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Gauge Dependence in Higher Derivative Quantum Gravity and the Conformal Anomaly Problem

I. L. Shapiro†

Department of Physics, Hiroshima University, Higashi-Hiroshima, 724, Japan.

A. G. Jacksenaev.

Tomsk State Pedagogical Institute, Tomsk, 634041,Russia.

Abstract. We examine the gauge dependence in higher derivative quantum gravity and find that the change of gauge condition leads to the variation of the effective action that is proportional to the conformal shift of the classical action. From this follows that one can not explain the appearance of the nonconformal counterterms in the Weyl gravity as the effect of a "bad" gauge. The last claim is the serious resoning in favour of the anomaly origin of Weyl gravity. Moreover we consider some new version of (induced) conformal gravity and find that the gauge dependence in this theory also has not relation to the existence of a nonconformal divergencies.

†e-mail adress: shapiro@fusion.sci.hiposhima-u.ac.jp,
On sabbatical leave from Tomsk State Pedagogical Institute, Tomsk, 634041,Russia

1 Introduction.

Higher derivative quantum gravity (HDQG) is, along with others, quite a useful model for the investigation of quantum gravitational effects. The main attractive feature of this model is the renormalizability [1,2] and the related opportunity to use the renormalization group methods for the study of the asymptotical behaviour of HDQG and the GUT models with HDQG [3 - 11]. At the same time HDQG is not unitary within the usual perturbation scheme. We shall not discuss the untarity problem here, restricting ourselves by pointing out the known referencies [1,4,5,13,14]. One can find the general review of HDQC in [12]. The unsolved problem of untarity doesn't allow one to consider HDQG as the fundamental and universal theory of gravity. At the same time one can use this theory as a model for the investigation of some aspects of quantum gravity. Present paper is devoted to the problem of the gauge dependence of the counterterms and to the discussion of the anomaly origin of the conformal version of the HDQG which is usually named as Weyl gravity. The last theory have the classical action of the form

$$S_W = \frac{1}{2\lambda}\int d^4x\sqrt{-g}\; C_{\alpha\beta\mu\nu}C^{\alpha\beta\mu\nu} + (surface\ \ terms), \qquad (1)$$

where $C_{\alpha\beta\mu\nu}$ is the Weyl tensor which is noted below as C. (1) is the particular case of the general action of HDQG.

$$S_{HD} = \int d^4x\sqrt{-g}\{\frac{1}{2\lambda}C_{\alpha\beta\mu\nu}C^{\alpha\beta\mu\nu} - \frac{\omega}{3\lambda}R^2 - \frac{1}{\kappa^2}(R-2\Lambda)\} + (surface\ \ terms). \qquad (2)$$

Note that the surface terms are necessary for the renormalizability (see, for example, discussion in [12]). ω, λ are the dimensionless couplings of the gravitational interaction. Both theories (1),(2) possess the general covariance. Moreover the Weyl gravity (1) is invariant under the conformal transformation

$$g_{\mu\nu} \to g'_{\mu\nu} = g_{\mu\nu}e^{2\sigma(x)}. \qquad (3)$$

The possible sourse of the anomaly is that the path integral, which correspond to the quantum theory, contains the divergencies and therefore one have to introduce the regularization scheme [25,26]. Since all known kinds of regularization violate one of the symmetries, the appearance of the anomaly is possible (see [15] for the review of the anomalies in quantum field theory). Such an arguments are usual in discussion of the anomaly origin of Weyl gravity. However there is no formal proof of the anomaly existence and the only known fact in a favour of this is the result of the one - loop counterterms calculation in [5].

According to power counting and general covariance considerations the possible counterterms in HDQG have the form:

$$\Delta S = \int d^4x\sqrt{-g}\{a_1 C_{\alpha\beta\mu\nu}C^{\alpha\beta\mu\nu} + a_2 R^2 + a_3 E + a_4 \Box R\}, \tag{4}$$

where a_i are some divergent constants (we suppose the use of the dimensional regularization), E is the Gauss - Bonnet topological invariant

$$E = R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2. \tag{5}$$

The one - loop counterterms in the theory (1) have been found in [5] in the form (4) with the nonzero a_2 and hence the conformal invariance is violated (as well as the multiplicative renormalizability). The counterterms of the form

$$\Delta S' = \int d^4x\sqrt{-g}\{a_5 R + a_6\} \tag{6}$$

are also possible in the theory (2) if the Einstein and cosmological terms occure in the classical action.

There is another competative explanation for the appearance of the nonconformal counterterm $\int d^4x\sqrt{-g}R^2$ in Weyl gravity. In fact our expectation to get the conformal invariant counterterms is based on the use of the background field method. So it is important to check the correctness of our use of this method. The gauge fixing term that have been introduced in [5] is of the form

$$S_{gf} = \frac{1}{2}\int d^4x\sqrt{-g}\chi_\mu Y^{\mu\nu}\chi_\nu, \tag{7}$$

where the general form of the background gauge and weight operator is

$$\chi_\mu = \nabla_\nu h^\nu_\mu - (\beta + \frac{1}{4})\nabla_\mu h, \tag{8}$$

$$Y^{\mu\nu} = \frac{1}{\alpha}(-g^{\mu\nu}\Box + \nabla^\nu\nabla^\mu - \gamma\nabla^\mu\nabla^\nu). \tag{9}$$

Here α, β, γ are gauge parameters, $h = h^\mu_\mu$ and the expansion of the metric $g_{\mu\nu}$ into the background $g_{\mu\nu}$ and quantum $h_{\mu\nu}$ parts

$$g_{\mu\nu} \to g'_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}$$

is supposed. The quantization of the theory (1) needs also the supplementary condition for fixing the conformal symmetry. In [5] this condition have been taken in the form $h = 0$. The consistent use of the background field method require the gauge condition to preserve the symmetries in the background fields sector. In the case under consideration the condition $h = 0$ does not violate the background conformal invariance. However we

can not wait for the conformal invariant result because of the conformal noncovariance of the operators (8),(9).

Thus we have two competative reasons for the appearance of the nonconformal counterterm $\int d^4x\sqrt{-g}R^2$ in Weyl gravity, and hence the question of conformal anomaly in Weyl gravity is not clear. Here we consider the second version and argue that the appearance of the nonconformal counterterm is not caused by the choice of gauge condition. In fact if a_2 (4) is not equal to zero due to the nonconformal structure of the operators (8),(9) then it is natural to suppose that a_2 will depend on the gauge parameters α, β, γ. Below it will be shown that the divergencies in the theory (1) do not depend on the gauge parameters and so we have a weighty reasoning in favour of the anomaly existence. Incidentally we consider the gauge dependence of the counterterms in a general HDQG (2) and separate the couplings of the theory into the essential and nonessential ones.

The second part of the paper is devoted to the study of the gauge dependence in a new conformal gravity theory. The action of this theory contains an additional dilaton field. This action can be regarded as the integration constant for the induced quantum gravity. Although there is not direct relation between the gauge dependence of the effective action and the conformal shift of the classical action in this new theory, the change of gauge fixing condition doesn't effect the possible nonconformal counterterms.

2 Gauge dependence of effective action in HDQG.

Let us start with the introduction of some notations which will be common for both theories (1) and (2). $\Gamma(\alpha_i)$ will means the value of the effective action, which corresponds to the arbitrary values of the gauge parameters α_i, and $\Gamma_m = \Gamma(\alpha_i^{(0)})$ is the value of the (minimal) effective action corresponding to some special values of the gauge parameters $\alpha_i^{(0)}$.

Our aim is to establish the gauge dependence of the effective action, that is the form of the function $\Gamma(\alpha_i)$. Note that the general form of this dependence is clear without any special calculations. It is well - known that the gauge dependence disappear on mass - shell (see, for example, [12] and [27] for the general and rigid prove), and this fact gives the key to the understanding of the problem. We can write

$$\Gamma(\alpha_i) = \Gamma_m + \int d^4x\sqrt{-g}\varepsilon^{\mu\nu}f_{\mu\nu}(\alpha_i), \tag{10}$$

where $\varepsilon^{\mu\nu} = \delta S/\delta g_{\mu\nu}$ is the extremal of the classical action S, and $f_{\mu\nu}(\alpha_i)$ is some unknoun function.

We are interesting here in the only divergent part of the effective action $\Gamma(\alpha_i)$. Since the divergencies are local and moreover the values of $\Gamma_m, \Gamma(\alpha_i), \varepsilon^{\mu\nu}$ have just the same dimension as the classical action, we get

$$f_{\mu\nu}(\alpha_i) = g_{\mu\nu} f(\alpha_i) \tag{11}$$

and therefore

$$\Gamma(\alpha_i) = \Gamma_m + \int d^4x \sqrt{-g} f(\alpha_i) g_{\mu\nu} \frac{\delta S}{\delta g_{\mu\nu}}. \tag{12}$$

Thus we find that the change of the gauge condition in the HDGT is equivalent to the conformal shift of the classical action S. This fact leads us to some important sequences. So we get that in the Weyl gravity (1) the effective action does not depend on the gauge parameters ans hence the only reason for the nonconformal counterterms is the conformal anomaly.

Let now say some words about the known possibility to reject the conformal anomaly, which was proposed in [5,9] (see also early papers [21,22] and the book [12]). Introducing the new scalar quantity $P[g_{\mu\nu}]$ which satisfy the equation

$$\Box P = \frac{1}{6} RP \tag{13}$$

we construct the conformal invariant metric

$$\bar{g}_{\mu\nu} = P^2[g]\, g_{\mu\nu}. \tag{14}$$

As far as $R(\bar{g}_{\mu\nu}) = 0$ one can kill the conformal noninvariant counterterms if the background metric $g_{\mu\nu}$ is substituted by $\bar{g}_{\mu\nu}$. In such a way we can restore the conformal invariance on the quantum level and so provide the renormalizability of the Weyl gravity. Of course the described method is formally uncorrect becouse in fact the anomaly exist. For example, we are not able to make the renormalization in terms of the bare fields $g_{\mu\nu}$ but only with the use of some initial conditions of eq.(13).

Now we use eq.(12) for the investigation of the gauge dependence in the general version of HDQG (2). It is known that

$$g_{\mu\nu} \frac{\delta(\sqrt{-g}R^2)}{\delta g_{\mu\nu}} = -6\sqrt{-g}(\Box R). \tag{15}$$

Since the conformal and surface terms in (2) don't contribute to the extremal, we obtain

$$\Gamma(\alpha_i) = \Gamma_m + \int d^4x \sqrt{-g} f(\alpha_i) \{-\frac{1}{\kappa^2}(R - 4\Lambda) - \frac{2\omega}{\lambda}\Box R\}. \tag{16}$$

And so, we can conclude, that the values of a_1, a_2, a_3 (4) do not depend on the gauge parameter values unlike a_4, a_5, a_6 do. Note, that one can evidently construct the new parameter $a_7 = a_6 + \Lambda a_5$ which is gauge independent.[1] Hence the combination $\kappa^2\Lambda$ is

[1] Of course, the values $\frac{\lambda}{4\omega}a_4 - \kappa^2 a_5$ and $\frac{\lambda}{4\omega}a_4 + \frac{\kappa^2}{\lambda}a_6$ are also gauge-independent

essential coupling constant (terminology of S.Weinberg [23]). The renormalization of this coupling is gauge-independent.

The eg.(13) is the general form of the gauge dependence of the theory (2). The explicit form of the function $f(\alpha, \beta, \gamma)$ is derived in the Appendix.

3 Gauge dependence in induced conformal gravity.

Let's fistly call in mind the main elements of the induced action derivation [19] (see also [18,20] and [12]).The starting point is the theory of free massless fields of spin 0 (with conformal coupling), $\frac{1}{2}$ and 1 in an external gravitational field. Vacuum guantum effects lead to the conformal anomaly trace of the energy-momentum tensor [17]

$$T =< T_\mu{}^\mu >= k_1 C^2 + k_2 E + k_3 \Box R, \tag{17}$$

where the values of $k_{1,2,3}$ are determined by the number of fields of different spin (see, for example [12]). (14) leads to the equation for the effective action

$$-\frac{2}{\sqrt{-g}} g_{\mu\nu} \frac{\delta\Gamma}{\delta g_{\mu\nu}} = T. \tag{18}$$

One can solve eq.(18) within the following scheme [18,19]. Put $g_{\mu\nu} = \bar{g}_{\mu\nu} exp(2\sigma)$, where $\bar{g}_{\mu\nu}(x)$ is the metric with fixed determinant, and $\sigma(x)$ is conformal factor. Then (18) takes the form

$$\frac{\partial \Gamma[\bar{g}_{\mu\nu} exp(2\sigma)]}{\partial\sigma} = e^{-4\sigma} T(\bar{g}_{\mu\nu} exp(2\sigma)) \tag{19}$$

which have the following nonlocal solution

$$\Gamma[g_{\mu\nu}] = S_c[g_{\mu\nu}] +$$
$$\int d^4x \sqrt{-g_x} d^4y \sqrt{-g_y} \{k_1 C^2 + \frac{1}{2}k_2(E - \frac{2}{3}\Box R)\}_x G(x,y) \{k_2(E - \frac{2}{3}\Box R)\}_y +$$
$$+ \int d^4x \sqrt{-g}(k_3 + \frac{2}{3}k_2) R^2 \tag{20},$$

where $G(x,y)$ is the Green function for the Hermitian conformal covariant fourth-order operator

$$\Delta = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}R\Box + \frac{1}{3}(\nabla^\mu R)\nabla_\mu$$
$$\Delta_x G(x,y) = \delta(x-y). \tag{21}$$

The solution (20) contains the arbitrary conformal invariant funtional S_c , which is the integration constant for the equation (18). Now we discuss the form of some possible terms in S_c. It is more suitable to rewrite (20) in a local form with the help of some additional scalar field. To make this one have to suppose that $S_c[g_{\mu\nu}]$ contains the structure

$$C^2\Delta^{-1}C^2 = \int d^4\sqrt{-g_x}d^4y\sqrt{-g_y}C_x^2G(x,y)C_y^2. \tag{22}$$

Then, taking into account the possible Weyl term, we get the folowing local form of S_c

$$S_c = \int d^4x\sqrt{-g}\{(q_1+q_2\varphi)C^2+\frac{1}{2}\varphi\Delta\varphi\}. \tag{23}$$

Here φ is an additional dimentionless spin 0 field with zero conformal weight, and q_1, q_2 are some constants (parameters of the action). Let substitute, for generality, the linear construction in (23) by the arbitrary function $q(\varphi)$, and we obtain the action which can be treated as some generalization of (1):

$$S_c = \int d^4x\sqrt{-g}\{q(\varphi)C^2+\frac{1}{2}\varphi\Delta\varphi\}. \tag{24}$$

Note, that the more general expression is possible to construct if inserting the second arbitrary function $p(\varphi)$ in front of second item in (24) .

$$S_c = \int d^4x\sqrt{-g}\{q(\varphi)C^2+\frac{1}{2}p(\varphi)\varphi\Delta\varphi\}. \tag{25}$$

However , it is clear that (25) differs from (24) only by some change of variable φ and function $g(\varphi)$ and therefore the case (24) is quite general .

As far as theory (24) has just the same symmetrics, as the (1) ones, all the reasons in fauvor of the anomaly existence are also valid here. The only distinction between (1) and (24) is that the gauge dependence of the effective action in (24) is not proportional to the conformal shift of the classical action and therefore the effective action of the theory is gauge fixing dependent.

Making the separation of the field variables into background and quantum ones

$$g_{\mu\nu} \to g'_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu},$$
$$\varphi \to \varphi' = \varphi + \sigma \tag{26}$$

we get the path integral presentation of the effective action

$$\Gamma[g_{\mu\nu},\varphi] = \int \mathcal{D}h_{\mu\nu}\mathcal{D}\sigma\mathcal{D}\bar{c}^{\tau}\mathcal{D}c_{\lambda}$$
$$\delta(h_{\mu\nu}g^{\mu\nu})expi\{S_c[g+h,\varphi+\sigma] - S'_{c,g}[g,\varphi]h - S'_{c,\varphi}[g,\varphi]\sigma -$$
$$S_c[g,\varphi] + S_{c,gf} + S_{c,gh}[\bar{c},c;g,\varphi]\}, \tag{27}$$

where $\bar{c}^{\tau}, c_{\lambda}$ are ghosts and the conformal gauge $h = h_{\mu\nu}g^{\mu\nu} = 0$ have been used . The general form of the gauge fixing form can be founded in analogy with the similar $d = 2$ case [16].

$$S_{gf} = \int d^4\sqrt{-g}\chi_\mu Y^{\mu\nu}\chi_\nu,$$
$$\chi_\mu = \alpha_1(\varphi)\nabla_\alpha h^\alpha_\mu + \alpha_2(\varphi)\nabla_\mu\sigma + \alpha_3(\varphi)(\nabla_\mu\varphi)\sigma + \alpha_4(\varphi)(\nabla_\tau\varphi)h^\tau_\mu,$$
$$Y^{\mu\nu} = \alpha_5(\varphi)g^{\mu\nu}\Box + \alpha_6(\varphi)\nabla^\mu\nabla^\nu + \alpha_7(\varphi)R^{\mu\nu} + \alpha_8(\varphi)Rg^{\mu\nu}. \tag{28}$$

Here $\alpha_{1,...,8}(\varphi)$ are gauge parameters which depend on the background field φ . Note , that the use of such a gauge (with the special choice of α_i) enables us to calculate the one-loop counterterms in the theory (24), and also in a general dilaton quantum gravity in four dimensions. Such a calculation will be the purpose of a separate paper. Here we consider only the gauge dependence of the effective action. The arguments of the previous section are also valid for the theory (24) and we obtain the equation, that is analogous to eq. (10).

$$\Gamma(\alpha_i) = \Gamma_m + \int d^4x\sqrt{-g}\{f^{(1)}_{\mu\nu}(\alpha_i)\frac{\delta S_c}{\delta g_{\mu\nu}} + f^{(2)}(\alpha_i)\frac{\delta S_c}{\delta\varphi}\}, \tag{29}$$

where $f^{(1)}_{\mu\nu}, f^{(2)}$ are some unknown functions of the gauge parameters . Γ_m is the value of Γ which correspons to some special (minimal) choice of gauge parameters. Then, due to the dimensional analysis we find $f^{(1)}_{\mu\nu} = f^{(1)}(\alpha_i)g_{\mu\nu}$, where $f^{(1)}(\alpha_i)$ is some dimensionless function . Hence the first item on the right of (29) is proportional to the conformal shift of S_c . Since the action S_c is conformal invariant we have

$$\Gamma(\alpha_i) = \Gamma_m + \int d^4x\sqrt{-g}f^{(2)}(\alpha_i)[\Delta\varphi + q'(\varphi)C^2], \tag{30}$$

where $q' = dq/d\varphi$.

Note that the difference $\Gamma(\alpha_i) - \Gamma_m$ is conformally invariant and therefore it is impossible to remove any conformal noninvariant counterterm by selection of the gauge parameters $\alpha_i(\varphi)$. Thus the gauge dependence in the theory (24) doesn't have relation to the conformal anomaly as well as in the Weyl gravity (1) and thetefore the theory (24) is expected to be nonrenormalizable due to the possible nonconformal divergencies.

Of course, one can use the trick with the conformal regulization, substituting the background metric $g_{\mu\nu}$ by $\bar{g}_{\mu\nu}$ (14). Then the anomaly gisappear, and all the possible counterterms may be removed by some reparametrization of the field φ and the function $q(\varphi)$. The variation of gauge parameters leads to the only change of this transformations. Now we shall restrict ourselves by the one-loop case where $f^2(\alpha_i) = O(\varepsilon^{-1})$, where $\varepsilon = n - 4$ is the parameter of dimensional regularization. Let the divergencies of Γ_m are removed by the transformation of the form

$$\varphi^{(0)} = \varphi + z_\varphi^m,$$
$$q^{(0)} = q + z_q^m. \qquad (31)$$

Then it is easy to see that the divergencies of $\Gamma(\alpha_i)$ are removed by the renormalization transformation, if z_φ^m and z_q^m in (31) are substituted by the values

$$z_\varphi^{\alpha_i} = z_\varphi^m + f(\alpha_i),$$
$$z_q^{\alpha_i} = z_q^m + \frac{1}{2} f(\alpha_i) q'(\varphi). \qquad (32)$$

4 Conclusion.

We have investigated the gauge dependence in two kinds of gravity theories. In HDQG the variation of the gauge condition leads to the change of the divergent part of the effective action that is proportional to the conformal shift of the classical action. The explicit form of the gauge dependence in general (nonconformal) HDQG have been found. Since the Weyl gravity is the conformal invariant version of HDQG the conformal shift of the classical action here is equal to zero and therefore it is not possible to relate the nonvanishing nonconformal counterterms of Ref.[5] with the effect of conformal noncovariant background gauge. So we recognize that the conformal anomaly in the theory (1) really exist. In a new, induced conformal gravity the variation of gauge condition is not equivalent to the conformal shift of classical action, but leads only to some change of the renormalization transformations. However, this change does not effect the possible anomaly counterterms.

5 Appendix

In this appendix we make the explicit calculation of the function $f_{\mu\nu}(\alpha_i)$ (10), using the method of [24] with some modifications which are necessary in higher derivative theories. The starting point is the theory with classical action S that is a functional of the fields g^n. The gauge transformations and the Noether identities have the form

$$\delta g^n = \nabla_\alpha^n \xi^\alpha,$$
$$\epsilon_n \nabla_\alpha^n = 0, \quad \epsilon_n = \frac{\delta S}{\delta g^n}. \qquad (A.1)$$

Let now introduce the gauge fixing action for the theory of HDQG is the form

$$S_{gf} = \frac{1}{2}\int d^4x \sqrt{-g}\chi_\mu Y^{\mu\nu}\chi_\nu,$$
$$\chi_\mu = \chi_\mu(\alpha_i), \quad Y^{\mu\nu} = Y^{\mu\nu}(\alpha_i). \qquad (A.2)$$

Here α_i are gauge parameters. As we have seen from eq.(10) the weight operator can also depends on a gauge parameters in higher derivative theories. The propagators of the fields g^n and ghosts are defined by the relations

$$F_{kn}\, G^{np} = \delta_k^p, \quad M_{\mu\beta} K^{\beta\lambda} = \delta_\mu^\lambda, \qquad (A.3)$$

where

$$F_{kn} = \frac{\delta^2 S}{\delta g^n \delta g^k} + \frac{\delta \chi_\mu}{\delta g^n} Y^{\mu\nu} \frac{\delta \chi_\nu}{\delta g^k},$$
$$M_{\mu\beta} = G_{\mu\nu} \frac{\delta \chi^\nu}{\delta g^n} \nabla_\beta^n. \qquad (A.4)$$

From (A.1) and (A.4) follows the Ward identity

$$\nabla_\alpha^p K^{\alpha\beta} - G^{kp} \frac{\delta \chi^\beta}{\delta g^k} = -G^{kp} \epsilon_n \frac{\delta \nabla_\alpha^n}{\delta g^k} K^{\alpha\beta}. \qquad (A.5)$$

In a one - loop approximation the effective action Γ is given by the expression

$$i\Gamma = -\frac{1}{2} Tr\, \ln\, F_{kn} + Tr\, \ln\, M_{\mu\beta} - \frac{1}{2} Tr\, \ln\, G^{kp}. \qquad (A.6)$$

Suppose that the weight operator $Y^{\mu\nu}$ and the gauge χ_μ depend on arbitrary parameter t. Taking the derivative of (A.6) on t and taking into account (A.5) we find

$$i\Gamma' = -\frac{1}{2} G^{nk} \epsilon_p \frac{\delta \nabla_\alpha^p}{\delta g^n} K^{\alpha\mu} [\frac{\delta \chi^\nu}{\delta g^k} (Y_{\mu\nu})'_t + 2Y_{\mu\nu} (\frac{\delta \chi^\nu}{\delta g^k})'_t], \qquad (A.7)$$

where the touch denote the derivative on t.

Now we can derive $\Gamma(\alpha_i)$ with the help of (A.7) and the general formula

$$i\Gamma(\alpha_i) = i\Gamma(\alpha_i^{(0)}) + \sum_{i=1}^{s} \int_{\alpha_i^0}^{\alpha_i} i \frac{\partial \Gamma(\alpha_1, ..., \alpha_i, \alpha_{i+1}^{(0)}, ..., \alpha_s^{(0)})}{\partial \alpha_i} d\alpha_i. \qquad (A.8)$$

The equations (A.7),(A.8) give the general form of the gauge parameters dependence of the effective action in higher derivative gravity theories (including the dilaton theories like (24)). Now we consider (as an example of such dependence) the concrete theory (1) with the gauge fixing term (7). Since we have used the conformal gauge $h = 0$ there are only two gauge parameters α and γ.

$$K^{\rho\sigma} = -\alpha g^{\rho\sigma} \Box^{-2} + \alpha(1 - \frac{1}{2\gamma}) \nabla^{(\rho} \nabla^{\sigma)} \Box^{-1}\,,$$

$$Y^{\mu\nu,\rho\sigma} = P^{\mu\nu,}{}_{\lambda\theta}[4(\alpha-\lambda)\delta^{\lambda\theta,\kappa\omega}\Box^2 - 4\nabla^\lambda\nabla^\theta\nabla^\kappa\nabla^\omega(\alpha+\lambda\frac{\alpha+3\lambda\gamma}{\lambda-6\lambda\gamma})+$$

$$+2\lambda g^{\lambda(\kappa}\Box\nabla^{\omega)}\nabla^\theta]P_{\kappa\omega,}{}^{\rho\sigma}\Box^{-4}\,,$$

$$P^{\mu\nu,}{}_{\lambda\theta} = \frac{1}{2}(\delta^\mu_\lambda\delta^\nu_\theta + \delta^\mu_\theta\delta^\nu_\lambda) - \frac{1}{4}g^{\mu\nu}g_{\lambda\theta}. \qquad (A.9)$$

The analysis of eq. (A.7) shows that the nonzero contributions to the divergences are given by the only universal traces which do not contain curvature. Note that in the main part of the article we obtain this result in the framework of consideration, which is based on power counting and on the locality of counterterms.

The minimal operator F_{kn} corresponds to the following values of the gauge parameters [5,11]

$$\alpha^{(0)} = \lambda, \qquad \gamma^{(0)} = \frac{2}{3}.$$

Performing the integration (A.8) and substituting (A.9) we obtain

$$i\Gamma(\alpha,\gamma) = i\Gamma(\alpha^{(0)},\gamma^{(0)}) + \varepsilon^{\mu\nu}f_{\mu\nu}(\alpha,\gamma)\,,$$

$$f_{\mu\nu}(\alpha,\gamma) = \frac{3}{2}g_{\mu\nu}[\frac{\lambda}{2}\ln(\frac{9\gamma\lambda^{5/2}}{2\alpha^{3/2}(\alpha-6\lambda\gamma)}) + \alpha - \lambda] \qquad (A.10)$$

So the explicit expression (A.10) is in a good accord with the corresponding result of the main text of the paper.

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