Incremental Diagnosis of Discrete-Event Systems

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1 Introduction

It is established that diagnosing dynamical systems, represented as discrete-event systems amounts to finding what happened to the system from existing observations. In this context, the diagnostic task consists in determining the trajectories (a sequence of states and events) compatible with the observations. The diagnosis is generally defined as resulting from the synchronization of the automaton modelling the behavior of the system with the automaton that represents the observations sent by the system during the diagnosis period.

In this article, we are interested in avoiding the global computation by slicing the automaton of observations and building the diagnosis on successive slices of observations.

We introduce the concept of *automata chain* to represent an automaton by a sequence of automata slices. We then provide the properties such an automata chain has to satisfy to be a *correct slicing* and define a *reconstruction* operation to get the global automaton back. We demonstrate that, given a correct slicing of the observations, we can compute a global and correct diagnosis from the reconstruction of a diagnosis automata chain.

2 Automata chain

In this section we introduce the concept of *automata chain* whose goal is to enable us to slice an automaton into pieces. We use the well-known definitions of automata (with I the set of initial states and F the set of final states), path and trajectories (paths between an initial and a final state). The main property of an automata chain (first bullet) is that a state is not allowed to appear in two distinct automata of the chain, except if it is a frontier state between two successive automata, i.e it is a final state of the former and an initial state of the later. More generally, if a state belongs to the ith automata and also to the jth automata, with $j > i$, it appears also in all the automata between the ith and the jth as a frontier state.

Definition 1 (Automata chain) *A sequence of automata* $(A^1, \ldots, A^n)$ *with* $A^i = (Q^i, E, T^i, I^i, F^i)$ *is called* automata chain*, and denoted* $\mathcal{E}_A$*, if:*

- $\forall i, j,\ j > i, \forall q,\ q \in Q^i \cap Q^j \Rightarrow q \in F^i \wedge q \in I^{i+1}$,
- $\forall i, j, \forall q, q'$*, if* $\{q, q'\} \subseteq Q^i \cap Q^j$ *then* $\forall p$*, path of* A^i *between* q *and* q'*,* p *is also a path of* A^j*.*

An automata chain is given in Figure 1.

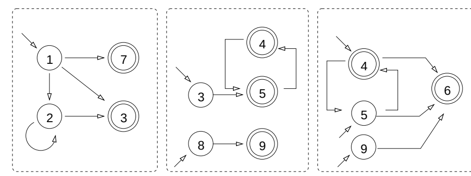

Figure 1: Chain of three automata

Let $\mathcal{E}_A$ be an automata chain $(A^1, \ldots, A^n)$. A trajectory of $\mathcal{E}_A$ is defined as being the ordered (from 1 to n) concatenation of n trajectories, one for each automaton. For instance, the path going from state 1 to state 6 through the states 3 and 5 is a trajectory of the automata chain of Figure 1. Conversely, the path going from state 8 to 6 through 9 is not a trajectory.

Definition 2 (Correct slicing) *Let A be an automaton and* $\mathcal{E}_A = (A^1, \ldots, A^n)$ *an automata chain.* $\mathcal{E}_A$ *is a* correct slicing *of A, denoted* $\mathcal{E}_A = Sli(A)$*, iff the set of trajectories of* $\mathcal{E}_A$ *is equal to the set of trajectories of A.*

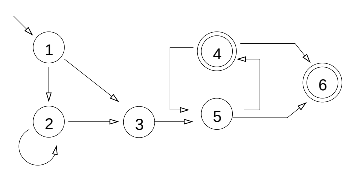

Figure 2: The chain in Figure 1 is a correct slicing of this automaton obtained by reconstruction (see Def. 3) of the chain

Definition 3 (Automaton reconstruction) *Let* $\mathcal{E}_A = (A^1, \ldots, A^n)$ *be an automata chain with* $A^i = (Q^i, E, T^i, I^i, F^i)$*. We call* reconstruction *of the chain* $\mathcal{E}_A$*, the simplified automaton obtained from* $(Q^1 \cup \ldots \cup Q^n, E, T^1 \cup \ldots \cup T^n, I^1, F^n)$.

Theorem 1 *Let A be an automaton and* $\mathcal{E}_A$ *an automata chain. If* $\mathcal{E}_A$ *is a correct slicing of A, then A is obtained by the reconstruction of* $\mathcal{E}_A$. The proof is not given here.

The reconstruction of $\mathcal{E}_A$ is denoted $Sli^{-1}(\mathcal{E}_A)$. If $\mathcal{E}_A$ is a correct slicing of A, then $A = Sli^{-1}(\mathcal{E}_A)$.

We call *prefix-closed automaton of* A (resp. *suffix-closed automaton of* A) denoted A^+ (resp. A^-) the automaton A whose all states are final (resp. initial). We denote $A^{\#}$, the automaton which is both prefix-closed and suffix-closed.

Definition 4 (Automata chain synchronization) *We call* synchronization of an automata chain $\mathcal{E}_A = (A^1, \ldots, A^n)$ with an automaton M *the sequence denoted* $\mathcal{E}_A \otimes M$ *defined by:* $\mathcal{E}_A \otimes M = (A^1 \otimes M^+, A^2 \otimes M^{\#}, \ldots, A^{n-1} \otimes M^{\#}, A^n \otimes M^-)$.

Theorem 2 *Let* $\mathcal{E}_A$ *be an automata chain and* M *an automaton, then* $\mathcal{E}_A \otimes M$ *is an automata chain and* $\mathcal{E}_A \otimes M$ *is a correct slicing of* $Sli^{-1}(\mathcal{E}_A) \otimes M$. The proof is not given.

3 Diagnosis by slices

Let us first recall the definitions used in the domain of discrete-event systems diagnosis where the model of the system is traditionnally represented by an automaton. The model of the system describes its behaviour and the trajectories of Mod represent the evolutions of the system.

Definition 5 (Model) *The* model of the system, *denoted* Mod*, is an automaton* $(Q^{Mod}, E^{Mod}, T^{Mod}, I^{Mod}, F^{Mod})$. I^{Mod} *is the set of possible states at* t_0*. All the states of the system may be final, thus* $F^{Mod} = Q^{Mod}$, $Mod^+ = Mod$ *and* $Mod^{\#} = Mod^-$.

Let us turn to observations and diagnosis definitions. Generally, we don't know the total order on the observations emitted by the system. Consequently, the observations are represented by an automaton, each trajectory of which represents a possible order of emission of the observations.

Definition 6 (Observations) *The* observations, *denoted* Obs_n*, is an automaton describing the observations emitted by the system during the period* $[t_0, t_n]$.

Definition 7 (Diagnosis) *The* diagnosis, *denoted* Δ_n*, is an automaton describing the possible trajectories on the model of the system compatible with the observations sent by the system during the period* $[t_0, t_n]$.

The diagnosis is defined (see [Sampath *et al.*, 1996]) as resulting from the synchronization of the automata representing the system model and the observations: $\Delta_n = Mod \otimes Obs_n$. Using Theo. 2, it is possible to compute the diagnosis by slices. The idea is to compute diagnosis slices, corresponding to observations slices. The global diagnosis can then be reconstructed from the diagnosis automata chain which is obtained.

Definition 8 (Diagnosis by slices - Diagnosis slice) *Let* Mod *be the system model. Let* $\mathcal{E}_{Obs_n} = (Obs^1, \ldots, Obs^n)$*, be a correct slicing of* Obs^n*, the observations emitted during the period* $[t_0, t_n]$*. The synchronization of* $\mathcal{E}_{Obs_n}$ *with* Mod*, i.e* $\mathcal{E}_{Obs_n} \otimes Mod = (Obs^1 \otimes Mod, Obs^2 \otimes Mod^{\#}, \ldots, Obs^n \otimes Mod^{\#})$ *is the* diagnosis by slices *of the system. It can be denoted by the diagnosis automata chain* $(\Delta^1, \ldots, \Delta^n)$*, where* Δ^i *is called the* i*th* diagnosis slice *of the system.*

It can be proved (using Theo. 2) that the diagnosis by slices of a system, here $\mathcal{E}_{Obs_n} \otimes Mod$, correctly represents the diagnosis computed on the global observations since the reconstruction of $\mathcal{E}_{Obs_n} \otimes Mod$ equals the global diagnosis:

Result 1 $\Delta_n = Mod \otimes Obs_n = Sli^{-1}(\mathcal{E}_{Obs_n} \otimes Mod)$

4 Incremental diagnosis

In the diagnosis by slices as presented above, the ith diagnosis slice, Δ^i, is computed independently from the others, by synchronizing the ith observation slice from the chain $\mathcal{E}_{Obs_n}$, Obs^i, with the system model $Mod^{\#}$. In the incremental synchronization, noted $\odot$, (see [Grastien *et al.*, 2005] for more details), the set of initial states of an automaton of the chain is restricted by the set of final states of its predecessor.

Theorem 3 *Let* $\mathcal{E}_A$ *be an automata chain and* M *be an automaton. We have* $Sli^{-1}(\mathcal{E}_A \odot M) = Sli^{-1}(\mathcal{E}_A \otimes M)$. The proof is not given.

Provided that $\mathcal{E}_{Obs_n} = (Obs^1, \ldots, Obs^n)$ is a correct slicing of Obs_n we have: $\Delta_n = Sli^{-1}(\mathcal{E}_{Obs_n} \odot Mod)$.

We note $\forall i,\ \mathcal{E}_{Obs_i} = (Obs^1, \ldots, Obs^i)$, the automata chain of the first i observations automata. Let $i < n$, and $\mathcal{E}_{\Delta_i} = (\Delta^1, \ldots, \Delta^i)$ the automata chain resulting from the incremental synchronisation of $\mathcal{E}_{Obs_i}$ with the system model Mod. We can incrementally compute $\mathcal{E}_{\Delta_{i+1}} = \mathcal{E}_{Obs_{i+1}} \odot Mod$ as follows:

Result 2 $\mathcal{E}_{\Delta_{i+1}} = (\Delta^1, \ldots, \Delta^i, \Delta^{i+1})$ *where* Δ^{i+1} *is the automaton* $(Obs^{i+1} \otimes Mod^{\#})$ *whose initial states are restricted by the set of final states of* Δ^i.

Let Obs_i be the automaton provided by the reconstruction operation on $\mathcal{E}_{Obs_i}$, and let Δ_i be the reconstruction of $\mathcal{E}_{\Delta_i}$.

Result 3 $\Delta_i = Obs_i \otimes Mod$.

5 Conclusion

In this paper, we formalized the computation by slices of diagnosis for discrete-event systems. We introduced and defined the concept of automata chain that enables us to handle slices of observations and slices of diagnosis rather than global observations and global diagnosis. In the diagnosis by slices, the ith diagnosis slice, Δ^i, is computed independently from the others. In [Grastien *et al.*, 2005], we show that this result can be instantiated to the case where the observation automaton is sliced according to time, according to temporal windows.

Our study exhibits the (non trivial) correctness properties that the observation slicing, in an automata chain, has to satisfy in order to guarantee the completeness of the diagnosis computation. This first step is then essential before considering the incrementality of on-line diagnosis computation.

References

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