Feature-Based Laser Scan Matching For Accurate and High Speed Mobile Robot Localization

A. A. Aghamohammadi, H. D. Taghirad, A. H. Tamjidi, and E. Mihankhah

*Advanced **R**obotics and **A**utomated **S**ystems (ARAS),*
Department of Electrical Engineering,
K. N. Toosi University of Technology

Abstract—**This paper introduces an accurate and high speed pose tracking method for mobile robots based on matching of extracted features from consecutive scans. The feature extraction algorithm proposed in this paper uses a global information of the whole scan data and local information around feature points. Uncertainty of each feature is represented using covariance matrices determined due to observation and quantization error. Taking into account each feature's uncertainty in pose shift calculation leads to an accurate estimation of robot pose. Experiments with low range URG_X002 laser range scanner illustrate the effectiveness of the proposed method for mobile robot localization.**

Index Terms—**autonomous mobile robot, localization, laser scan matching, feature uncertainty**

I. INTRODUCTION

FAST and accurate localization algorithm is one of the most important requirements in mobile robotics. Consequently, the subjects of localization and mapping have justifiably received considerable attention in the last 15 years, [1-4]. Standard methods often use odometric sensors for relative localization which is also known as dead reckoning. It is well known that the errors due to dead reckoning are accumulating over time, and one possible method to assist with localization is to use two-dimensional range finders such as laser range finders (LRF) [5], or ring of ultrasonic range sensors [6]. However, using laser range finders is more accurate compared to ultrasonic range sensors.

Usually, two consecutive laser scans are matched to obtain an estimate of the relative displacement of the robot. Scan matching algorithms can be categorized based on their association method, i.e. point to point or feature to feature. In point-wise scan matching algorithms, which are implemented in many cases to solve the matching problem, usually two scans are compared directly. Different routines are developed to use point to point matching approaches such as the iterative closet point (ICP), iterative matching range point (IMPR) [7], and the popular iterative dual correspondence (IDC) [7]. Pfister *et al.* proposed recently an interesting new approach to point-wise scan matching [8]. This algorithm weights the contribution of each scan point to the overall matching error according to its uncertainty. Although their implementation results are impressive, its main drawback is its high computation cost. Moreover, in matching algorithms it is assumed that corresponding points of two scans captured from two different poses of robot are successfully matched. This assumption is generally not true and especially in a point-wise scan matching algorithm, this is an optimistic assumption. SLIP method [9] uses a probabilistic distance metric to robust rejection of outliers and establishes point-wise correspondences. Metric-based ICP (MbICP) [10] presents a metric-based matching algorithm by point-wise matching dense range scans. It uses geometric distance that takes into account the translation and rotation of a robot simultaneously. Histogram based approaches, e.g. [11], use a special representation of the scanned data for matching of two consecutive scans. These algorithms are not robust with respect to the sensor noise and they require high computation cost. Some other approaches are also proposed for matching which utilize either active or passive artificial landmarks [12].

Above mentioned algorithms are subjected to high computation cost and imprecision, particularly in case of self-localization for high speed robot in large environments. Due to the iterative calculations required to obtain optimal scan matching in these algorithms, computational complexity are high, and in order of $O(n^k)$, in which $k > 1$ and n is the number of scan points. In point-wise approaches, n is approximately two orders of magnitude more than that of feature-based methods.

Recently Lingemann *et al.* [13] proposed a promising algorithm (HAYAI) to solve self-localization problem of high speed robots. Although this method is a fast and feature based method for scan matching, it suffers from the lack of satisfying robustness property of feature extraction and it is well-suited for high range sensors. In this paper the idea of this approach is borrowed for the scan matching and similar arguments are given regarding the computation cost and precision of feature based scan matching. However, by introducing appropriate algorithm for extracting features, and modeling existence features' uncertainty, the robustness of the method is improved significantly, even for low range sensors. The main idea behind the proposed algorithm in this paper is to address the issue of feature matching extracted from two consecutive scans, such that they can be integrated based on their covariance, in order to form accurate displacement estimation. Analyzing different sources of errors which affect displacement estimation process, appropriate uncertainty

models is developed for each feature extracted from the scan. Incorporating these models in displacement estimation, results in more accurate estimations. This paper is organized as follows: section 2 describes the feature extraction and matching algorithms in detail. Section 3 aims to calculate optimal pose shift of robot between two consecutive scans using matched feature pairs and their covariance. Section 4 demonstrates the experimental results and the concluding results are given in last section.

II. FEATURE EXTRACTION AND MATCHING

Some techniques of data analysis are applied to the scan S, provided by the scanner, in order to extract features from the scan. Desired features are divided into two types: features corresponding to the jump-edges and those corresponding to the corners detected in the scan. Formal notation for the scan S is given by:

$$S = \left(p_i\right)_{i=1,\dots,n} \quad , p_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix}$$

In order to detect jump-edges in scan S, scan points are divided into some clusters consisting of consecutive scan points in their natural order. As a result, every cluster features a dedicated start point (p_i) and an end point (p_j), in which, i and j are indices of points in whole scan S, and also $i < j$. Therefore, k'th cluster is defined by:

$$c_k = \left\{ p_m \,\middle|\, p_m \in S, i \leq m \leq j \right\}$$

Clustering procedure is as following: $p_1 \in c_1$ and for all points in scan like p_i $(i=1,\dots,n)$ the distance $d_i = \|p_{i+1} - p_i\|$ is calculated, then

$$p_{i+1} \in c_k \quad if \quad \left(p_i \in c_k, d_i \leq d_{th}\right)$$
$$p_{i+1} \in c_{k+1} \quad if \quad \left(p_i \in c_k, d_i > d_{th}\right)$$

In which, d_{th} is the maximum distance between two consecutive points within a cluster. The start and end point of each cluster are good candidates for being a feature. On the other hand all features have to be invariant with respect to robot's displacement. Thus, only invariant points are reliable for being selected as a feature and thus must be chosen. There are two cases in which one can see variation in end-cluster features:

Case 1) Clusters correlated to objects partially occluded by the other object. The start or the end point of a cluster, which is established by occlusion, is a growing (variant) point with respect to robot's displacement, and is not a good feature as illustrated in Fig 1. Therefore, the feature selection can be found by the following routine, given p_i the start or end point of cluster c_k,

if $\left|\theta_i - \theta_{i-1}\right| < c\theta_{th}$ and $r_i > r_{i-1}$ and p_i is start of $c_k \Rightarrow p_i$ is not a feature

if $\left|\theta_{i+1} - \theta_i\right| < c\theta_{th}$ and $r_i > r_{i+1}$ and p_i is end of $c_k \Rightarrow p_i$ is not a feature

otherwise $\Rightarrow p_i$ is a feature

Where $\theta_i = \tan^{-1}\left(y_i / x_i\right)$ and $r_i = \sqrt{x_i^2 + y_i^2}$ represent p_i in polar coordinates. According to the above conditions, if the
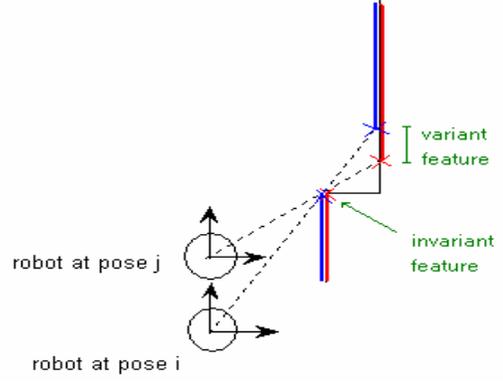
Fig. 1. Lines in black represent the environment, blue and red lines are the acquired scans from pose i and j, respectively. Upper feature is varying with respect to robot's displacement, but the other feature is an invariant one.

angle difference between two clusters is more than $c\theta_{th}$, no occlusion has happened. $c\theta_{th}$ is often equal to the sensor's angular resolution.

Case 2) When the start or the end of a cluster established due to sensors low range and not to a real feature in the environment. As illustrated in Fig. 2 low range sensors are subjected to this problem more than high range sensors. So if p_i is the start or end of a cluster one can say:

if $r_i > c r_s$ $\quad p_i$ is not a feature

otherwise $\quad p_i$ is a feature

In which r_s is the maximum range of laser scanner sensor. $c = 0.9$ can be a reasonable value.

Second class of features corresponds to the corners within the clusters. In [13] filters with small lengths are introduced to operate on the sequence of distance values (r_i) of points in the scan. Because of their small length, this method suffers from the lack of global sight on the scan data and some global information is missed. For example it may choose some features on flat walls due to scan noise. Therefore, in this paper it is proposed that for detecting corner features, at first a line fitting algorithm is applied as a global filter. In the literature several approaches are developed to solve the problem of extracting line segments from a sequence of points, with different speed and accuracy. Reference [14] has compared the state-of-art methods for line extracting, and
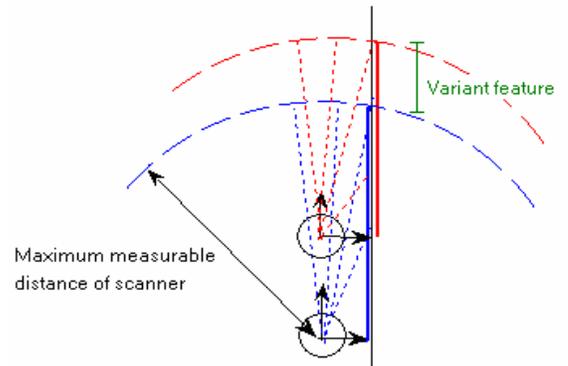
Fig. 2. End cluster feature, established due to sensor low range, is variant with respect to robot's displacement.

reported that for real-time applications, split-and-merge

algorithm is the best choice by its fair correctness and superior speed. Thus, split-and-merge algorithm is used in here for line fitting within each cluster. Due to the primary clustering, merging stage is often not required, and if required it is very fast.

The parameter tuple specifying a line reads as: $l_q = \left[\alpha_q, n_q, len_q \right]$, where, α_q is the angle between line and x axis; n_q is the number of associated scan points in line l_q, and len_q is the length of l_q. If intersection of two successive lines,
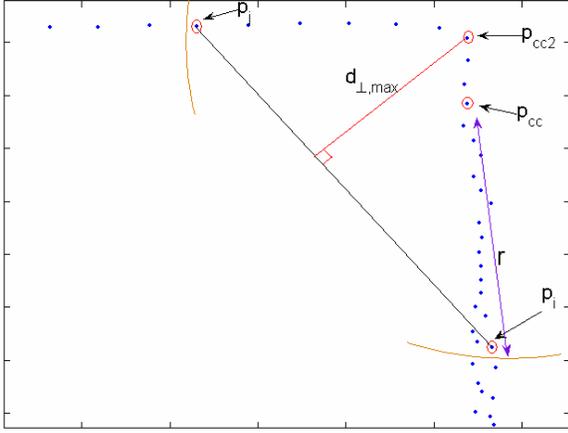
Fig. 3. Blue dots represent a part of scan around p_{cc}. p_i and p_j are scan points, which have distance r from p_{cc} and p_{cc1} is the point produces the maximum perpendicular distance from the line connecting p_i to p_j.

l_q and l_{q+1}, satisfies two following conditions, p_{cc}, which is the end point of l_q, is introduced as a candidate corner feature.

Condition1) $|\alpha_{q+1} - \alpha_q|$ exceeds the minimum threshold $\Delta\alpha_{th}$.

Condition2) For both l_q and l_{q+1}, either $len_q > len_{th}$ or $n > n_{th}$. len_{th} is the minimum acceptable length and n_{th} is the minimum acceptable number of points of lines intersected at this corner. p_{cc} may be not the exact corner point. Thus, using local information around p_{cc}, the following algorithm tries to find the nearest point to exact corner feature.

First, two points $p_i \in l_q$ and $p_j \in l_{q+1}$ are selected which have distance r to p_{cc} (see figure 3). r is proportional to len_q and len_{q+1}. p_i and p_j are connected by a straight line and for all scan points, p_m, lying between p_i and p_j, $i < m < j$, distance $d_{\perp,m}$ to this line is calculated. The scan point, produces the maximum distance, is named p_{cc_2}. If $p_{cc_2} = p_{cc}$, procedure is terminated and p_{cc} is selected as a corner feature. If $p_{cc_2} \neq p_{cc}$, then $p_{cc} = p_{cc_2}$, r is decreased and the above procedure is recursively called until $p_{cc_2} = p_{cc}$.

Finally, each corner feature is specified by $[p_{cc}, \alpha_{pre}, \alpha_{next}]$, in which p_{cc} is the feature point in scan and cc is its index in the whole scan. α_{pre} and α_{next} are angles of two consecutive

lines established this corner. Moreover, each start(end)-cluster feature is specified by point p_{cc} of scan and $\alpha_{pre}(\alpha_{next})$ which is the angle of line connecting to that.

After extracting features from two consecutive scans, matching algorithm, based on a dissimilarity function, has to be calculated. Owing to the fact that features' topology can not change fundamentally from one scan to the other, below dissimilarity function between p_i and p'_i, two features from two consecutive scans, is introduced as:

$$d(p_i, p'_j) = \left\| p_i - p'_j \right\|_2 + B$$

If $\left| \alpha_{next_i} - \alpha'_{next_j} \right|$ or $\left| \alpha_{pre_i} - \alpha'_{pre_j} \right|$ (which one exists) exceeds a maximum threshold, the p_i and p'_i are not matched, and B becomes infinity, otherwise $B = 0$.

After dissimilarity matrix is constructed, at each step the minimum value element of this matrix is chosen, its corresponding row and column are omitted and correlated features are matched, until all rows and columns of dissimilarity matrix are omitted or the minimum value of elements exceeds a maximum matching distance.

III. POSE SHIFT CALCULATION

This section aims to find proper uncertainty model for each of extracted features. Uncertainty of each feature has two major causes, rooted in physical properties of laser range finder: measurement process noise and quantized nature of rays' angles. First phenomenon that affects range sensing is measurement process noise in each scan point, $p_i \in S$, which is calculated as follows:

$$e_{ob} = p_{i_n} - p_i \qquad (1)$$

in which,

$$p_{i_n} = \left(r_i + e_{r_i} \right) \begin{pmatrix} \cos(\theta_i + e_\theta) \\ \sin(\theta_i + e_\theta) \end{pmatrix}$$

$$p_i = r_i \begin{pmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{pmatrix} \qquad (2)$$

p_{i_n} is a noisy measurement of point p_i. e_θ is a zero mean Gaussian noise with small standard deviation, represents the i'th direction's angle uncertainty. e_{r_i} is a noise added to true range of p_i. It comprises of two terms, $e_{r_i} = ar_i + b$, in which r_i is the measured range of p_i, a is a zero mean Gaussian random variable and b is the representation of a bias noise exists in range measurement process, which modeled by an additive Gaussian noise with mean value equals to b_0. These values can be obtained by statistical analysis of measurement data. Approximating $\cos(e_\theta) = 1$, $\sin(e_\theta) = e_\theta$, $e_\theta e_r \approx 0$ yields:

$$p_{i_n} = (r_i + ar_i + b) \begin{pmatrix} \cos(\theta_i) - e_\theta \sin(\theta_i) \\ \sin(\theta_i) + e_\theta \cos(\theta_i) \end{pmatrix} \Rightarrow$$

$$e_{ob} = r_i e_\theta \begin{pmatrix} -\sin(\theta_i) \\ \cos(\theta_i) \end{pmatrix} + (ar_i + b) \begin{pmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{pmatrix} \qquad (3)$$

Second source of error in considering a scan point as a

feature, is a quantized nature of laser scanner rays' angles. This issue causes that the point p_i, considered as a feature point, not necessarily be the same physical feature in the environment. Particularly, when incidence angle between laser scanner's i'th ray and surface of object related to p_i is near to zero or 180 degrees, this error is much considerable.

Quantization error for start(end)-cluster features is collinear with the boundary tangent of the object [8], on which p_i is selected (see fig. 4).
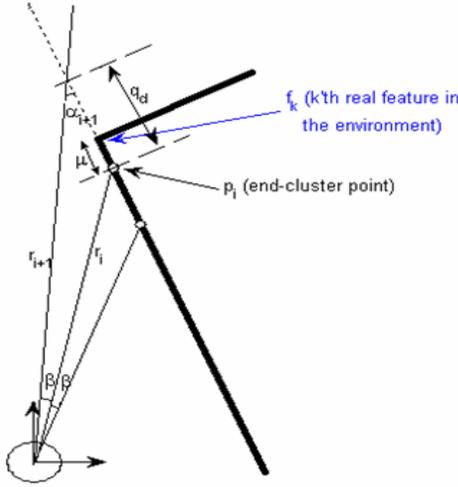
Fig. 4. p_i is an end-cluster point, and f_k is a real feature in the environment. α_{i+1} is the incidence angle between ray $i+1$ and direction of wall, on which p_i lies.

$$e_q = \mu \vec{t} \tag{4}$$

In which $\vec{t}$ is a unit vector collinear with fitted line to scan points around p_i and μ is a random variable represents quantization error's quantity. For an end-cluster feature, $t = \begin{bmatrix} \cos(\gamma) & \sin(\gamma) \end{bmatrix}^T$ and for a start cluster feature, $t = -\begin{bmatrix} \cos(\gamma) & \sin(\gamma) \end{bmatrix}^T$, where:

$$\gamma = \text{atan} 2 \left(p_e^y - p_s^y, p_e^x - p_s^x \right). \tag{5}$$

$p_s = [p_s^x \quad p_s^y]$ and $p_e = [p_e^x \quad p_e^y]$ are leading and ending points of the fitted line around p_i. Thus if f_k is the true position of the k'th feature in the environment in robot's coordinate and p_i is the selected point of the scan as the end of a cluster, we have:

$$f_k = p_i + e_{ob_i} + e_{q_i} \tag{6}$$

Accounting for the fact that the expected value of the modeled errors are nonzero, considered position for the k'th feature, $\hat{p}_k$, should differ from p_i

$$\hat{f}_k = E(f_k) = p_i + E(e_{ob}) + E(e_q) \tag{7}$$

Where $E(\cdot)$ is the expected value operator and:

$$E(e_{ob}) = r_i E(e_\theta) \begin{pmatrix} -\sin(\theta_i) \\ \cos(\theta_i) \end{pmatrix} + (E(a)r_i + E(b)) \begin{pmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{pmatrix} = \tag{8}$$

$$E(b) \begin{pmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{pmatrix} = b_0 \begin{pmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{pmatrix}$$

Figure 4 illustrates that q_d is the maximum value of μ and can be calculated by the formula:

$$q_d = \frac{\sin(\beta)}{\sin(\alpha_{i+1})} r_i \tag{9}$$

In which β is the separation angle of the laser scanner's rays and α_{i+1} is the incidence angle between fitted line to scan points at p_i and (i+1)'th ray of LRF. Due to the unknown shape of objects in the environment, it is reasonable to assume that μ is a random variable with uniform distribution, so:

$$E(e_q) = E(\mu \vec{t}_i) = E(\mu) \vec{t}_i = \frac{q_{d_i}}{2} \vec{t}_i \tag{10}$$

After considering $\hat{f}_k$ as the k'th feature, its error covariance should be calculated as:

$$f_k = \hat{f}_k + \tilde{e}_{ob_i} + \tilde{e}_{q_i} \tag{11}$$

Where

$$\tilde{e}_{ob_i} = e_{ob_i} - E(e_{ob_i})$$
$$\tilde{e}_{q_i} = e_{q_i} - E(e_{q_i}) \tag{12}$$

Assuming that $\tilde{e}_{ob_i}$ and $\tilde{e}_{q_i}$ are independent:

$$Cov(f_k) = E\left((f_k - \hat{f}_k)^T (f_k - \hat{f}_k) \right) = Cov(\tilde{e}_{ob_i}) + Cov(\tilde{e}q_i) \tag{13}$$

Again assuming that e_θ, a and b are independent equation8 results into,

$$Cov(\tilde{e}_{ob_i}) = r_i \sigma_\theta^2 \begin{pmatrix} \sin^2(\theta_i) & -\sin(\theta_i)\cos(\theta_i) \\ -\sin(\theta_i)\cos(\theta_i) & \cos^2(\theta_i) \end{pmatrix} \tag{14}$$

$$+ \left(\sigma_a^2 r_i^2 + \sigma_b^2 \right) \begin{pmatrix} \cos^2(\theta_i) & \sin(\theta_i)\cos(\theta_i) \\ \sin(\theta_i)\cos(\theta_i) & \cos^2(\theta_i) \end{pmatrix}$$

in which, σ_θ is the standard deviation of the theta measurement noise, σ_a that for the range measurement noise, and σ_b is the bias standard deviation. The covariance of the quantization error can be calculated as follows:

$$Cov\left(\tilde{e}_{q_i} \right) = E\left[\left(\mu_i \vec{t}_i - \frac{q_{d_i}}{2} \vec{t}_i \right) \left(\mu_i \vec{t}_i - \frac{q_{d_i}}{2} \vec{t}_i \right)^T \right] = \tag{15}$$

$$E\left[\left(\mu_i - \frac{q_{d_i}}{2} \right)^2 \right] \vec{t}_i \vec{t}_i^T = \frac{q_{d_i}^2}{3} \begin{pmatrix} \cos^2(\gamma_i) & \cos(\gamma_i)\sin(\gamma_i) \\ \cos(\gamma_i)\sin(\gamma_i) & \sin^2(\gamma_i) \end{pmatrix}$$

Using feature measurements and their corresponding covariance, the algorithm aims to calculate robot's displacement between scans. Displacement considered here, is composed of a translation, T, followed by a rotation, R. To find optimal T and R, the following error function should be minimized [16].

$$E = \sum_{k=1}^{m} \left(\hat{f}_k^{new} - (R \hat{f}_k^{pre} + T) \right)^T C_{\Delta \hat{f}_k}^{-1} \left(\hat{f}_k^{new} - (R \hat{f}_k^{pre} + T) \right) \tag{16}$$

$\hat{f}_k^{pre}$ and $\hat{f}_k^{new}$ are the k'th pair of matched features and m is the number of such pairs. $C_{\Delta\hat{f}_k}$ represents the amount of error that uncertainty of innovation vector, $\hat{f}_k^{new} - (R\hat{f}_k^{pre} + T)$, imposes to the pose shift calculation. Assuming that errors in two different scans are independent, we have,

$$C_{\Delta\hat{f}_k} = Cov\left(\tilde{e}'_{ob} + \tilde{e}'_q\right) + RCov\left(\tilde{e}_{ob} + \tilde{e}_q\right)R^T \qquad (17)$$

By substitution of Eq. 17 into Eq. 16 we reach to an error function that should be minimized. Sequential quadratic programming (SQP) method is used to solve the above nonlinear optimization. Since in feature-based approach the number of points taken into account in optimization procedure is often reduced to 0.01 of number of points used in point based approach, SQP method produces an accurate solution for R and T in a small portion of the computing time.

IV. EXPERIMENTS

Proposed method was implemented on the Melon robot, which is equipped with a Hokuyo URG_X002 Laser range scanner. The maximum measurable distance of this scanner is 4095mm. It's angular resolution per step is (180/512)=0.3515 degrees. Noise/maximum-range ratio in these laser scanners, are considerably more than high range sensors. In the experiments the following values are used by statistical analysis of measurement data: $\sigma_a = 5 \times 10^{-3}$, $\sigma_b = 1mm$ and $\sigma_\theta = 10^{-3}$ degree. All reported graphs are scaled in centimeters.
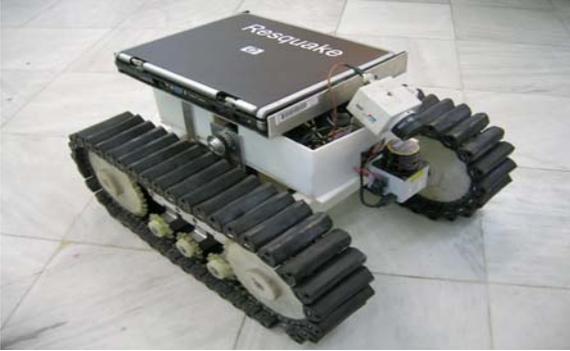
Fig. 4. Melon, the mobile robot equipped with low range laser scanner

Fig. 5 shows the extracted features of 74th scan along with corresponding ellipsoids. The ellipsoids are illustrated by a scaling factor of 10, indicating the 99 percent confidence region of each feature's covariance. There are two ellipsoids calculated in each scan for every feature, which are illustrated in two colors. The observation uncertainty ellipsoid (OUE) represents the measurement noise covariance of each feature, while the whole uncertainty ellipsoid (WUE) represents the whole covariance of the feature, consisting of the observation and quantization errors. The relative size of these ellipsoids reveals the contribution of the different sources of errors in the pose shift estimation. As it is seen in this figure the extracted features correspond to the real features in the environment. The growing start(end)-cluster points are effectively excluded from the selected features, and no features on flat walls are selected. Matching of the extracted

features from scan numbers 74 and 75 is shown in Fig. 6. As it is shown in this figure, the extracted features, which are shown by stars, are well matched and related to each other with connecting lines.
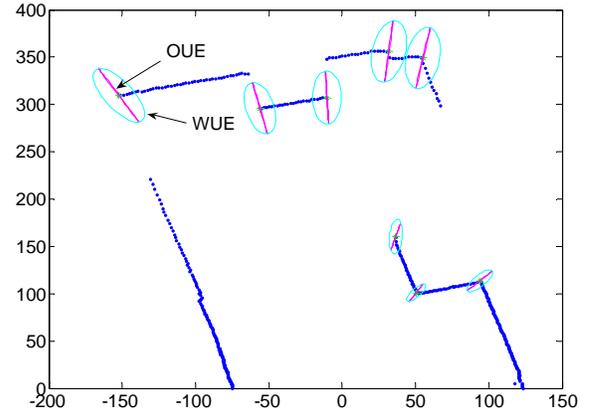
Fig. 5. Extracted features from scan number 74. Ellipsoids in magenta represent the measurement covariance (OUE) of each feature and cyan ellipsoids (WUE) related to whole covariance of each feature.

Figure 7 shows the map constructed by superimposing 193 scans gagged during drive in an L shape corridor of our lab. It should be mentioned that the only sensor used for localization is laser range scanner and none of the map improving algorithms such as global relaxation [16] is considered in this experiment. For the sake of comparison, we also implemented the HAYAI method and the same map produced with this method. Due to feature extraction and matching methods in HAYAI, this method uses some filters. In the case of high length filters, information relative to the details in scan is omitted, so only if there are features correspond to big jump-edges or features correspond to sharp corners with high length lines in the scan, it has a chance to choose appropriate features. However, even in the presence of such features, due to high length filtering, position of scan points varies and error between selected point as feature and exact position of feature and features' uncertainty will increase. After all, in low range LRFs probability of such features' existence is considerably low. Therefore, small length filters should be used, (which is the case in [13]) but this case suffers from the lack of robust feature extraction and inaccurate localization in noisy scans. Besides, weakness of this method to block variant features, and discrimination between effects of features with high and low uncertainty in localization are other reason for the inaccuracy.

As it is shown in figure 7, although there are small details in the environment, map is relatively well extracted by the proposed method. This is mainly due to the structure of the proposed method, in which basically no data reduction filtering is used, and therefore, the small size walls are well extracted in the map. In addition, omitting variant features and optimally incorporating features regarding their uncertainty lead to accurate superimposing raw scans data. The cost of the significant improvement in the accuracy is mainly the relative computation time. On a 1.1 GHZ processor, Processing 193 scans by our method needed 5.67 seconds which is approximately five times of that for HAYAI procedure. However, the amount of time required is still much lower

compared to that of histogram based or point-based method (like IDC) and the algorithm is fast enough for real-time localization in high speed robots. A comparison between HAYAI and other algorithms speed is reported in [13].

V. CONCLUSIONS

This paper describes a feature based matching algorithm in which features are incorporated in robot pose calculation, based on their uncertainty. Total uncertainty of each individual feature is represented by calculating the observation and quantization covariance. Optimal pose displacement estimation is calculated by minimizing the objective function derived from maximum likelihood concept. The feature extraction algorithm proposed in this paper uses a global filtering of the data, using a line fitting algorithm, in which a split-and-merge method is used. In order to accurately estimate the features, local information is used after global filtering stage. Matching of the features for consecutive scans is accomplished by analyzing the parameters specify each feature. The method is implemented on a mobile robot equipped with small range URG_X002 laser range scanner, and the experimental results verifies the effectiveness of the proposed method in feature extraction, mapping, and robot localization.

REFERENCES

[1] R. C. Smith and P. Cheeseman, .On the representation and estimation of spatial uncertainty, Int. J. of Robotics Research, vol. 5, no. 4, pp. 56.68, 1986.

[2] S. Thrun, D. Fox, and W. Burgard, A Probabilistic Approach to Concurrent Mapping and Localization for Mobile Robots, Machine Learning, vol. 31, pp. 29–53, 1998.

[3] S.I. Roumeliotis and G.A. Bekey, Bayesian estimation and Kalman filtering: A unified framework for Mobile Robot Localization, in Proc. IEEE Int. Conf. on Robotics and Automation, San Fransisco, CA, April 24-28 2000, pp. 2985–2992.

[4] J. Leonard and H. Durrant-Whyte, Mobile robot localization by tracking geometry beacons, IEEE Trans. on Robotics and Automation, vol. 7, no. 3, pp. 376–382, June 1991.

[5] M. Amann, T. Bosch, M. Lescure, R. Myllyl¨a, and M. Rioux, .Laser Ranging: A critical review of usual techniques for distance measurement, Opt. Eng., vol. 40, no. 1, pp. 10.19, Jan. 2001.

[6] J. Crowley, .World modeling and position estimation for a mobile robot using ultrasonic ranging,. in Proc. IEEE Int. Conf. on Robotics and Automation, 1989, pp. 674.680.

[7] F. Lu and E. Milios, Robot Pose Estimation in Unknown Environments by Matching 2D Range Scans, J. of Intelligent and Robotic Systems, vol. 20, pp. 249–275, 1997.

[8] S. Pfister, K. Kriechbaum, S. Roumeliotis, J. Burdick, Weighted range sensor matching algorithms for mobile robot displacement estimation, in: Proceedings of the IEEE International Conference on Robotics and Automation (ICRA'02), 2002, pp. 667–1674.

[9] AJensen, B.; Siegwart, R. "Scan alignment with probabilistic distance metric", in Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems, 2004.

[10] J. Minguez, F. Lamiraux, and L. Montesano, "Metric-based scan matching algorithms for mobile robot displacement estimation," in Proceedings of the IEEE International Conference on Robotics and Automation (ICRA), Barcelona, Spain, 2005.

[11] J. H. Moon, B. J. You, H. Kim, S. R. Oh, Fast Markov localization using angle-histogram, in: Proceedings of the 11th IEEE International Conference on Advanced Robotics (ICAR'03), Coimbra, Portugal, 2003, pp. 411–416.

[12] J.V. Bitterling, S. K¨upper, B. Mertsching, Incremental feature-based pose tracking for a mobile robot, in: Proceedings of the 11th IEEE
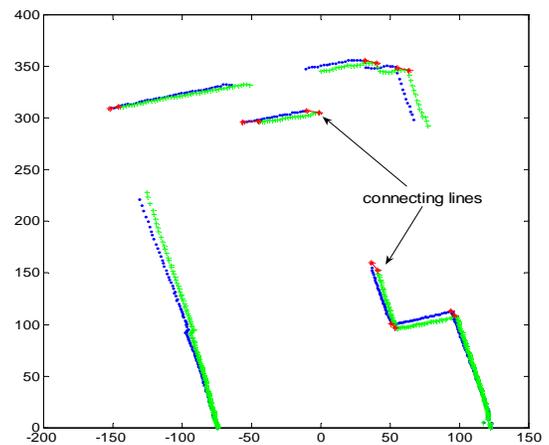
Fig. 6. Reference scan is shown by '•' and the new scan is shown by '+'. Matched features are specified by red stars and connected to each other.

Fig. 7. Constructed map using proposed method
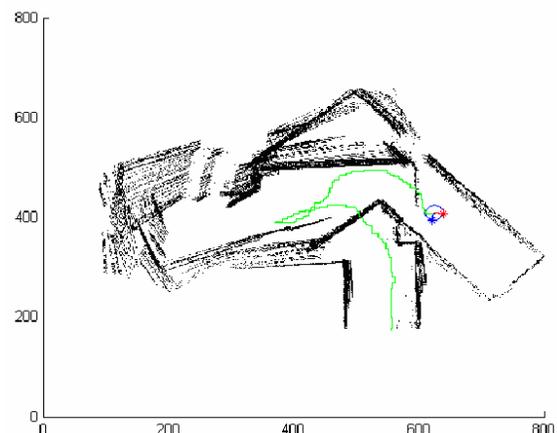
Fig. 8. Constructed map using HAYAI method

International Conference on Advanced Robotics (ICAR'03), Coimbra, Portugal, 2003, pp. 1038–1043.

[13] K. Lingemann, H. Surmann, A. Nuchter, and J. Hertzberg, High-speed laser localization for mobile robots, Journal of Robotics and Autonomous Systems, vol. 51, no. 4, pp. 275–296, 2005.

[14] V. Nguyen, A. Martinelli, N. Tomatis, and R. Siegwart, A Comparison of Line Extraction Algorithms using 2D Laser Rangefinder for Indoor Mobile Robotics, in Proc. 2005 IEEE Int. Conf. on Intelligent Robots and Systems, 2005.

[15] Coleman, T.F. and Y. Li, An Interior, Trust Region Approach for Nonlinear Minimization Subject to Bounds, SIAM Journal on Optimization, Vol. 6, pp. 418-445, 1996.

[16] F. Lu, E. Milios, Globally consistent range scan alignment for environment mapping, Autonomous. Robots 4 (1997) 333–349.