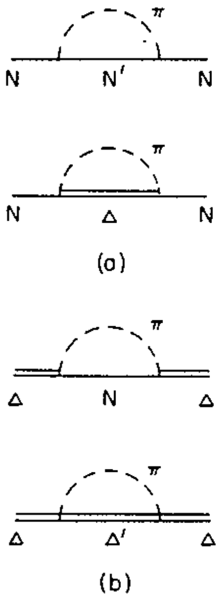


Fig. 6.3

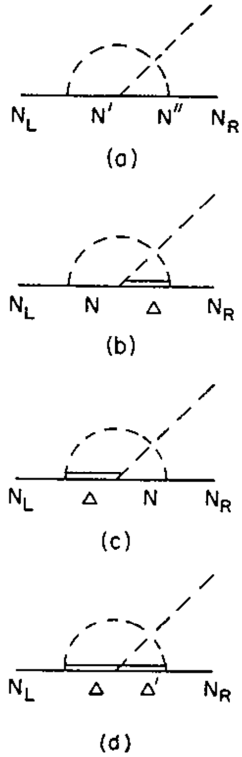


Fig. 6.4

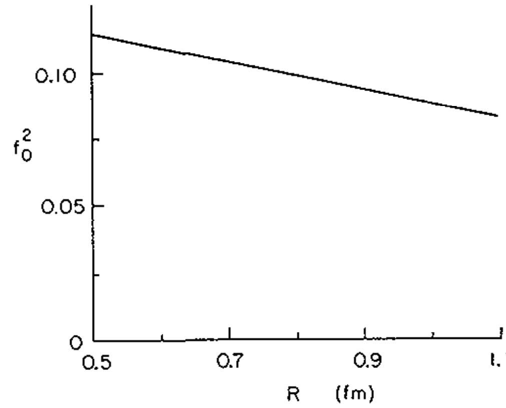


Fig. 6.5

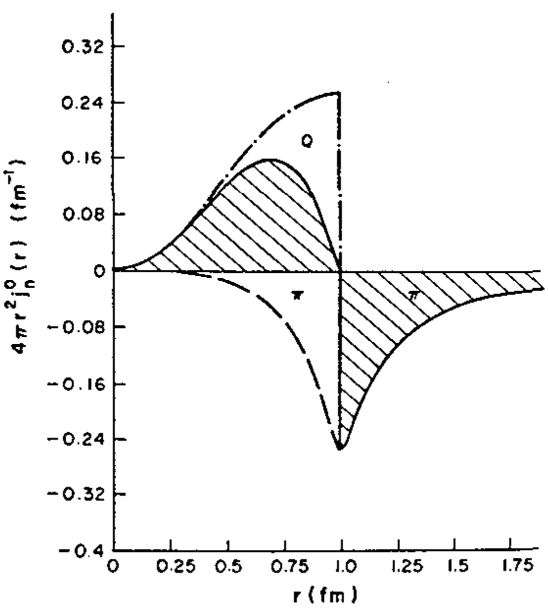
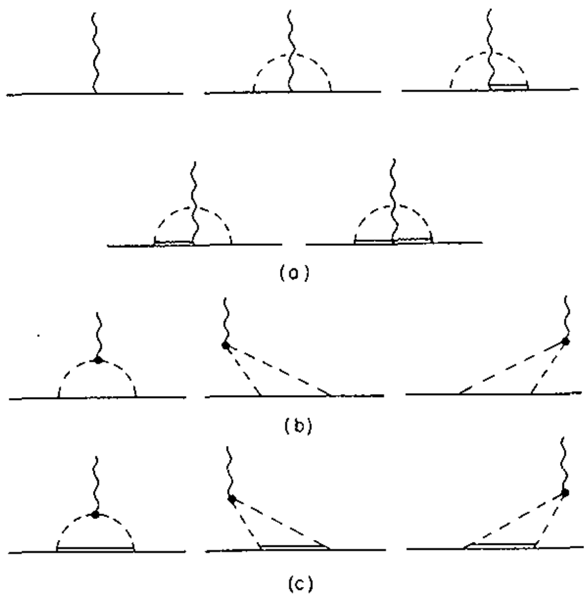


Fig. 6.6



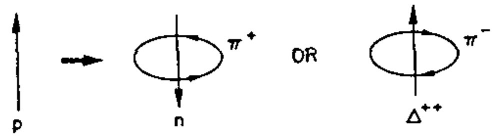


Fig. 6.8

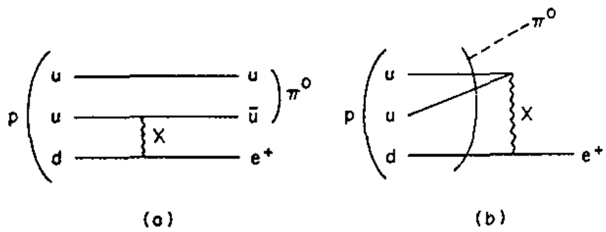


Fig. 6.9

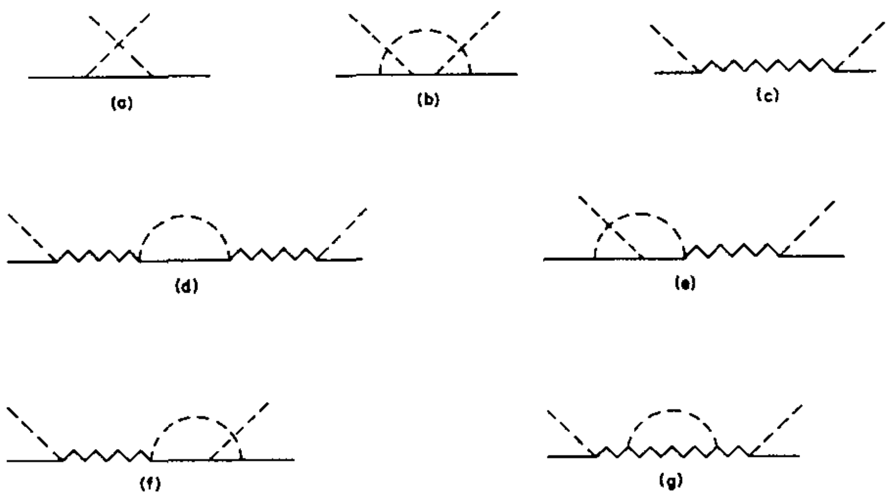


Fig. 6.10

7. TOWARDS A NEW VIEW OF NUCLEAR PHYSICS

In the preceding sections we have attempted to put together a thorough, and as far as possible, objective review of bag models, chiral symmetry and the applications to single hadron properties. This task was made relatively easy by the fact that the successes described in Section 6 are the culmination of many years of theoretical effort. On the other hand, there have been only a few tentative steps made towards our ultimate goal of defining a consistent, unified picture of nuclear and particle physics. It is our aim in this section to present a blatantly optimistic view of how this search may go. If we achieve nothing more than generating an interest in the nuclear community in tackling some old problems in a new framework this review will have succeeded.

In view of the successes of chiral bag models it seems a natural next step to attempt to derive the properties of many-nucleon systems from the same starting point. That is, each nucleon should be treated as a relatively large quark bag with a rather thin pion cloud. In contrast with the conventional models of the N-N interaction we see little room or necessity for vector mesons. To explain this consider the early 60's picture of the nucleon anomalous moment. Basically this was interpreted in a vector dominance model as the photon coupling to a ρ-meson which is then absorbed by the nucleon through the interaction

$$\mathcal{L}_{\rho NN} = g_{\rho NN}\left(\frac{1+K_V}{2M}\right) \psi^\dagger (\underline{\sigma}\times\underline{q})\cdot\underline{\rho}\ \tau_3\ \psi\ , \tag{7.1}$$

where K_V=3.7 is the isovector nucleon anomalous moment ($\mu_p-\mu_n-1$). In Eq. (7.1) we have shown only the non-relativistic limit of the anomalous coupling ($\rho_\mu\sigma^{\mu\nu}q_\nu$). If the direct vector coupling ($\rho_\mu\gamma^\mu$) to both the

ρ and ω is also included one has at least a qualitative explanation of the neutron and proton charge distributions too (Hof 63).

By the mid 60's one had an alternate explanation of μ_p/μ_n based on the static quark model (Beg+ 64). The 70's saw the refinement of the harmonic oscillator quark model calculations — still non-relativistic, but "QCD motivated". Also in the 70's came the bag models, which predicted μ_p/μ_n correctly *without vector mesons* and furthermore (Section 2.5) explained why the non-relativistic quark models worked. Most recently we have seen the development of chiral bag models, and particularly the CBM, which improved the overall agreement with data for the nucleon octet without altering any of the earlier successes. Once again there was no need for vector meson contributions.

In the triplet state the $\bar{q}q$ interaction associated with one-gluon exchange is strongly repulsive. Thus, unlike the pion, it is quite likely that the vector mesons are large ($R \sim 1.0$ fm in the MIT bag model — DeG+ 75). Their large mass implies that virtual vector mesons should have ranges of a few tenths of a fermi about the bag. Since the sharp bag surface in the MIT model is in any case a phenomenological simplification, it seems to make little physical sense to talk of virtual vector mesons (with a propagator like $(q^2-m_\rho^2)^{-1}$) about the bag. It would be more physically reasonable to treat such terms as virtual $\bar{q}q$ excitations in the nucleon bag — i.e. "sea quarks". Finally we might observe that even if one agrees to include vector mesons as a working hypothesis, the ρ (for example) could only couple through two pions, and therefore with a soft form-factor. In that case its effects would be very small (Nis 81, AT 82).

The moral of all this is that quite different theoretical pictures can often reproduce a limited data set. One's preference for a particular model must be determined not just by convenience but also by the range of phenomena with which it is consistent. We saw in Section 2.3 that the MIT bag model embodies by construction the concept of asymptotic freedom, suggested by deep inelastic scattering, as well as confining the quarks and gluons. It is consistent with how we believe the solution of QCD should look. Supplemented with a pion field it also incorporates the SU(2) × SU(2) symmetry of QCD. A mechanism has been suggested by which the pion could develop from QCD as a Goldstone boson associated with dynamical symmetry breaking. In short the chiral bag models are consistent with a great deal of data ranging from high energy electron and neutrino scattering down to static properties like magnetic moments. They also match our theoretical prejudices. In the form described in Section 6, namely the CBM, it is quite straightforward to make calculations.

For all these reasons it seems to us absolutely compelling that we begin the long job of replacing the old meson exchange picture by one in which the internal structure of the nucleon is taken seriously. Naturally for several years it will not be possible to duplicate the quality of fits achieved over more than 20 years work, by hundreds of theorists, culminating in the Paris potential (Vin 82). Nevertheless the rewards in the long term will be great. For example, one might hope for a new and deeper understanding of nuclear matter and phenomena like pion condensation associated with high density.

7.1. The Nucleon-Nucleon Force

Attempts to understand the nucleon-nucleon force have probably occupied more man-years of effort than almost any other single scientific problem—except perhaps the creation of better weapons. Through the application of sophisticated techniques relying on analytic properties of scattering amplitudes, the Paris group has arrived at a remarkably accurate description of the N-N force in free space (Vin 82). The claim is often made that the N-N potential is known to distances of order 0.8 fm on the basis of such calculations. There are some fascinating questions connected with the analytic behaviour of wave functions and scattering amplitudes in a theory with confinement (Wol 82). Eventually one would hope to put together the concepts of QCD and dispersion relations. However, for the present we simply note that there are conceptual problems to be overcome. In particular, in a collision of two bags of radius one fermi it would appear self-evident that quark degrees of freedom could be significant inside two fermis.

A number of attempts have already been made to derive a N-N interaction from quark models. From the introductory discussion to Section 7 it should be clear that rederivations of heavy boson exchange on the basis of quark interchange (Web 80, Web 81) do not seem realistic This does not mean that one will not have *effective* isovector-vector, isoscalar-vector exchanges and so on when two bags overlap. It simply means that (as for the anomalous magnetic moments of the nucleons) these are better treated directly in terms of quarks. Consequently our discussion will centre on work like that of DeTar (DeT 78-80) and also of Harvey (Har 81).

There is already enough excellent work on the short range N-N force in quark models that a full review of that alone would not be out of place. Our purpose in this section is merely to outline briefly that work which we find most promising. Unfortunately this discussion can not be considered complete.

7.1.1. *The short range force in a bag model*

The pioneering work in the application of the MIT bag model to the N-N force is that of DeTar (DeT 78-80). Although much of his work was very sophisticated, involving calculations in a deformed bag, in fact a major finding was that the deformation made little difference. One can understand his essential results on the basis of a spherical bag approximation.

Briefly then it is supposed that once two nucleon bags overlap sufficiently they coalesce to form a 6-quark bag.* Although it is no longer correct to think of the quark clusters in such a bag as nucleons, DeTar was nevertheless able to calculate the total energy of the system as a function of the separation between the clusters. The difference between the total energy of the 6-quark system and two nucleon masses was compared with conventional N-N potentials. While there is no rigorous justification for comparing this energy with conventional N-N potentials, in fact there are many similarities. In particular there is a repulsive core of about 300 MeV, which arises from the colour-magnetic one-gluon exchange interaction.

As mentioned above it would divert us too much to review DeTar's work in detail. Instead let us sketch how the calculation would

*The model of DeTar says nothing about the N-N force outside this coalescence radius.

proceed in the spherical approximation. That is, when the six quarks are in the same bag it is assumed to be spherical. (In practice this gives fairly reliable answers.) Then the left and right clusters have wave functions

$$q_L = q_S - \sqrt{\mu}\, q_P \ ,$$

$$q_R = q_S + \sqrt{\mu}\, q_P \ , \qquad (7.2)$$

where q_S is the $1s_{1/2}$ and q_P (an odd function of z) the $1p_{3/2}$ state in the same large spherical bag. (From our discussion in Section 2 we recall that a state with $j \neq 1/2$ can satisfy the *n.l.b.c.* only in an angle-averaged sense.) The parameter $\mu\ \varepsilon(0,1)$ determines the average separation of the left and right clusters. It serves as a variational parameter in the sense we now describe.

For given μ one can calculate the parameter δ,

$$\delta = \frac{2\mu^{1/2}(1+\mu)}{1+\mu^2} \int q_S^+(\underline{x})\ q_P(\underline{x})\ z\ d^3x \ , \qquad (7.3)$$

which corresponds to the internucleon distance at large separations, and in any case serves as a measure of the cluster separation. Given some value of the Lagrange multiplier C, and a separation δ_0 of interest, one can evaluate

$$H(C,\delta_0;\mu) = \langle H_{MIT} + C(\delta-\delta_0)\rangle \ . \qquad (7.4)$$

For fixed C and δ_0 one can minimise Eq. (7.4) as a function of μ. By varying C, $\delta(\mu)$ at the minimum can be made equal to δ_0. The expectation value of the MIT Hamiltonian at this constrained minimum is called $E(\delta_0)$. By repeating the whole process for a new δ_0 one can actually map out the function $E(\delta_0)$. Note that this calculation is complicated by the fact that for each μ, $\langle H_{MIT}\rangle$ can only be evaluated subject to the *n.l.b.c.* — so that R can also vary with δ_0 in principle. Fortunately the radius of the 6-quark bag is essentially independent of δ_0 — i.e. about 1.3 fm.

In Fig. 7.1 we show the value of $E(\delta)$ calculated by DeTar in a number of spin-isospin channels. The repulsive core which we mentioned earlier is clear. However, so is the very strong attraction at slightly larger separations. The latter seems to be the result of a cancellation that doesn't quite happen. A slight reduction of α_c from 0.55 to 0.36 (consistent with the CBM description of the Δ-N mass splitting — see Section 6.3.1) essentially kills this attraction without significantly affecting the repulsive core (DeT 80).

Given the obvious qualitative similarities between Fig. 7.1 and conventional N-N potentials it is rather disappointing that not much more has been done. The next stage requires some dynamical scheme for bringing bags together, letting them coalesce and fission again. Unfortunately no realistic method of calculating this has yet been formulated. This is certainly a very important problem to resolve.

7.1.2. *Nucleon-nucleon force in the non-relativistic quark model*

In view of the success of the non-relativistic quark model (NRQM) in hadronic spectroscopy (Isg 80), it is quite natural to consider extending it to treat the scattering of two composite hadrons. Moreover, because the model is essentially non-relativistic, the standard nuclear technique for scattering of two clusters (resonating group method) can readily be applied. The group theory is a little more complicated by the extra colour degree of freedom, but these details have all been worked out by Harvey (Har 80, Har 81). For our present purposes it is sufficient to realize that once two composite nucleons overlap, it is not enough to consider just N-N configurations. There will also be a Δ-Δ component as well as a "hidden-colour" C-$\overline{C}$ configuration.

Harvey's first work (Har 81a, Har 81b), like that of DeTar in the bag model, involved simply calculating the total energy of the system as a function of the inter-cluster distance. As we mentioned in Section 7.1.1 there is no compelling reason for comparing this with phenomenological N-N potentials since the effective interaction in a quark model would be highly non-local. Nevertheless in DeTar's work this procedure did produce strong, short range repulsion, and it was therefore quite disturbing when Harvey found no such effect. Indeed the energy of his 6-quark system at zero separation (r=0) was very close to $2m_N$. The reason for this difference seems to be DeTar's insistence on having all six quarks in the $1s_{1/2}$ orbit (s^6) at r=0, whereas Harvey had quite a large s^4 p^2 (hidden colour) component. However, it must also be pointed out that the definition of "separation" in these two calculations is quite different. Whereas DeTar's definition actually means the separation between peaks in the matter distribution, Harvey's is the distance between the origins for two sets of basis functions. Thus, "zero separation" may not be the same in the two calculations (DeT 82).

Recent work by Arima and collaborators has suggested an explanation for this apparent discrepancy (Oka+ 81, OY 80). The essential problem was already discussed in Section 2. That is, the NRQM consists of a one gluon exchange potential and a recipe for restricting the space in which the diagonalisation is to be performed. Furthermore, only the baryon spectrum with respect to the nucleon is fitted—the nucleon mass itself is put in by hand. Arima *et al.* (Oka+ 81) used the quark cluster model of Oka and Yazaki (OY 80) with Harvey's Hamiltonian to confirm his results in the six-quark system. However, they found that if $2\hbar\omega$ excitations were included in a variational calculation of the nucleon mass itself, then the effective nucleon mass would be lowered by 540 MeV. In that case the six-quark system would again be appreciably heavier than $2m_N$ at r=0 — a net repulsion of 760 and 850 MeV in the (S,T) = (1,0) and (0,1) channels respectively. Clearly one needs to formulate an unambiguous truncation procedure that is equivalent in a system of three and six quarks.

A much more sophisticated program, which was begun recently by Harvey and LeTourneux (Har 81c)*, involves a direct solution of the Schrödinger equation (Lib 77, WS 80)

$$H\,\psi(\underline{x}) = E\,\psi(\underline{x}) \ , \tag{7.5}$$

where

$$H = \sum_i T_i + \sum_{i<j} \sum_{a=1}^{8} \lambda_i^a \lambda_j^a \, F(r_{ij}) \ , \tag{7.6}$$

is the NRQM Hamiltonian (Isg 80). The radial form of the potential is a harmonic oscillator

$$F(r) = Br^2 \ , \tag{7.7}$$

where B < 0 guarantees confinement for a colourless hadron [see Eq. (2.82)]. As usual, the solution $\psi(\underline{x})$ is constructed in terms of a set of anti-symmetrised cluster wave functions $\phi_\alpha(\underline{x},\underline{r})$ describing two 3-quark clusters separated by a distance $\underline{r}$,

$$\psi(\underline{x}) = \sum_\alpha \int d\underline{r} \ \phi_\alpha(\underline{x},\underline{r}) \ f_\alpha(\underline{r}) \ . \tag{7.8}$$

The solution of the Griffin-Hill-Wheeler equations (OL 80) for $f_\alpha(\underline{r})$ (with appropriate boundary conditions)

$$\int d\underline{r}\,[H_{\alpha'\alpha}(\underline{r}',\underline{r}) - E\,N_{\alpha'\alpha}(\underline{r}',\underline{r})]\,f_\alpha(\underline{r}) = 0 \ , \tag{7.9}$$

yields the N-N phase shifts. (The function $N_{\alpha'\alpha}$ is simply the overlap of two clusters located at $\underline{r}$ and $\underline{r}'$.)

With the addition of a long range pion-like interaction (for which there is no compelling theoretical argument in the NRQM!), Harvey was

*See also the very similar, recent work by Faessler and collaborators (Fae+ 82).

able to obtain quite a good qualitative fit to the $^3S_1$ N-N phase shifts. Significantly this fit reproduced the change of sign at about 250 MeV. Thus the model clearly does incorporate a repulsive short range interaction.

The major advantage of this approach is that one can directly follow the collision of two clusters without assumptions about the radial configuration at r=0 (e.g. s^6 only). There are, however, a number of fundamental objections to overcome. As observed by Greenberg and Lipkin the NRQM gives rise to unobserved, strong van der Waals forces between hadrons — in contradiction with experiment (GL 81). On a more technical level we have already recorded the ambiguity in restricting the harmonic oscillator space in which the diagonalisation should be carried out — in the 3- and 6-quark systems. Finally, the treatment of the quarks as non-relativistic is fundamental to this method. They necessarily have a mass of about 360 MeV — one third of the average N and Δ masses. Thus each cluster has a dynamical mass of 1080 MeV at all inter-cluster separations — unaffected by the dynamics. This is clearly a crude approximation, and as Harvey has observed could be removed only in a truly relativistic treatment. That is a major challenge for the future.

7.1.3. *The long range force*

Unlike the NRQM where unobserved van der Waals forces occur naturally at large distances, in the naive bag model there is no interaction at all for non-overlapping bags. Of course, the static spherical bag is an idealisation and in reality one would expect to deal with a finite surface thickness and surface fluctuations. However, it is probably reasonable to ignore this fuzziness in first order. Then the only mechanism for interaction in the region $r > 2R$ is pion exchange. For this the chiral bag models are ideally suited.

The first discussion of the long range N-N force generated by pion coupling at the bag surface was that of Gross (Gro 79). Following the first paper of Brown and Rho he considered the interaction between bags resulting from a linear combination of pseudoscalar (λ) and pseudovector ($1-\lambda$) pion coupling at the surface. He showed that this gave rise to an $NN\pi$ vertex function

$$\Gamma(q^2) \sim j_0(qR) + (3\lambda-2)j_2(qR) \ , \qquad (7.10)$$

which reduces to that of the CBM [see Eq. (6.21)] in the case $\lambda=1$. Moreover, he observed—as many others have done since—that *this form factor did not alter the radial dependence of the OPE force for* $r > 2R$, because $\Gamma(q^2)$ is an entire function of q^2.

If for the moment we suppose that the OPE interaction can be calculated using the interaction Hamiltonian (6.24), even when two bags overlap, then the CBM form-factor will cut down the OPE potential for $r < 2R$. It is interesting to see what evidence there is to support the existence of such a form-factor. Clearly the matter will be complicated by the tendency of ρ-meson like exchanges at short distance to also damp the OPE. Nevertheless, by using experimental data to construct the Fermi invariant amplitudes for N-N scattering (Gol+ 60, BJ 76), and taking the appropriate linear combination of amplitudes to isolate the isovector-pseudoscalar pole term, Gersten was able to pick out the one-pion-exchange contribution (Ger 81). The data are consistent with a form-factor of the CBM type with a radius between 0.65 and 1.0 fm—although it is only the initial slope that is determined.

In another attempt to see such effects, Gersten and Thomas (GT 81) looked for specific partial waves in which the first iterated OPE Born term was a good approximation to the two-pion-exchange box diagram—

namely $^3D_2$, ε_2, $^3G_3$ and $^3G_4$. (One can not consider L too high or else the form-factor has no effect at all.) Unfortunately the experimental determination of the $^3G_3$ and $^3G_4$ phase shifts is not good. But for both $^3D_2$ and ε_3 a bag radius $R \sim 0.8$ fm produces a good fit to the data.

However, the fundamental question in all this is what happens to the one and two pion exchange force when the two bags do overlap. More specifically, how much must the bags overlap before the "Cheshire bag approximation"* breaks down? It may well be that the answer to this question is quite a lot! From DeTar's work (Section 7.1.1) we know *that (with $\alpha_c \sim 0.36$) nothing very dramatic happens when two bags begin* to overlap. Moreover the NNπ coupling strength goes as $g_A/2f$, and g_A depends on the spin-isospin structure, *not* on the radial size of a hadron (or quark cluster). Finally, as we have argued, the pion is not excluded from the bag interior (although it may have a somewhat different mass there). Thus, even with an individual nucleon of radius (0.8-1.0) fm, it is conceivable that the usual OPE plus TPE potential is not too far wrong down to (1.0-1.3) fm. The challenge in the next years will be to turn qualitative statements like "not too far wrong" into a quantitative theory.

For the present, one attractive, phenomenological option is to extend the old Feshbach-Lomon boundary condition model (LF 68), to include NΔ and $\Delta\Delta$ (and perhaps even C-C) components outside the boundary radius R_o (Lom 81). Inside the boundary radius one would describe the system purely as six quarks (Hog+ 80, Kis 81, Mil 82). Naively one might identify

*The "Cheshire bag approximation" is a term coined by Fritz Coester to *describe the use of the CBM Hamiltonian even when two bags overlap.* Like Lewis Carroll's Cheshire cat, there is nothing to the bag except a "smile".

the boundary radius R_0 with the size of a 6-quark bag (i.e. about 20% bigger than the nucleon bag). For the backward electro-disintegration of the deuteron, Kisslinger has shown that the quarks can make an important contribution—particularly at high momentum transfer ($q^2 > 10\ fm^{-2}$). The elastic deuteron form-factor seems to scale as expected for a 6-quark bag at high momentum transfer and there has been a similar success for the deep inelastic structure function—with about a 6% admixture of the 6-quark component (BF 80). Even at very low energy, such as the circular polarization in thermal neutron capture (DO 81), it has been suggested that the quark contribution could be crucial.

7.1.4. *Nucleon-antinucleon scattering*

With the expectation of large quantities of high quality data from LEAR in the near future, there is a renewed interest in the $N\bar{N}$ system. Conventionally one obtains the $N\bar{N}$ potential from that for NN by G-parity. One simply changes the sign of the N-N-meson coupling constant for those mesons of odd G-parity (π, ω, etc.). Thus the strong, short range repulsion generated by ω-exchange in the N-N system becomes a very strongly attractive potential for $N\bar{N}$—which can support many bound states. Clearly in the case of large composite N and $\bar{N}$ even this feature of the $N\bar{N}$ interaction may be in doubt. However, our present interest is not with that problem, but rather with the major ambiguity of any potential model, namely the effect of annihilation. The annihilation in the $N\bar{N}$ system is in fact so strong that the deeply bound states mentioned above would be unobservably broad (MT 76). This unfortunate conclusion can only be avoided if for some reason, a) the annihilation potential is extremely short range, or b) strongly state dependent, or c) the optical model treatment is invalid.

It was noticed by Wilets and collaborators (Wil+ 81) that the bag model should yield a fairly definite idea of the shape of the annihilation potential. Before the bags overlap there is no annihilation at all. When the bags do overlap the process,

$$q\bar{q} \rightarrow \text{gluon} \tag{7.11}$$

becomes possible, and the remaining four quarks and gluon will arrange themselves into mesons. The probability for the process (7.11) obviously depends on the amount of overlap of the N and $\bar{N}$ bags. Thus, although a perturbative calculation based on (7.11) would not be expected to yield the correct magnitude of the annihilation process, one might expect the geometry to be well represented. Just as DeTar found nothing dramatic when two nucleon bags start to overlap, so Wilets *et al.* found little annihlation at $r=2R$. Most of the strength of the annihilation seems to occur in the region $r \in (0.5R,R)$. From their extensive analysis of the presently available $p\bar{p}$ scattering data Wilets *et al.* found a range of bag radius parameters between 0.7 and 1.0 fm, with the overall best fit 0.86 ± 0.06 fm. This is in excellent agreement with the radius expected in the CBM, as we discussed in Section 6. We can expect to hear much more about this problem in the next few years.

7.1.5. *Exotic states*

It is an unavoidable consequence of the bag model that not only will three-quark (3q) baryons exist, but in fact any colour singlet combination — 6q, 4q $\bar{q}$ etc. Were such states to be discovered as relatively long-lived identifiable particles, it would be a real triumph for QCD. Much theoretical effort has been devoted to calculating the spins, parities and masses of such states (Joh 75, Jaf 77, WL 78, Mul+ 79, Mul 80). Obviously it was very tempting to attribute the rapid energy dependence observed in $\Delta\sigma_L$ and $\Delta\sigma_T$ at the Argonne ZGS (Apr+ 80) to such a dibaryon resonance—certainly the energy regions coincided.

However, the dibaryon example reveals the essential problem of almost all exotics. The structure in the $^3F_3$ N-N channel coincides with the opening of the N-Δ p-wave, and the inclusion of this coupled channel alone can qualitatively reproduce the observed structure (Bet+ 82). In order to reach this conclusion one must perform rather complicated three-body calculations (involving two nucleons and a pion), which decently respect unitarity. The moral of the story is simply that when an exotic is connected with several open channels it can not be discussed in isolation. One rather simple attempt to deal with this is the P-matrix formalism of Jaffe and Low (JL 79). Using this, it has been suggested that indeed a number of B=0 and B=2 exotics would not be expected to produce dramatic effects in π-π and N-N phase shifts (Low 79). However, one would ideally like to see a consistent, unitary, coupled channels calculation. At least for those cases where pion production is significant (like the dibaryons) the CBM should provide the basis for such a treatment.

One very important exception is the doubly strange Λ-Λ bag, which is actually predicted to be bound by about 80 MeV (Jaf 77) and therefore to have no strong decay channels. The experimental observation of this state would be very exciting but it has not yet been seen (Car 78, Pau 82). One possible reason for its non-appearance is provided by the chiral bag models. For example, in the CBM the pionic self-energy contribution is of order -130 MeV for the Λ (Thé 82, TT 82). But the di-lambda would be some 30% larger (because of the *n.l.b.c.*). Because the pionic self-energy decreases like $R^{3\cdot 5}$ as R increases, one would naively expect the pion self-energy for the di-lambda to be cut in half. That alone would be enough to unbind the di-lambda and make it rather difficult to see. A more refined calculation of the pionic corrections to the exotics is presently underway (MT 82b).

In closing this very brief discussion of N-N forces we recall that in Sections 7.1.1 and 7.1.2 we reviewed two attempts to describe the short distance N-N force in terms of quarks—either in the NRQM or the bag model. However, at no time did we discuss corrections associated with chiral symmetry (because neither DeTar nor Harvey considered this). Nevertheless, for exactly the reasons we have just outlined for the di-lambda, the inclusion of pion self-energies will tend to provide some short range repulsion! This will be true for the CBM and bags of the MIT size, although a similar point was made by Vento *et al.* in the context of the little hedgehog (Ven+ 81, see Section 5.3.2).

7.2. Symmetry Breaking as a Clue

Ultimately one might hope to start from a microscopic model of the nucleon (including chiral symmetry) and derive a precision fit to N-N scattering data. But, as we hope is clear from the discussion in Section 7.1 such a precision fit is a long way off. Moreover, it would be stretching one's hopes too far to expect to convince unbelievers that a quark level description is necessary on the basis of even an excellent fit to N-N data alone. Nevertheless the situation is not as bad as it may first appear—there are more subtle avenues of attack.

We have come to hold symmetry principles rather dear in nuclear and particle physics, and violations of any fundamental symmetry are studied in great detail. It is not unreasonable to expect that the new view of nuclear physics proposed here should have something new to say about symmetry violation. It is conceivable that predictions of symmetry violation made in our present, crude models might survive the improvements necessary to obtain quantitative fits to nuclear data. We might even hope to find cases where the quark model suggests a new and beautifully simple explanation for a problem that has hitherto been a puzzle for conventional nuclear theory. In this section we briefly report on one example of each kind. Although these are the only ones of which we are aware at present, the reader is graciously invited to find more!

7.2.1. *Charge symmetry violation in OPE*

Whether or not a symmetry is fundamental depends, of course, on one's point of view. In a quark model it is quite apparent that conventional isospin is an accidental symmetry. Indeed the u and d quark masses are typically of order 5 and 10 MeV respectively (Wei 77, BT 82) so SU(2) is badly broken at the Lagrangian level (see Section 7.2.2). However, these masses are much smaller than the eigenvalue of the Dirac equation for a light, confined quark [if $w_{u/R} \sim 400$ MeV, $w_{d/R} \sim 402$ MeV — see Eqs. (2.89)-(2.91)] — the constituent quark mass (Section 2.4). Thus the microscopic breaking of the symmetry gets hidden and isospin looks good at the hadronic level.

Since charge symmetry is a special case of isospin invariance, corresponding to rotations by 180° about the y-axis in isospin space (HM 79), it is clearly no longer "fundamental". Nevertheless there is a great deal of experimental activity presently aimed at finding charge symmetry violation (CSV) in the N-N system (Dav+ 81). So far there is no clear indication of CSV there. The classical case which has been studied at length is the $^1S_0$ scattering length. At present the best experimental values for nn and pp are -18.6 ± 0.6 fm (Gab+ 79) and -17.1 ± 0.2 fm (Gur+ 80; after Coulomb corrections) respectively. While this apparently indicates a small CSV, there is considerable discussion of the meaning of the errors quoted.

In a recent LAMPF experiment Hollas and co-workers failed to see a charge-symmetry-violating forward-backward asymmetry in the process $np \rightarrow d\pi^0$ at a level of 0.5% (Hol+ 81). The most sensitive tests so far should come from experiments presently underway at both IUCF and TRIUMF, where one is looking for a small difference in the position of the zero in P and A in np elastic scattering (Dav+ 81).

Conventional theoretical models for CSV typically involve ρ-ω and π-η mixing in a one-boson-exchange picture (HM 79). The presence of such mixing is a result of the u-d mass difference mentioned earlier (LS 79). However, in view of our discussion of the short and medium range N-N force in Section 7.1, it is not obvious that such mixing for real mesons has anything to do with N-N scattering. It would seem more appropriate to directly calculate N-N scattering in one of the ways discussed in Section 7.1 using $m_u \neq m_d$ directly. This has not yet been done.

What has been looked at is the possibility of a direct source of CSV in the OPE interaction caused by $m_u \neq m_d$ (Tho+ 81b). Because of the explicit appearance of quarks and pions in the Lagrangian density, and its excellent convergence properties, the CBM is ideally suited to this problem. We recall from Eq. (5.103) that the pion-nucleon coupling had strength $g_A/2f$, where g_A is the axial charge of the nucleon calculated in the bag model. In Section 3.3.1 we calculated g_A explicitly for the MIT bag model and showed why it gave such an improvement over the naive quark model. The presence of the lower piece of the Dirac spinor for the quark gave a maximum suppression of about 34% of the non-relativistic value (5/3) in the case $m_{quark} = 0$ [Eq. (3.36)]. Of course, in the non-relativistic limit of infinite quark mass the lower component vanishes and the value of 5/3 is restored. If one has two masses in between the ultra-relativistic and non-realtivistic limits, the suppression factor will be smaller, and hence g_A larger, for the heavier of the two.

In particular, if m_d is (4-5) MeV heavier than m_u—as we require in order to fit the n-p mass difference (BT 82)—then g_A will be larger for the d- than the u-quark. If we consider π^0 coupling to the n and p, it should now be clear that the coupling to the neutron will be larger than that to the proton, because the former contains more d-quarks. In fact, using the spin-flavour wave functions

$$|p\uparrow\rangle_{s-f} = u_1 u_2 d_3(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow)/\sqrt{6} \ ,$$

$$|n\uparrow\rangle_{s-f} = d_1 d_2 u_3(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow)/\sqrt{6} \ , \qquad (7.12)$$

for distinguishable u- and d-quarks one can easily show that

$$\frac{g_A^n}{g_A^p} = 1 + \frac{3}{5}\,\delta \ , \qquad (7.13)$$

where $(1-\delta)$ is the ratio of g_A for a single u-quark to that for a single d-quark. Using the results of Golowich and collaborators (Gol 75, Don+ 75) we find $\delta = 0.64\%$ for $(m_d - m_u) = 5$ MeV, and hence g_A^n/g_A^p is greater than one by 0.4%.

This is outside the level of accuracy for present neutral current experiments. However one may hope to see this effect through the difference in $f_{\pi^0 nn}$ and $f_{\pi^0 pp}$, implied by Eq. (5.103), viz:

$$\sqrt{4\pi}\ f/m_\pi = g_A/2f \ .$$

Clearly we expect that the $nn\pi^0$ coupling constant should be about 0.4% bigger than that for $pp\pi^0$ — in direct violation of charge symmetry. For the N-N scattering length this implies $|a_{nn}| - |a_{pp}^{\text{No Coul}}| = +0.3$ fm (Tho+ 81b), which is in the same direction as experiment but a little small. (Although we stress again that the experimental numbers are not conclusive.) Other systems in which we might hope to see this CSV include the decay widths of the Δ, and the forward-backward asymmetry in $np \to d\pi^0$ — which may be enhanced for an appropriate polarization observable.

7.2.2. *The ^3He-^3H mass difference — a new perspective*

Within the framework of non-relativistic potential theory the three-nucleon system has been amenable to exact solution for about a decade. As we observed in Section 1 the discrepancy between the experimental binding energy of the triton and that obtained with realistic potentials has usually been attributed to relativistic or off-shell effects. However, a much more disturbing problem is the failure to fit the ^3H-^3He mass difference. After removing the n-p mass difference there is a residual 760 keV splitting between these mirror nuclei. Potential model calculations using charge independent forces give typically 640 keV and never more than 680 keV — see the Proceedings of the TRIUMF workshop (Dav+ 81). The remaining 80 keV has been a mystery for at least 15 years. If one takes all possible sources of CSV in a conventional OBE potential model, and they all add coherently with maximum permissible strength one can just about get the 80 keV. However, it is not a very compelling explanation.

In order to see what a quark level description would imply for the same problem, we first need to review the n-p mass difference itself.

The calculation of the electromagnetic shift in the bag model is rather complicated (Des+ 77) but the answer can be understood quite simply. Within about 10%

$$\Delta M_{e-m} = \sum_{i<j} \frac{Q_i Q_j}{R} , \qquad (7.14)$$

where the bag radius R is a measure of the average interquark distance. For $(\Delta E^p_{e-m} - \Delta E^n_{e-m})$ this gives about 0.5 MeV (with R=1 fm), in agreement with Deshpande *et al.* Note that this effect acts in the *wrong* way, tending to make the proton heavier than the neutron.

The only freedom in the bag model description is to take the u- and d-quarks to have different masses. With a u-quark mass about (4-5) MeV less than that of the d-quark the necessary 1.79 MeV mass difference (1.29 MeV experimental plus 0.5 MeV from electromagnetic effects) can be explained (BT 82). About 80% of the shift is simply associated with the change in quark eigenfrequency [see Eqs. (2.89)-(2.91)], and the rest with the change in the colour magnetic term (Section 2.2.2).

Next we recall that ^{3}He is one of the most dense nuclear systems available. Its point nucleon distribution has an rms radius of only 1.6 fm. With the nucleon itself having a radius of about 1.0 fm, it is highly likely that in a random snapshot of the nucleus we shall find two nucleons overlapping. Thus one obvious difference between ^{3}H and ^{3}He is that with some probability, P, we shall find the contents of two neutrons in one bag in the former, whereas in the latter we would find two protons. *The essential point is that the mass splitting between a 2p-bag and a 2n-bag is not* $2(m_p - m_n)$.

First the *n.l.b.c.* implies that the radius of a 6-quark bag is bigger than that of a 3-quark bag. We recall from Section 7.1.1 that

DeTar found $R_6 \sim 1.3$ fm, compared with $R_3 \sim 1.0$ fm. (In general one can show that $R \sim M^{1/3}$, with M the mass of the multiquark system.) Therefore we find at once a 30% reduction in the n-p mass splitting caused by $m_u \neq m_d$. In addition, a simple calculation with Eq. (7.14) shows that even allowing for the increase in average interquark separation, the Coulomb splitting increases in the wrong direction. The net result is that the 2n and 2p bags are split by only 0.9 MeV, instead of $2(m_n - m_p) = 2.6$ MeV. Alternatively, the effective n-p mass difference for the fraction of time, P, that the bags overlap is only 0.45 MeV.

A probability P of 10% would therefore suffice to explain the 80 keV discrepancy [(2.6-0.9/2) × 10% ≈ 80 keV]. This is a perfectly reasonable probability and indeed if we assume that when the centre of one bag is within R_3 of the centre of another they have coalesced, one obtains a probability $(1.0/1.6)^3 = 24\%$ for ^{3}He. It is clearly difficult to make this argument more quantitative at the present time, but the A=3 system does provide a beautiful example of just how different the quark model perspective may be—even for a familiar problem. Further work along these lines is presently being carried out (TG 82) to see to what extent such ideas can contribute to an explanation of the famous Nolen-Schiffer anomaly (NS 69).

7.3. The Nuclear Many-Body Problem

As there is no published calculation of the properties of a many-nucleon system near nuclear matter density (ρ_o) in the sort of model which we have presented, this will be a brief section. (We exclude from the present discussion the very high density limit of quark matter, where there are no individual bags at all.) Nevertheless it does seem appropriate to collect together some of the ideas which may eventually be applied to the problem.

In a very stimulating attempt to understand how a system of finite size bags might behave, Baym introduced the idea of percolation (Bay 79). To introduce the concept, consider an infinite array of cubit children's blocks, some of which are copper and some wooden. If they are arranged at random there is a critical percentage of the blocks (P_c = 31%) which must be copper in order to *guarantee* that there is an infinite conducting chain through the array. If instead of being cubic we have spheres arranged on a regular lattice, P_c is 15 ± 1.5%. Finally for conducting spheres only, arranged at random through space, the critical percentage of space which must be occupied by spheres is 34%.

The analogy is of course that if two bags touch we expect that the quarks (i.e. a colour current) will be able to flow between them. (This was exactly the assumption made by DeTar—see Section 7.1.2.) Consequently, in infinite nuclear matter above a certain critical density (ρ_c), we expect that there should be at least one infinite conducting chain along which the quarks flow freely. This free flow of quarks is known as "percolation". Since the volume of a spherical bag is just ($4\pi R^3/3$), we expect that

$$\rho_c = 0.34/\left(\frac{4\pi}{3}R^3\right) , \qquad (7.15)$$

and hence (with ρ_o = 0.17 fm^{-3}), ρ_c is ($1/2\rho_o$, ρ_o, $1.4\rho_o$) for R = (1.0, 0.8, 0.7) fm respectively.

We see that in the centre of a large nucleus like ^{208}Pb, any acceptable nucleon bag radius (following the considerations of Section 6, $R \geqslant 0.8$ fm) will imply the presence of conducting chains. More to the point, for a radius near the MIT value ($R \sim 1.0$ fm) ρ_c is of order $\rho_o/2$, and even the nuclear surface should contain such chains. Such is our ignorance at present that it is not even clear whether this would have observable consequences! Qualitatively at least, it does seem easier to reconcile the success of the conventional shell model for valence nucleons with a somewhat smaller bag radius — say $R \sim 0.8$ fm. In that case $\rho_c \sim \rho_0$ and one would expect little effect in the nuclear surface where $\rho \sim \rho_0/2$. On the other hand, one might expect that single particle ideas could fail in the nuclear interior.

7.3.1. *Dense nuclear matter*

There has been considerable theoretical and experimental interest in the past few years in the possibility of exotic phenomena at densities higher than ρ_0 — phenomena like pion condensation and Lee-Wick matter. Chiral symmetry plays a crucial role in the conventional description of such processes. Indeed the σ-model, which we described at length in Section 4.4 is the starting point for most of the work in this area (LW 74, Bay 78, Cam 78, Mey 81). Clearly if we are to be concerned about effects of the finite size of the nucleon in the centre or even the surface of finite nuclei, it is unthinkable to ignore such effects at densities twice that of nuclear matter or greater! Indeed it would seem that pion condensation or Lee-Wick matter in the usual scenario of point-like nucleons with spin-isospin ordering is quite unlikely. Nevertheless the phenomenon which replaces it, namely overlapping bags with a free flow of colour through linked bags may be more interesting!

Incidentally, if it makes sense to talk of finite size nucleons exchanging pions even when they overlap a little (as discussed in Section 7.1), the CBM should provide an admirable successor to the σ-model. As we observed in Sections 5 and 6, it naturally incorporates the Δ-degree of freedom on the same footing as the nucleon. One does not have to put in $g_A \neq 1$ by hand (as in the σ-model). Finally in its linearised form the CBM is a rapidly convergent renormalisable theory and one does not have the ambiguities of using a tree-level Lagrangian in a many-body system. Self-energy corrections are meaningful in the CBM. In the final part of this section we wish to outline a new approach to the nuclear many problem designed to exploit these advantages of the CBM.

7.3.2. *The e^s formalism—a generalisation*

In attempting to solve for the properties of a many-body system for a given Hamiltonian it is essential that one use a technique which allows for systematic improvement. The coupled cluster expansion, or e^s formalism, has played this role in conventional nuclear theory (Coe 69, Kum+ 78, ZE 79). While making no attempt at a serious review of the formalism (the quoted articles fulfil that purpose) it is worthwhile to outline its essential features here. Given a many-body Hamiltonian

$$H = H_o + V \ , \tag{7.16}$$

where V includes all two-body interactions, the linked cluster expansion amounts to writing the exact eigenfunction of H, namely Ψ, as

$$\Psi = e^s \ \Phi \ , \tag{7.17}$$

where Φ is a slater determinant describing the non-interacting Fermi gas.

If we define creation operators for particles and holes ($a^+(x)$, $b^+(x)$ respectively) in the usual way, the operator s is

$$s = \sum_{n>1} s_n \ , \tag{7.18}$$

$$s_n = \frac{1}{(n!)^2}\int dx_1 \ldots dx_n \int dy_n \ldots dy_1 \ a^+(x_1) \ldots a^+(x_n) \times$$

$$\times \ b^+(y_n) \ldots b^+(y_1) \ s_n(x_n, \ldots x_1; y_1, \ldots y_n) \ . \tag{7.19}$$

Clearly s_n is related to the amplitude for creating n particle-hole pairs. What is less obvious is that it is the amplitude for creating *correlated* particle-hole pairs. This is crucial in a low-density system because one can prove rigorously that the importance of the n'th order piece goes as $(h^3\rho)^{n-1}$ where ρ is the density and h a "healing distance"—related to the range of the two-body interaction. With $h\sim 1$ fm and $\rho_0\sim 0.17$ fm^{-3}, one has a systematically convergent expansion at nuclear matter density. For completeness we note that in the case of pure two-body interactions in infinite nuclear matter, the total energy can be calculated entirely in terms of s_2 (s_1=0 by translational invariance). That is the total energy per particle is given by (Coe 69)

$$(E/A) = \rho_0{}^{-1}(\Phi|H|\Phi) + \frac{1}{4}\int dk \int dp \; dP(p|V|k) \; s(k,p;P) \; , \qquad (7.20)$$

where

$$s_2(k_1k_2\,;\,p_2p_1) = \delta(k_1+k_2-p_2-p_1) \; s(k,p;P) \; . \qquad (7.21)$$

Of course, in order to obtain s_2 one must solve a set of coupled cluster equations involving all amplitudes $\{s_n\}$. These equations are easily obtained by noting that

$$H\,\Psi = E\,\Psi \; , \qquad (7.22)$$

and by Eq. (7.17)

$$e^{-S}\,H\,e^{S}\;\Phi = E\;\Phi \; . \qquad (7.23)$$

But any particle or hole destruction operator, d, acting on Φ gives zero, so that

$$<\Phi|d\;e^{-S}\;H\;e^{S}|\Phi> = 0 \; . \qquad (7.24)$$

More generally,

$$<\Phi|b(y_1)\;\ldots\;b(y_n)\;a(x_n)\;\ldots\;a(x_1)\;e^{-S}\;H\;e^{S}|\Phi> = 0, \forall n \qquad (7.25)$$

which are the coupled cluster equations. After truncation at some order N (because of the proof of convergence noted above) one obtains a closed set of non-linear integral equations. The convergence of the iterative solution of those equations can be formally established for certain conditions on V.

In recent years we have come to realize the importance of the Δ in nuclear physics. A suitable generalisation of the e^S formalism to include the Δ explicitly was recently developed by Coester (Coe 81) for the Betz-Lee model (BL 81). In their model the only pion emission and absorption allowed are the processes $\Delta \leftrightarrow N\pi$. In such a simple field theory there is no renormalisation of the nucleon, but the properties of the Δ, and hence the intermediate range N-N force, will be density dependent.

The excellent convergence properties of the CBM, and the fact that the Δ (and other B=1 resonances) appears so naturally there, have prompted us to develop a linked cluster expansion including pion degrees of freedom explicitly (Coe+ 82). Formally all that is required is to replace Eqs. (7.18) and (7.19) by

$$s = \sum_{n,m\geq 1} s_{n,m} , \tag{7.26}$$

where

$$s_{n,m} = \frac{1}{m!}\frac{1}{(n!)^2}\int dk_1 \dots dk_m \int dx_1 \dots dx_n \int dy_n \dots dy_1 \times$$
$$\times \alpha^+(k_1) \dots \alpha^+(k_m)\, a^+(x_1) \dots a^+(x_n)\, b^+(y_n) \dots b^+(y_1) \times$$
$$\times s_{n,m}(k_1 \dots k_m; x_n \dots x_1; y_1 \dots y_n) . \tag{7.27}$$

In Eq. (7.27) $\alpha^+(k_1)$ creates a pion of momentum and isospin k_1 and $s_{n,m}$ is, of course, the amplitude for creating m pions and n particle-hole pairs all correlated. The generalisation of Eq. (7.25) to obtain the new coupled cluster equations is obvious.

Of course, in order to obtain equations which one can solve numerically one must again be able to justify a truncation at some maximum value of n *and* m. The cut-off in n will again be justified in terms of powers of $(h^3\rho)$. However, the cut-off in number of pions is a unique feature of the CBM and its justification was presented in Section 6. We expect that retaining all five amplitudes with m and n $\leqslant 2$ should be sufficient at nuclear matter density (Coe+ 82).

Unfortunately there are no numerical results available yet from this formalism, so one can not judge yet whether it will throw any new light on the nuclear many-body problem. Nevertheless there are solid physical reasons for believing that it might. Because the nucleon bag is relatively large, we have seen that the NNπ form-factor $(3j_1(kR)/kR)$ is quite soft. An equivalent dipole, $(k^2+\Lambda^2)^{-1}$, would have a range parameter $\Lambda \sim 640/R$ MeV (with R in fm). Thus the cut-off in all renormalisation integrals is of the order of the fermi momentum ($k_F \sim 275$ MeV/c). In such an intermediate situation one might expect that the properties of the many-body system as a function of density would be inextricably linked with the renormalisation process. This problem does not appear to have been seriously addressed before.

We can not conclude this section without a note of caution. There are many more subtleties in describing a system of composite nucleons than we have been able to address. The e^S formalism deals with the creation of N, Δ, ... obeying standard fermion anti-commutation relations and dressed with a pion cloud. As we have argued in the earlier sections, it is possible that for a bag radius in the lower range of that permitted in a chiral bag model ($R \sim 0.8$-0.9 fm), this may be a reasonable approximation even up to nuclear matter density. However, it must break down as the density increases and the quarks begin to percolate. It becomes increasingly difficult to assign a meaning to exchange terms, for example, as the density goes up. If we are lucky, we will begin to learn how to formulate this problem in a respectable way in the next few years. It is a noble endeavour!

Figure Caption

Fig. 7.1. Interaction energy of a spherical 6-quark bag as a function of the separation parameter δ — from DeT 78.

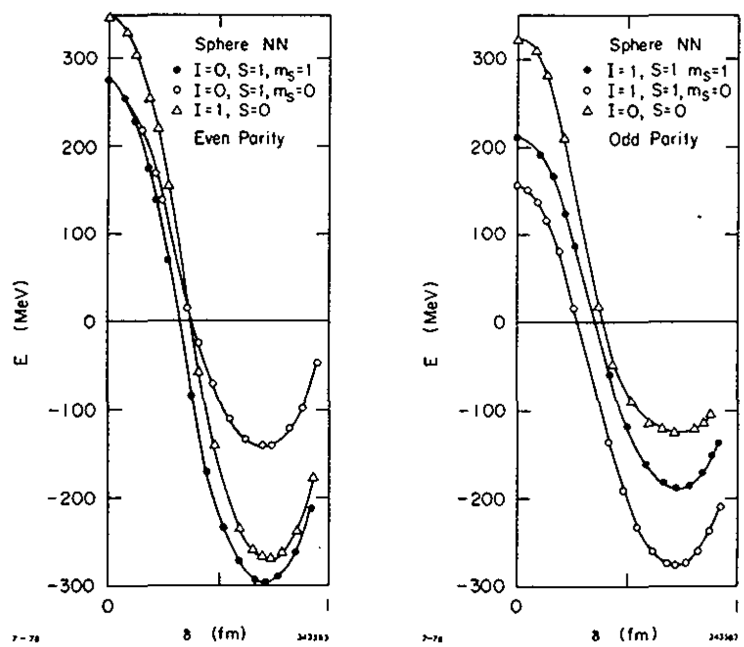


Fig. 7.1

8. CONCLUSION

This is a moment of dramatic change in our conception of nuclear physics. In the next decade the impact of the discoveries made by our colleagues in high energy physics will have to be reconciled with the conventional view of the nucleus. At the present stage we can only begin to guess at how much richer and more fascinating our subject may be. Amongst the admittedly crude models available to us in this detective work, we argued that the MIT bag model is a promising place to start. In particular, we outlined the ideas which have led a number of investigators to believe that it may have many of the properties of the eventual solution of QCD (incorporating both confinement and asymptotic freedom very concisely). For this reason we gave a detailed summary of the model, its underlying assumptions, its solutions, its predictions for the properties of single hadrons, and finally its unresolved problems.

Next we explained the concept of chiral symmetry and why it must be broken in nature—even though it is exact in pure QCD. The linear σ-model was used as the classic example of a spontaneously broken symmetry—with the appearance of the pion as a Goldstone boson. On a more fundamental level we mentioned the possibility that the pion may be the result of dynamical symmetry breaking caused by the strongly attractive one-gluon-exchange force in that channel. In that case its appearance would be independent of the usual mechanism for confinement. Then we reviewed the various attempts which have been made over the last three years to make a bag model incorporating chiral symmetry.

We saw that the cloudy bag model (CBM) in particular has produced a number of striking results for the properties of single hadrons—e.g.

the neutron electric form-factor, the magnetic moments of the neutron, proton and other members of the nucleon octet, and finally the proton lifetime. The CBM has led to a new and deeper understanding of the Δ-resonance which, like all the other baryons, enters in a natural, unified manner consistent with chiral symmetry. It was possible to transform the Lagrangian of the CBM so that it is a generalisation of the Weinberg Lagrangian and naturally incorporates the Weinberg-Tomozawa relationship for low energy pion scattering. Most significant for nuclear physics applications are the excellent convergence properties of the CBM. For example the bare NNπ coupling constant is renormalised by less than about 10% for any bag radius bigger than (0.7-0.8) fm.

Armed with a chiral bag model which had proven so successful in one-body systems, we made some observations in the last section about the N-N interaction and the nuclear many-body problem. Clearly that discussion was by far the most speculative. However, we did suggest that with a little subtlety one might, even now, be able to see some hints of the quark sub-structure in processes involving symmetry violation.

In order to be useful to the community a review must not only point out the achievements of a particular model, but also its faults and problems — the cutting edge of research often lies there. We have tried to pinpoint such problems throughout the review, but let us stress a few of the major questions again. One would be to firmly establish a relationship between the MIT bag model, soliton bag models and QCD. Of course, the nature and origin of the pion itself (particularly in relation to QCD) is an absolutely crucial question to answer. The formal problems associated with doing many-body calculations in a dense system of composite nucleons are formidable, but must be addressed. Finally there is a whole set of questions of a more technical nature, such as how to include recoil corrections, whether the CBM ideas can be generalised to SU(3) × SU(3), and so on. There is no shortage of work or challenge, and this whole review should be considered an invitation to take part.

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The seeds for this review were planted some six years ago by an experimental colleague, Martin Salomon, who asked whether the quark model might not have something to say about low energy pion physics— such as that investigated at TRIUMF. At the time I scoffed, but fortunately not too hard!

The present article began as a series of informal lectures at the University of Alberta in the summer of 1980, at the invitation of Gordon Greeniaus. These notes were initially put together as a set of five lectures for the Australian Institute of Physics, Nuclear and Particle Physics vacation school in May 1981, and first appeared as a Melbourne University preprint (UM/PP-81-29). It is a pleasure to thank Bruce McKellar for his hospitality during that visit, as well as Jack Sample who made the leave possible. Most of my colleagues at TRIUMF sat through an updated presentation of these lectures last fall and I am grateful to those who not only listened but made valuable comments. I would particularly like to thank Erich Vogt for his enthusiam and encouragement in the preparation of this much more complete manuscript.

Needless to say my understanding of the matters discussed here is largely the result of discussion with many colleagues. I am particularly indebted to my collaborators in the development of the CBM, Gerry Miller and Serge Théberge. I would also like to acknowledge the collaboration of L.R. Dodd, R.F. Alvarez-Estrada, M. Betz, A. Gersten, B.H.J. McKellar, A. Rinat and A. Rosenthal. In addition I have benefitted particularly from discussions with A. Arima, G.E. Brown, R. Cahill, C. Carlson, C. DeTar, T. Ericson, R. Goldflam, T. Goldman, J. Greben, P. Guichon, E. Henley, R. Jaffe, K. Johnson, F. Khanna, F. Myhrer, J. Niskanen, E. Oset, N. Weiss, L. Wilets and R. Woloshyn.

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APPENDIX I

Throughout these notes we follow the conventions of Bjorken and Drell (BD 64).

$$\beta = \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} ; \quad \underline{\gamma} = \begin{pmatrix} 0 & \underline{\sigma} \\ -\underline{\sigma} & 0 \end{pmatrix} , \tag{I.1}$$

$$x^\mu \text{ is a contravariant vector} - (x^0,x^1,x^2,x^3) = (t,\underline{x}), \tag{I.2a}$$

$$g^{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix} , \tag{I.2b}$$

$$\underline{\gamma} = \gamma^0 \, \underline{\alpha} , \tag{I.3}$$

$$\gamma_5 = \gamma^5 = i \, \gamma^0 \, \gamma^1 \, \gamma^2 \, \gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \tag{I.4}$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu,\gamma^\nu] , \tag{I.5a}$$

so that

$$\sigma^{ij} = \begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix} , \tag{I.5b}$$

$$\sigma^{oi} = i \, \alpha^i = i\begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix} , \tag{I.5c}$$

$$\{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu} , \tag{I.6}$$

$$\not{p} = \gamma_\mu p^\mu = \gamma^\mu p_\mu = i\not{\partial} . \tag{I.7}$$

The Dirac equation is

$$(\not{p} - m) \; u(p,s) = 0 ,$$

$$\bar{u}(p,s) \; (\not{p} - m) = 0 , \tag{I.8}$$

where

$$\bar{u} = u^+\gamma^0 . \tag{I.9}$$

To conclude this section on notation we briefly review a useful classification scheme for non-relativistic angular momentum eigenfunctions

$$|\ell \; \tfrac{1}{2} \; j \; \mu> \; \equiv \; |\chi_\kappa^\mu> = \sum_m C\,{}^{(\mu-m)}_{\ell}\,{}^{m}_{\frac{1}{2}}\,{}^{\mu}_{j} \, |\tfrac{1}{2} \, m>|\ell(\mu-m)> . \tag{I.10}$$

If we define

$$k = \underline{\sigma}\cdot\underline{\ell} + 1 , \tag{I.11}$$

then, because $\underline{\sigma}\cdot\underline{\ell}$ has eigenvalues $\{j(j+1) - \ell(\ell+1) - 3/4\}$, k has eigenvalues κ

$$k\ \chi_\kappa^\mu = -\kappa\ \chi_\kappa^\mu\ , \qquad (I.12)$$

with

$$\kappa = \ell\ , \quad j = \ell - \frac{1}{2}\ ,$$

$$\kappa = -\ell-1\ , \quad j = \ell + \frac{1}{2}\ , \qquad (I.13)$$

Thus κ alone specifies ℓ and j, for example

$$s_{1/2} \text{ is } \kappa = -1\ ,$$

$$p_{1/2} \text{ is } \kappa = +1\ ,$$

$$p_{3/2} \text{ is } \kappa = -2\ , \qquad (I.14)$$

and so on.

In conclusion we note that $(\sigma\cdot\hat{r})^2 = +1$, and $\underline{\sigma}\cdot\hat{r}$ is pseudoscalar, thus

$$\sigma\cdot\hat{r}\ \chi_\kappa^\mu = -\chi_{-\kappa}^\mu\ . \qquad (I.15)$$

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