Measuring conflict and agreement between two prioritized belief bases

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Abstract

In this paper, we investigate the relationship between two prioritized knowledge bases by measuring both the conflict and the agreement between them. First of all, a quantity of conflict and two quantities of agreement are defined. The former is shown to be a generalization of the Dalal distance. The latter are, respectively, a quantity of strong agreement which measures the amount of information on which two belief bases "totally" agree, and a quantity of weak agreement which measures the amount of information that is believed by one source but is unknown to the other. All three quantity measures are based on the *weighted prime implicant*, which represents beliefs in a prioritized belief base. We then define a degree of conflict and two degrees of agreement based on our quantity of conflict and the quantities of agreement. We also consider the impact of these measures on belief merging and information source ordering.

1 Introduction

In the belief revision and belief merging literature, the well-known Dalal distance known as the Hamming distance between interpretations [Dalal, 1988], plays a key role in the notion of minimal change. The Dalal distance between two interpretations models how many atoms are in conflict, so it measures only the quantity of conflict between them. Hunter has defined a degree of conflict between two knowledge bases based on the Dalal distance [Hunter, 2004].

In recent years, relationships between two knowledge bases have been defined by measures of information and contradiction. In [Konieczny *et al.*, 2003], a degree of contradiction and a degree of ignorance were defined and they can be used to order the sources of information. If a knowledge base has a high degree of contradiction and a low degree of ignorance, then it has a low order. In [Hunter, 2002], some compromise relations between two knowledge bases were defined according to the quantities of conflict information and total information in them.

In all the relationships described above, the quantity of conflict information between two knowledge bases was focused upon. However, in reality, when establishing the relationships between two agents, not only information in conflict, but also information in agreement should be considered. The quantities of conflict and agreement can affect each other. Considering two agents with low quantity of conflict between them, our perception of the degree of conflict between them will be further weakened if they have a lot in common. Furthermore, when two agents have no information in conflict, it is useful to consider the agreement between them.

We use two quantities of agreement; one is called the quantity of strong agreement which measures the information that both agents "totally" agree with, and the other is called the quantity of weak agreement which measures the information that is believed by one source but is *unknown* to the other. Both quantities will influence the degree of conflict, but their influences are different. Intuitively, the quantity of strong agreement will have more influence on the degree of conflict than the quantity of weak agreement. To illustrate, let us consider the following three knowledge bases: $B_1 = \{\phi, \psi\}$, $B_2 = \{\neg\phi, \psi\}$, and $B_3 = \{\neg\phi\}$. B_1 is in conflict with both B_2 and B_3. B_1 and B_2 strongly agree on ψ, whilst B_1 only weakly agrees with B_3 on ψ. Clearly the degree of conflict between B_1 and B_2 should be smaller than that between B_1 and B_3 because there is a topic that both B_1 and B_2 agree opon. However, when defining the degree of conflict [Hunter, 2004], Hunter did not distinguish the influences of strong agreement and weak agreement. To accompany the degree of conflict, we can define a degree of strong agreement and a degree of weak agreement.

In this paper, we will measure the conflict and agreement between two *prioritized* knowledge bases, where the priority of a formula, based on necessity degrees, is handled in possibilistic logic. It is well-known that priority plays an important role in inconsistency handling, belief revision and belief merging [Gärdenfors, 1988; Konieczny & Ramon, 1998; Benferhat *et al.*, 2002]. Possibilistic logic provides a good framework for expressing priority. We first define the *weighted prime implicant* (WPI), which is a generalization of the regular *prime implicant* to possibilistic logic. Then, the measures of conflict and agreement will be defined by WPIs.

This paper is organized as follows. Section 2 gives some preliminaries. We then define the weighted prime implicant and measures of conflict and agreement in Section 3. Section 4 discusses the impact of the measures of conflict and agreement. Finally we conclude the paper in Section 5.

2 Preliminaries

Classical logic: In this paper, we consider a propositional language $\mathcal{L}_{PS}$ from a finite set PS of propositional symbols. The classical consequence relation is denoted as $\vdash$. An interpretation is a total function from PS to $\{true, false\}$. Ω is the set of all possible interpretations. An interpretation w is a model of a formula ϕ iff $w(\phi) = true$. p, q, r,... represent atoms in PS. A literal is an atom p or its negation $\neg p$. We will denote literals by $l, l_1, ..$ and formulae in $\mathcal{L}_{PS}$ by ϕ, ψ, γ,... For each formula ϕ, we use $M(\phi)$ to denote its set of models. A *classical knowledge base B* is a finite set of propositional formulae. B is consistent iff there exists an interpretation w such that $w(\phi) = true$ for all $\phi \in B$. A *clause* C is a *disjunction* of literals: $C = l_1 \vee ... \vee l_n$ and its dual clause, or *term* D, is a *conjunction* of literals: $D = l_1 \wedge ... \wedge l_n$.

Possibilistic Logic [Dubois *et al.*, 1994]: Possibilistic logic is a weighted logic where each classical logic formula is associated with a number in $[0, 1]$, a necessity degree, which represents the extent to which the formula is true. A possibilistic knowledge base (**PKB**) is the set of possibilistic formulae of the form $B = \{(\phi_i,\ \alpha_i) : i = 1, ..., n\}$. *Possibilistic formula* $(\phi_i,\ \alpha_i)$ means that the necessity degree of ϕ_i is at least equal to α_i. A classical knowledge base $B = \{\phi_i : i = 1, ..., n\}$ corresponds to a PKB $B' = \{(\phi_i, 1) : i = 1, ..., n\}$. In this paper, we consider only PKBs where every formula ϕ is a classical propositional formula. The classical base associated with B is denoted as B^*, namely $B^* = \{\phi_i | (\phi_i,\ \alpha_i) \in B\}$. A PKB is consistent iff its classical base is consistent.

Semantically, the most basic and important notion is *possibility distribution* π: $\Omega \rightarrow [0, 1]$. $\pi(\omega)$ represents the possibility degree of interpretation ω with available beliefs. From *possibility distribution* π, two measures can be determined, one is the possibility degree of formula ϕ, defined as $\Pi(\phi) = max\{\pi(\omega) : \omega \models \phi\}$, the other is the necessity degree of formula ϕ, defined as $N(\phi) = 1 - \Pi(\neg\phi)$. The possibility measure is max-decomposable, i.e. $\Pi(\phi \vee \psi) = max(\Pi(\phi), \Pi(\psi))$. Whilst the necessity measure is min-decomposable, i.e. $N(\phi \wedge \psi) = min(N(\phi), \Pi(\psi))$.

Definition 1 *[Dubois* et al.*, 1994] Let* B *be a PKB, and* $\alpha \in [0, 1]$*. The* α-cut *of* B *is* $B_{\geq\alpha} = \{\phi \in B^* | (\phi, a) \in B \text{ and } a \geq \alpha\}$.

Definition 2 *[Dubois* et al.*, 1994] Let* B *be a PKB. A formula* ϕ *is said to be a* possibilistic consequence *of* B *to degree* a*, denoted by* $B \vdash_\pi (\phi, a)$*, iff the following conditions hold: (1)* $B_{\geq a}$ *is consistent; (2)* $B_{\geq a} \vdash \phi$*; (3)* $\forall b > a$*,* $B_{\geq b} \nvdash \phi$.

3 Measures of Conflicts and Agreements

3.1 Weighted prime implicant

In this section, we will define and discuss weighted prime implicants of PKBs.

A term D is an implicant of formula ϕ iff $D \vdash \phi$ and D does not contain two complementary literals.

Definition 3 *[Cadoli & Donini, 1997]* A prime implicant *of knowledge base B is an implicant D of B such that for every other implicant* D' *of B,* $D \nvdash D'$.

Prime implicants are often used in knowledge compilation [Cadoli & Donini, 1997] to make the deduction tractable. Suppose D_1, ..., D_k are all the prime implicants of B, we have $B \vdash \phi$, for any ϕ iff for every prime implicant D_i, $D_i \vdash \phi$.

Now we define weighted prime implicants of a PKB. Let us first define weighted prime implicants for PKB $B = \{(\phi_1, a_1), ..., (\phi_n, a_n)\}$ where ϕ_i are clauses. For a more general PKB, we can decompose it to an equivalent PKB whose formulae are clauses by the min-decomposability of necessity measures, i.e., $N(\wedge_{i=1,k}\phi_i) \geq m \Leftrightarrow \forall i, N(\phi_i) \geq m$.

Let $B = \{(\phi_1, a_1), ..., (\phi_n, a_n)\}$ be a PKB where ϕ_i are clauses. A weighted implicant of B is $D = \{(\psi_1, b_1), ..., (\psi_k, b_k)\}$, a PKB, such that $D \vdash_\pi B$, where ψ_i are literals. Let D and D' be two weighted implicants of B, D is said to be more *specific* than D' iff $D \neq D'$, $D'^* \subseteq D^*$ and $\forall (\psi_i, a_i) \in D, \exists (\psi_i, b_i) \in D'$ with $b_i \leq a_i$.

Definition 4 *Let* $B = \{(\phi_1, a_1), ..., (\phi_n, a_n)\}$ *be a PKB where* ϕ_i *are clauses. A weighted prime implicant (***WPI***) of* B *is* D *such that*

1. *D is a weighted implicant of B*
2. *$\nexists\, D'$ of B such that D is more specific than D'.*

Let us look at an example to illustrate how to construct WPIs.

Example 1 *Let* $B = \{(p, 0.8), (q \vee r, 0.5), (q \vee \neg s, 0.6)\}$ *be a PKB. The WPIs of B are* $D_1 = \{(p, 0.8), (q, 0.6)\}$, $D_2 = \{(p, 0.8), (r, 0.5), (\neg s, 0.6)\}$, *and* $D_3 = \{(p, 0.8), (q, 0.5), (\neg s, 0.6)\}$.

The WPI generalizes the prime implicant.

Proposition 1 *Let* $B = \{(\phi_1, 1), ..., (\phi_n, 1)\}$ *be a PKB which contains formulae with weight 1, i.e.,* B *is a classical knowledge base. Then* D *is a WPI of* B *iff* D *is a prime implicant of* B.

However, given PKB B, if D is a WPI of B, then D^* is not necessarily a prime implicant of B^*. A counterexample can be found in Example 1, where D_3 is a WPI, but $D_3^* = \{p, q, \neg s\}$ is not a prime implicant of B^*.

The following proposition says that WPIs can be used to compile a PKB.

Proposition 2 *Let B be a PKB. If* D_1,...,D_n *are all the WPIs of B, then for any formula* ϕ*, we have,*

$$B \vdash_\pi (\phi, a) \; iff \; D_i \vdash_\pi (\phi, a), \; for \; all \; D_i.$$

Next we give some justification for the WPI.

First of all, to measure information in a single classical knowledge base (this knowledge base may be inconsistent), most of the current methods are based on the *models* of the knowledge base [Hunter, 2002; Lozinskii, 1994]. In [Hunter, 2002], the degree of inconsistency is measured based on the *model* of an inconsistent knowledge base in the framework of quasi-classical logic. In [Lozinskii, 1994], a *quasi-model* of an inconsistent knowledge base, which is a maximal consistent subbase of the knowledge base, is defined to measure information for inconsistent sets. By Definition 4, each WPI can be interpreted as a partial truth assignment. Suppose p is an atom and D is a WPI, then $(p, a) \in D$ means that there is an argument for p in D with certainty degree a, and $(\neg p, b) \in D$ means that there is an argument against p in D with certainty degree b, while $\phi \notin D^*$ means the truth value of ϕ is undetermined in D. By Proposition 2, a WPI can be viewed as a *partial model* of a possibilistic knowledge base. This is consistent with the methods in [Hunter, 2002; Lozinskii, 1994].

Second, when all the formulae in a PKB have the same weight 1, a WPI is a prime implicant. In classical logic, a classical model is often used to define the distance between two knowledge bases [Dalal, 1988]. However, classical models are not suitable for us to define the quantities of agreement between knowledge bases because a classical model must assign a truth value to every atom in the knowledge bases. Let us look at the example in the introduction again. The only model for B_1 is $w = \{\phi, \psi\}$ and there are two models for B_3, i.e., $w_1 = \{\neg\phi, \psi\}$ and $w_2 = \{\neg\phi, \neg\psi\}$. B_1 and B_3 weakly agree on ψ because only B_1 supports ψ. However, by comparing w with w_1 or comparing w with w_2 we cannot get such a conclusion. In contrast, a prime implicant can be viewed as a *partial* truth assignment. That is, only some of the atoms are assigned truth values. Given a prime implicant D of B, a three-value semantics can be associated with it as follows:

$$v_D(p) = \begin{cases} true & \text{if } D \vdash p, \\ false & \text{if } D \vdash \neg p, \\ undetermined & \text{otherwise.} \end{cases} \quad (1)$$

In the example, B_1 has one prime implicant $D_1 = \{\phi, \psi\}$ and B_3 has one prime implicant $D_2 = \{\neg\phi\}$, where D_2 does not contain any information on ψ; so the quantity of weak agreement between D_1 and D_2 is 1. As a consequence, the weak agreement between B_1 and B_3 is 1, which is consistent with our analysis above.

3.2 Quantity of conflict and quantities of agreement

In this section, we will measure the quantities of conflict and agreement between two PKBs based on the WPI.

First we define the quantity of conflict between two WPIs.

Definition 5 *Let B_1 and B_2 be two PKBs. Suppose C and D are WPIs of B_1 and B_2 respectively, then the* quantity of conflict between C and D *is defined as*

$$q_{Con}(C, D) = \sum_{(p,a)\in C \ and \ (\neg p,b)\in D} min(a, b). \quad (2)$$

When all the weights are 1, $q_{Con}(C, D)$ is the cardinality of the set of atoms which are in conflict in $C \cup D$.

Definition 6 *Let B_1 and B_2 be two PKBs. Suppose $\mathcal{C}$ and $\mathcal{D}$ are the sets of WPIs of B_1 and B_2 respectively, then the* quantity of conflict between B_1 and B_2 *is defined as*

$$Q_{Con}(B_1, B_2) = min\{q_{Con}(C, D) | C \in \mathcal{C}, \ D \in \mathcal{D}\}. \quad (3)$$

The quantity of conflict between B_1 and B_2 measures the minimum amount of information in conflict between them.

Example 2 *Let $B_1 = \{(\neg p, 0.8), (\neg q \vee r, 0.6)\}$ and $B_2 = \{(p \vee \neg r, 0.7), (q, 0.5)\}$ be two PKBs. The WPIs of B_1 are $C_1 = \{(\neg p, 0.8), (\neg q, 0.6)\}$ and $C_2 = \{(\neg p, 0.8), (r, 0.6)\}$, and the WPIs of B_2 are $D_1 = \{(p, 0.7), (q, 0.5)\}$ and $D_2 = \{(\neg r, 0.7), (q, 0.5)\}$. It is easy to calculate that $q_{Con}(C_1, D_1) = 1.2$, $q_{Con}(C_1, D_2) = 0.5$, $q_{Con}(C_2, D_1) = 0.7$, $q_{Con}(C_2, D_2) = 0.6$. Therefore, the quantity of conflict between B_1 and B_2 is 0.5.*

Proposition 3 *Let B, B_1 and B_2 be three PKBs. If $B_1 \subseteq B_2$, then $Q_{Con}(B, B_1) \leq Q_{Con}(B, B_2)$.*

Proposition 3 tells us that the quantity of conflict between two PKBs will increase (not strictly) when one of them has some new information added.

Let X be a set of classical propositional formulae. Let $I(X)$ be the set of interpretations of X delineated by the atoms used in X (i.e. $I(X) = 2^{Atoms(X)}$, where $Atom(X)$ denotes the set of atoms appearing in X). Let $M(X, Y)$ be the set of **models** of X that are in $I(Y)$. That is, $M(X, Y) = \{w \models \wedge X | w \in I(Y)\}$. The Dalal distance [Dalal, 1988] between two models w_i, w_j of a classical formula is the Hamming distance between them, i.e., $Dalal(w_i, w_j) = |w_i - w_j| + |w_j - w_i|$.

Proposition 4 *Let B_1 and B_2 be two consistent classical knowledge bases. Let $Dalal\ (B_1, B_2) = min \{Dalal(w_i, w_j) | w_i \in M(B_1, B_1 \cup B_2), w_j \in M(B_2, B_1 \cup B_2)\}$. Then we have*

$$Q_{Con}(B_1, B_2) = Dalal(B_1, B_2).$$

Proposition 4 is very important, because it reveals that our quantity of conflict coincides with the Dalal distance in classical logic. Therefore, the quantity of conflict $Q_{Con}(B_1, B_2)$ can be taken as a generalization of the Dalal distance.

Definition 7 *Let B_1 and B_2 be two PKBs. Suppose C and D are WPIs of B_1 and B_2 respectively, then the* quantity of strong agreement between C and D *is defined as*

$$q_{SA}(C, D) = \sum_{(p,a)\in C, \ (p,b)\in D} min(a, b), \quad (4)$$

When all the weights are 1, $q_{Con}(C, D)$ is the cardinality of the set of literals that are in both C and D.

Definition 8 *Let B_1 and B_2 be two PKBs. Suppose $\mathcal{C}$ and $\mathcal{D}$ are the sets of WPIs of B_1 and B_2 respectively, then the* quantity of strong agreement between B_1 and B_2 *is defined as*

$$Q_{SA}(B_1, B_2) = max\{q_{SA}(C, D) | C \in \mathcal{C}, \ D \in \mathcal{D}\}. \quad (5)$$

The quantity of strong agreement between B_1 and B_2 measures how much information is supported by both B_1 and B_2.

Example 3 *(Continue Example 2) By Equation 4, we have $q_{SA}(C_1, D_1) = q_{SA}(C_1, D_2) = 0$ and $q_{SA}(C_2, D_1) = q_{SA}(C_2, D_2) = 0$. Therefore, the quantity of strong agreement between B_1 and B_2 is $Q_{SA}(B_1, B_2) = 0$.*

Definition 9 *Let B_1 and B_2 be two PKBs. Suppose C and D are WPIs of B_1 and B_2 respectively, then the* quantity of weak agreement between C and D *is defined as*

$$q_{WA}(C, D) = \sum_{(p_i,a_i)\in C\cup D, p_i \notin C^* \cap D^* \ and \ \neg p_i \notin C^* \cup D^*} a_i. \quad (6)$$

When all the weights are 1, $q_{Con}(C, D)$ is the cardinality of the set of literals which are in only one of C or D.

Definition 10 *Let B_1 and B_2 be two PBKs. Suppose $\mathcal{C}$ and $\mathcal{D}$ are the sets of WPIs of B_1 and B_2 respectively, then the* quantity of weak agreement between B_1 and B_2 *is defined as*

$$Q_{WA}(B_1, B_2) = max\{q_{WA}(C, D)|C{\in}\mathcal{C},\ D{\in}\mathcal{D}\}. \quad (7)$$

The quantity of weak agreement between B_1 and B_2 measures the information supported by only one knowledge base and *unknown* to the other.

Example 4 *(Continue Example 2) By Equation 6, we have $q_{WA}(C_1, D_1) = 0$, $q_{WA}(C_1, D_2) = 1.5$, and $q_{WA}(C_2, D_1) = 1.1$, $q_{WA}(C_2, D_2) = 1.3$. Therefore, the quantity of weak agreement between B_1 and B_2 is $Q_{WA}(B_1, B_2) = 1.5$.*

The function Q_{WA} is not monotonic with regard to the subset relation, as shown below

Example 5 *Let $B_1 = \{(p, 1)\}$ and $B_2 = \{(q, 1)\}$, then $Q_{WA}(B_1, B_2) = 2$. However, the quantity of weak agreement between $B_3 = \{(p, 1), (p \to \neg q, 1)\}$ and B_2 is $Q_{WA}(B_2, B_3) = 1$, where $B_1{\subseteq}B_3$.*

Based on the quantity of conflict and quantities of agreement, we can define the following relationships between two knowledge bases B_1 and B_2 as

- B_1 and B_2 are said to be *totally in conflict* iff $Q_c(B_1, B_2) > 0$ and $Q_{SA}(B_1, B_2) = Q_{WA}(B_1, B_2) = 0$.
- B_1 and B_2 are *totally in agreement* iff $Q_{Con}(B_1, B_2) = Q_{WA}(B_1, B_2) = 0$ and $Q_{SA}(B_1, B_2) > 0$.
- B_1 and B_2 are *partially in conflict* iff $Q_{Con}(B_1, B_2) > 0$ and $Q_{SA}(B_1, B_2) + Q_{WA}(B_1, B_2) > 0$.

3.3 Degree of conflict and degrees of agreement

In this subsection, we will define a degree of conflict and two degrees of agreement between two PKBs. The degree of conflict measures the extent to which two knowledge bases are in conflict. It was first introduced in [Hunter, 2004] to measure the believability of arguments.

Definition 11 *Let B_1 and B_2 be two self-consistent knowledge bases, and $Dalal(B_1, B_2)$ be the Dalal distance between B_1 and B_2. The* degree of conflict between B_1 and B_2*, denoted as $C(B_1, B_2)$, is defined as follows:*

$$C(B_1, B_2) = \frac{Dalal(B_1, B_2)}{log_2(|I(B_1{\cup}B_2)|} \quad (8)$$

Although this definition gives a method to measure the degree of conflict, it can sometimes overestimate the degree of conflict between two knowledge bases, because it doesn't distinguish the influences of strong agreement and weak agreement. For example, let us consider two pairs of knowledge bases (B_1, B_2) and (B_1', B_2), where $B_1 = \{p, q, r\}$, $B_2 = \{\neg p, q, r\}$ and $B_1' = \{p\}$. Although the quantity of conflict between B_1 and B_2 is 1, the quantity of strongly agreement between them is 2. This means B_1 and B_2 have more in agreement than in conflict. In contrast, although the quantity of conflict between B_1' and B_2 is also 1, but with $D_{SA}(B_1', B_2) = 0$ and $D_{WA}(B_1', B_2) = 2$, the degree of conflict between B_1' and B_2 should be higher than that between B_1 and B_2. However, by Equation 8, $C(B_1, B_2) = C(B_1', B_2) = 1/3$. This is not reasonable.

We propose the following revised definition of the degree of conflict.

Definition 12 *Let B_1 and B_2 be two PKBs. Let C and D be WPIs of B_1 and B_2 respectively. $Atom_C(C, D)$ denotes the cardinality of the set of atoms which are in conflict in $C{\cup}D$. Then the* degree of conflict between C and D *is defined as*

$$d_{Con}(C, D) = \frac{q_{Con}(C, D)}{Atom_C(C, D) + q_{SA}(C, D) + \lambda q_{WA}(C, D)}, \quad (9)$$

where $\lambda \in (0, 1]$ is used to weaken the influence of the quantity of weak agreement on the degree of conflict. In the following, we always assume that $\lambda = 0.5$, that is, the quantity of weak agreement only has "half" as much the influence on the degree of conflict as the quantity of strong agreement.

Definition 13 *Let B_1 and B_2 be two PKBs. Suppose $\mathcal{C}$ and $\mathcal{D}$ are the sets of WPIs of B_1 and B_2 respectively, then the* degree of conflict between B_1 and B_2 *is defined as*

$$D_{Con}(B_1, B_2) = min\{d_{Con}(C, D)|C{\in}\mathcal{C},\ D{\in}\mathcal{D}\}. \quad (10)$$

The advantage of our degree of conflict can be seen from the following example.

Example 6 *Let us consider a dialogue between three people John, Mary, and Gary. They are discussing "whether Italy is the best football team in the world "(p) and "whether the best forwards are in Brazil" (q). John says "I think Italy is the best football team in the world and the best forwards are in Brazil", Mary says "No, I think France is the best team, but I agree with you that the best forwards are in Brazil", and Gary says "No, I think France is the best team". So the knowledge bases are $John = \{p, q\}$, $Mary = \{\neg p, q\}$ and $Gary = \{\neg p\}$. By Equation 8, we have $C(John, Mary) = C(John, Gary) = 1/2$. This is not reasonable, because John and Mary agree on q, the degree of conflict between them should be less than the degree of conflict between John and Gary. In contrast, we have $D_c(John, Mary) = 1/2$ and $D_{Con}(John, Gary) = 2/3$, so $D_c(John, Mary) < D_c(John, Gary)$.*

Proposition 5 *Let B_1, B_2 be two classical knowledge bases. Suppose $C(B_1, B_2)$ and $D_{Con}(B_1, B_2)$ are the degrees of conflict defined by Definition 11 and Definition 13 respectively. Then $C(B_1, B_2){\geq}D_{Con}(B_1, B_2)$.*

Similarly, we can define the degree of strong agreement. We hold that the influence of the quantity of conflict on the degree of strong agreement is more than that of the quantity of weak agreement.

Definition 14 *Let B_1 and B_2 be two PKBs. Let C and D be WPIs of B_1 and B_2 respectively. $Atom_{SA}(C, D)$ denotes the cardinality of the set of atoms which are included in both C and D. Then the* degree of strong agreement between C and

D is defined as

$$d_{SA}(C,D)=\frac{q_{SA}(C,D)}{Atom_{SA}(C,D)+q_{Con}(C,D)+\lambda q_{WA}(C,D)}, \quad (11)$$

where $\lambda \in (0,1]$ is used to weaken the influence of the quantity of weak agreement on the degree of conflict. As in Definition 12, we usually take $\lambda = 0.5$.

Definition 15 *Let B_1 and B_2 be two PKBs. Suppose $\mathcal{C}$ and $\mathcal{D}$ are the sets of weighted prime implicants of B_1 and B_2 respectively, then the degree of strong agreement between B_1 and B_2 is defined as*

$$D_{SA}(B_1,B_2)=max\{d_{SA}(C,D)|C\in\mathcal{C},\ D\in\mathcal{D}\}. \quad (12)$$

Example 7 *Let $B_1 = \{(p,0.8),(q\vee r,0.4),(p\rightarrow s,0.5)\}$ and $B_2 = \{(p\vee\neg r,0.8),(q,0.6),(\neg s,0.7)\}$. The WPIs of B_1 are $C_1 = \{(p,0.8),(q,0.4),(s,0.5)\}$ and $C_2 = \{(p,0.8),(r,0.4),\ (s,0.5)\}$, and the WPIs of B_2 are $D_1 = \{(p,0.8),(q,0.6),(\neg s,0.7)\}$ and $D_2 = \{(\neg r,0.8),(q,0.6),(\neg s,0.7)\}$. So $d_{SA}(C_1,D_1) = 0.48$, $d_{SA}(C_1,D_2) = 0.17$, $d_{SA}(C_2,D_1) = 0.4$, $d_{SA}(C_2,D_2) = 0$. Therefore, $D_{SA}(B_1,B_2) = 0.48$.*

The degrees of conflict and strong agreement are related to each other.

Proposition 6 *Let B_1 and B_2 be two PKBs. Then their degree of conflict and degree of strong agreement cannot be greater than 0.5 at the same time, i.e., if $D_{Con}(B_1,B_2) > 0.5$, then $D_{SA}(B_1,B_2)\leq 0.5$.*

We can also define the degree of weak agreement.

Definition 16 *Let B_1 and B_2 be two PKBs. Let C and D be WPIs of B_1 and B_2 respectively. $Atom_{WA}(C,D)$ denotes the cardinality of the set of atoms which are included in only one of C and D but not both. Then the* degree of weak agreement between C and D *is defined as*

$$d_{WA}(C,D)=\frac{q_{WA}(C,D)}{Atom_{WA}(C,D)+q_{Con}(C,D)+q_{SA}(C,D)}, \quad (13)$$

In Definition 16, the quantity of conflict and quantity of strong agreement have the same influence on the degree of weak agreement. When both B_1 and B_2 are classical knowledge bases, we have $d_{WA}(C,D)=\frac{Atom_{WA}(C,D)}{|Atom(C\cup D)|}$.

Definition 17 *Let B_1 and B_2 be two PKBs. Suppose $\mathcal{C}$ and $\mathcal{D}$ are the sets of WPIs of B_1 and B_2 respectively, then the degree of weak agreement between B_1 and B_2 is defined as*

$$D_{WA}(B_1,B_2)=max\{d_{WA}(C,D)|C\in\mathcal{C},\ D\in\mathcal{D}\}. \quad (14)$$

Example 8 *(Continue Example 7) By Definition 16, we have $d_{WA}(C_1,D_1) = 0$, $d_{WA}(C_1,D_2) = 0.55$, $d_{WA}(C_2,D_1) = 0.3$, $d_{WA}(C_2,D_2) = 0.48$. So the degree of weak agreement between B_1 and B_2 is 0.55.*

Proposition 7 *Let B_1 and B_2 be two possibilistic knowledge bases. If $d_{WA}(B_1,B_2) > 0.5$, then $d_{Con}(B_1,B_2) < 0.5$ and $d_{SA}(B_1,B_2) < 0.5$.*

Proposition 7 shows that if the degree of weak agreement between two knowledge bases is large, i.e., greater than 0.5, then both the degree of conflict and degree of strong agreement between them should be small, i.e., less than 0.5.

4 Impact of Measures of Conflict and Agreement

4.1 Choice of combination operators

Many operators have been proposed for merging PKBs. Given two PKBs B_1 and B_2 with possibility distributions π_{B_1} and π_{B_2} respectively, the semantic results of their combination by a T-norm tn and a T-conorm ct are

$$\forall\omega,\ \pi_{tn}(\omega)=tn(\pi_{B_1}(\omega),\pi_{B_2}(\omega)), \quad (15)$$
$$\forall\omega,\ \pi_{ct}(\omega)=ct(\pi_{B_1}(\omega),\pi_{B_2}(\omega)). \quad (16)$$

The syntactic results associated with π_{tn} and π_{ct} are respectively the following PKBs [Benferhat *et al.*, 2002]:

$$B_{tn} = B_1\cup B_2\cup\{(\phi_i\vee\psi_j,\ ct(\alpha_i,\beta_j))|(\phi_i,\ \alpha_i)\in B_1 \text{ and } (\psi_j,\ \beta_j)\in B_2\}, \quad (17)$$

$$B_{ct} = \{(\phi_i\vee\psi_j,\ tn(\alpha_i,\ \beta_j))|(\phi_i,\ \alpha_i)\in B_1 \text{ and } (\psi_j,\ \beta_j)\in B_2\}. \quad (18)$$

If we require that the result of the combination be a consistent knowledge base, then the T-norm based operator cannot be used when there is a conflict between B_1 and B_2. In this case, we can only use a T-conorm based operator.

Typical T-norm operators are the minimum operator, the product operator and the *Łukasiewicz T-norm* ($tn_Ł$ for short) ($max(0,a+b-1)$). The duality relation respectively yields the following T-conorm: the maximum operator, the *probabilistic sum* (ct_{ps} for short) ($a+b-ab$), and the *bounded sum* (ct_{bs} for short) ($min(1,a+b)$).

Although some criteria to choose between merging operators have been given in [Benferhat *et al.*, 1997; 2002], these criteria are not enough.

Suppose two PKBs B_1 and B_2 are consistent, then the degree of conflict between them must be 0 and at least one of the degrees of agreement is greater than 0. If the degree of strong agreement between B_1 and B_2 is very high, then B_1 and B_2 share beliefs on most of the topics. In this case, it is advisable to combine them using an operator with higher *reinforcement* effect, for example, the *Łukasiewicz t-norm* max(0,a+b-1). However, if the degree of strong agreement between B_1 and B_2 is low and the degree of weak agreement between them is very high, it is advisable to combine them using the minimum operator which does not have any *reinforcement* effect.

Suppose B_1 and B_2 are in conflict, we usually use a T-conorm to combine them. When the degree of conflict between B_1 and B_2 is very high, then B_1 and B_2 have mostly different beliefs and we can choose the "bounded sum" operator which has a high *counteract* effect. On the other hand, if the degree of conflict between B_1 and B_2 is very low, we can choose the maximum which does not have any *counteract* effect.

More formally, we have the following criteria to choose between merging operators.

Merging operators selection criterion: Let $\oplus_1$ and $\oplus_2$ be two operators applied to merge A and B, and C and D respectively, then for all a, $b\in[0,1]$,
(1) $\oplus_1(a,b)\leq\oplus_2(a,b)$ if $0 < D_{Con}(A,B) < D_{Con}(C,D)$
(2) $\oplus_1(a,b)\geq\oplus_2(a,b)$ if $D_{Con}(A,B) = 0$ and $D_{SA}(A,B) < D_{SA}(C,D)$.

Example 9 *Let* $B_1 = \{(p, 0.6), (q \vee \neg r, 0.7), (s, 0.6)\}$ *and* $B_2 = \{(p, 0.5), (q, 0.4), (s, 0.4)\}$*, where* $D_{Con}(B_1, B_2) = 0$ *and* $D_{SA}(B_1, B_2) = 0.43$*. The merging operator here should be the* product *operator, and the result of merging is* $B = \{(p, 0.6), (q\vee\neg r, 0.7), (s, 0.6), (p, 0.5), (q, 0.4), (s, 0.4), (p, 0.8), (p\vee q, 0.76), (p\vee s, 0.76), (p\vee q\vee\neg r, 0.85), (q\vee\neg r, 0.88), (q\vee\neg r\vee s, 0.88), (p\vee s, 0.8), (q\vee s, 0.76), (s, 0.76)\}$*. However, if we use a* Lukasiewicz t-norm*, the result of merging is* $B' = \{\{(p, 0.6), (q\vee\neg r, 0.7), (s, 0.6), (p, 0.5), (q, 0.4), (s, 0.4), (p, 1), (p\vee q, 1), (p\vee s, 1), (p\vee q\vee\neg r, 1), (q\vee\neg r, 1), (q\vee\neg r\vee s, 1), (p\vee s, 1), (q\vee s, 1), (s, 1)\}\}$*. In* B'*, the weights of formulae* p *and* s *are reinforced to 1. However, the certainty degrees of* p *and* s *are not high in both* B_1 *and* B_2*. Moreover,* B_1 *and* B_2 *are not in strong agreement with each other because* $D_{SA}(B_1, B_2) = 0.43$*. So it is not reasonable to increase the weights of* p *and* s *to the highest certainty degree 1. In contrast, in* B*,* p *and* s *have certainty degrees of 0.8 and 0.76 respectively. Therefore the result of the product operator reflects the reinforcement of* B_1 *and* B_2 *more accurately than that of the* Lukasiewicz t-norm.

4.2 Ordering sources

In this section, we define an ordering relation to compare different knowledge bases based on the degree of conflict.

Definition 18 *Let* B_i*,* B_j*,* B *be three PKBs. A closeness relation* $\preceq_B$ *with regard to* B *is defined as.*

$$B_i \preceq_B B_j \ iff \ D_{Con}(B_j, B) \leq D_{Con}(B_i, B)$$

B_j is closer to B than B_i to B ($B_i \preceq_B B_j$) iff B_j has less quantity of conflict and more quantities of agreement with B than B_i. If $B_i \preceq_B B_j$, then we may view B_j is less problematic or more reliable than B_i with regard to B.

Example 10 *Let* $B_1 = \{(\neg p, 0.8), (\neg q, 0.5), (\neg r \vee s, 0.7)\}$*,* $B_2 = \{(\neg p, 0.8), (\neg q, 0.5), (\neg r, 1), (s, 0.7)\}$*, and* $B = \{(p\vee q, 0.8), (\neg s \vee r, 1)\}$*. Since* $D_{Con}(B_1, B) = 0.22 < 0.41 = D_{Con}(B_2, B)$*, so* $B_2 \preceq_B B_1$*.*

5 Conclusion

The main contribution of this work is that we not only measure the conflict between two prioritized knowledge bases but also measure their agreement in two ways. We defined the quantity of conflict and two quantities of agreement. The quantity of conflict is a generalization of the Dalal distance. We then defined the degree of conflict and degrees of agreement based on both the quantity of conflict and the quantities of agreement. We have shown that the definition of degree of conflict is more reasonable than that defined in [Hunter, 2004]. The measures of conflict and agreement can be very useful in many applications, such as belief merging, argumentation and heterogeneous source integration and management. Another potential application of the measures of conflict is to belief revision as we have shown that the quantity of conflict generalizes the Dalal distance.

We didn't touch the computational issue in this paper. It is clear that the computations of measures of conflict and agreement defined in Section 3.2 and 3.3 depend on the computation of WPIs. It has been shown in [Bittencourt *et al.*, 2004] that the computation of the set of prime implicants of a formula represented by *conjunctive normal form* is NP-$complete$ using a transformation algorithm in [Bittencourt *et al.*, 2003]. Given a PKB $B = \{(\phi_1, a_1), ..., (\phi_n, a_n)\}$ where ϕ_i are clauses, it is expected that the computation of all the WPIs of B is also NP-$complete$ by generalizing the transformation algorithm. This problem will be discussed in a future paper.

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