LOGIC MODELLING OF COGNITIVE REASONING*

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Abstract

Logic modelling is presented as an approach for exploring cognitive reasoning. The notion of mental construction and execution of propositional models is introduced. A model is constructed through inclusions and exclusions of assertions and assumptions about the task. A constructed model is executed in a logical control structure. Formal rules of inference are argued to be an essential feature of this architecture. A few examples are given for purpose of illustration.

1. Introduction

In this paper we present and discuss elements of a logic modelling framework for the study of human reasoning. There are three assumptions involved in developing logic modelling. Firstly, by using the concept of modelling we want to emphasize the computational hypothesis about cognitive processing. Also, computational studies of deductive reasoning are seldom seen, though such studies might throw further light on issues both in AI and cognitive psychology. Secondly, as denoted by using the concept of logic we intend to study and analyze cognitive reasoning in a computational framework which is close to formal logic. Thus, we use logic as a modelling language. Thirdly, we assume that reasoning is a rational process. This is a controversial hypothesis.

Basically, there are three competing hypotheses about the relation between logic and human reasoning |I|. They rest on different interpretations of the following observation. Suppose we have a set of premises, $\{p_1, \ldots, p_m\}$. We give the set to a theorem-prover which generates an inference, S_1. We also give the set to a subject who generates another inference, i_2. Suppose i_1 implies that i_3 is false. This might indicate that the subject has understood the task, but uses a faulty reasoning strategy [2, 3). This is the "ill-logical" hypothesis [2). Alternatively, the subject may not use valid rules of inference at all. In this interpretation, the subject is "non-logical" [2, 4) The third interpretation means that i_3 is made because the subject's understanding is different [l, 3]. Thus, the inference is perfectly valid, but generated from another set of premises, $\{p_1, \ldots, p_m\}$.

As pointed out by Smedslund [5] it is impossible to tell whether or not a subject is "logical" without assuming that he/she has understood the task correctly, and vice versa. We know fairly well what it means to be logical, but, what does it mean to understand correctly? As mentioned, it means that i'i and i_2 both are valid only if they were derived from different sets of premises. If the sets were identical, then the subject's "logic" is "ill" or possibly not there at all.

In our view, understanding means to construct a mental set of the external premises as they are perceived and interpreted. Obviously, such a construction might not be a direct one-to-one mapping of the external premises. Thus, a model might be "incorrect" in the sense of not being a mapping, but "correct" in the sense of being consistent to the reasoner's knowledge.

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Logic modelling is an attempt to explore such a relation between task environment and mental model within a computational framework. In the next sections, we present logic modelling in some detail and introduce the notion of propositional models as a major component in reasoning.

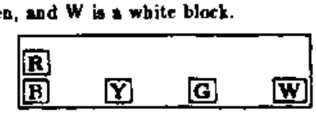
2. Constructing and running propositional models

The following notation is used throughout the discussion. We will assume an implicit $\forall$ quantification and use $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, and $\neg$ for conjunction, disjunction, conditional, biconditional, and negation. Variables will be denoted by x, y, z, etc. and constants by a beginning upper case letter. A ***proposition*** is either an ***assertion***, e.g., $P(x,y)$ or a ***rule*** (we also refer to it as an ***assumption***), e.g., $P(x,y) \wedge Q(y,z) \rightarrow R(x,z)$. A model is a set of assertions and rules.

The mechanisms behind the construction process are knowledge and attention. The goal is to select a set of relevant propositions that correspond to the external reasoning task. These propositions can enter from memory either as assertions or as assumptions. If the construction is driven by knowledge the reasoning effort can be reduced compared to the cases where the construction cannot rely on domain knowledge. The construction of the perceived premises is continued until the model is acceptable (or executable).

The only logical aspect of importance in the construction process is consistency. That is, we assume that an acceptable model is also a consistent model. Naturally, a model can be revised and made inconsistent with an earlier version.

Let us give a very simple illustration. Consider the following situation in the blocks world, where R is a red, B is a blue, Y is a yellow, G is a green, and W is a white block.



A propositional model of this scene could be constructed as follows,

Model 1

$$On(Red, Blue) \quad (1)$$
$$Left(Blue, Yellow) \quad (2)$$
$$Left(Yellow, Green) \quad (3)$$
$$Left(Green, White) \quad (4)$$
$$Left(x,y) \wedge Left(y,z) \rightarrow Left(x,z) \quad (5)$$
$$On(x,y) \wedge Left(y,z) \rightarrow Left(x,z) \quad (6)$$

In this model (1) to (4) correspond to the representation of the objects, whereas (5) and (6) are introduced rules (or assumptions) that might be relevant for the scene. For example, the rule in (5) states that an object, x, is to the left of another object, z, if x is to the left of an object, y, which is to the left of z.

The following propositions constitute a different model,

Model 2

$$Front(Red, Blue) \quad (7)$$
$$Left(Blue, Yellow) \quad (2)$$
$$Left(Yellow, Green) \quad (3)$$
$$Left(Green, White) \quad (4)$$

$$Left(x,y) \wedge Left(y,z) \rightarrow Left(x,z) \quad (5)$$
$$Front(x,y) \wedge Left(y,z) \rightarrow Front(x,z) \quad (8)$$

This model indicates quite another understanding of the scene as shown in (7) and (8). Notice that the two models are not isomorphic. If the assumption (8) was revised to,

Model 3

$$Front(x,y) \wedge Left(y,z) \rightarrow Left(x,z) \quad (9)$$

they would be isomorphic but still quite different. That is, there is a one-to-one map between model 1 and 3 but not between 1 and 2, and vice versa. Also, there is no direct map between 2 and 3 despite the fact that they share the same "view".

We assume that a constructed model is executed within a control structure based on logic. There are several important design issues involved in such an interpretation. For instance, how propositions are matched (e.g., unification), how they are manipulated (e.g., Modus ponens and Modus tollens), when they are manipulated (e.g., depth- or breadth-first), etc.. In the following we assume an interpretation that is a mixture of the control found in production systems |6| and logic programming |7|. Let us exemplify how a model can be run.

Suppose the following question (or goal) is included in the example models. *Is the blue block to the left of the white block?* The matching process will produce the following list of applicable instantiated rules for model 1 in a first cycle,

Forward inferences: Assertions	Rule
$Left(Blue,Yellow) \wedge Left(Yellow,Green) \rightarrow Left(Blue,Green)$	(5)
$Left(Yellow,Green) \wedge Left(Green,White) \rightarrow Left(Yellow,White)$	(5)
$On(Red,Blue) \wedge Left(Blue,Yellow) \rightarrow Left(Red,Yellow)$	(6)

Backward inferences: Subgoals	Rule
$Left(Blue,y) \wedge Left(y,White) \rightarrow Left(Blue,White)$	(5)
$On(Blue,y) \wedge Left(y,White) \rightarrow Left(Blue,White)$	(6)

An obvious solution to this decision problem is simply to execute all the instantiations in parallel. However, this is not very realistic from a cognitive point of view. We assume a control that *(i)* uses unification, *(ii)* is basically forward driven and *(iii)* executes a model in a depth-first manner.

Let us continue the examples. As can be verified, our example models (1,2 and 3) can conclude that the blue block is to the left of the white. Even though the models differ markedly, they can conclude the very same thing. It is trivial that two rcasoners can make the same conclusion with or without the same understanding.

Suppose we give another goal. *It the red block to the left of the green block?* Using a forward inference strategy or some combination, it can be verified that model 1 and 3 will succeed and conclude that it is true that the red block is to the left of the green block. The second model cannot resolve the goal and will conclude the negation of the statement by failure to prove. Naturally, this outcome means that the understanding is quite different. However, and most important, the inferences are all valid.

This discussion puts the issue about logic and reasoning in yet another perspective. For example, suppose that two subjects construct isomorphic models. Their answers to abstract questions (e.g., *It there a relation between the red and the green block?)* should then be isomorphic too. If they are not, we can conclude that at least one of them cannot be modelled by logic. However, it is not clear how we can study such a perspective. Let us start by analyzing reasoning tasks that have been empirically studied.

3. Form and content in reasoning about a rule

A classical task in studies of deductive reasoning is the four-card problem invented by Wason [8| and also extensively investigated by others. Several versions have been studied, but we focus on the following two.

You are presented with four cards showing, respectively, A, D, 4, 7, and you know from previous experience that every card, of which these are a subset, has a letter on one side and a number on the other side. You are then given this rule about the four cards in front of you: *// a card hat a vowel on one tide, then it hat an even number on the other tide.* You are told that the task is to tell which of the cards that need to be turned over in order to find out whether the rule is true or false. The most frequent answers are "A and 4" and "only A". In one study only 5 out of 128 subjects gave the answer A and 7 which is regarded as the correct answer. (3).

In another version the objects are four envelopes: a sealed, an unsealed, an envelope with a 4p stamp, and an envelope with a 5p stamp. The task is to test the rule: *// an envelope it sealed, then it hat a 5p stamp on it.* In one study, 22 out of 25 subjects picked the sealed envelope and the envelope with a 4p stamp on it, which is considered to be the correct answer |3|. Thus, the results are in sharp contrast.

If the two tasks are isomorphic, the result might, on the surface, indicate that human reasoners do not use rules of inference, but only domain-specific operators. In the following we try to show how the results could be interpreted in a logic modelling framework.

Let *Cara(x,y)* denote an object in the "card" task with *z* as the perceived symbol or value and with y as the hidden symbol. Let *Env{z,y)* be the corresponding proposition for the "envelope" task. Thus, we can represent the "card" task as composed of four reasoning objects, e.g., *Cara(A,turn(A)).* The "envelope" task can be represented analogously, e.g., *Env(Sealed,turn{Sealed)),* where *turn* is a function that gives the hidden value in both cases.

Suppose instead that we represent a card as, *Card(z,y,z),* where *z* is the perceived value on the side y, that is shown, and *z* is the hidden value on the other side. Likewise, *Env(x,* y, *z)* could represent the objects in the "envelope" task. For example, *Env(Sealed, Reverse, turn[Sealed))* represents the envelope which has its reverse side turned up. There is no ambiguity in this representation since every reasoner knows that to see if an envelope is sealed one has to get the reverse side. Similarly, stamps are always placed on the frontside. Consider now the "card" task. Should we represent the A as *Cara\A, Front, turn{A)), Cara\A, Back,turn(A)), Card(A, Front, turn{F)ront)),* etc.! Thus, it is possible to argue that this representation shows that the two tasks do not share the same form since we would have,

$$Card(x,Front,turn(x)) \leftrightarrow Card(x,Back,turn(x))$$
$$Env(x,Front,turn(x)) \leftrightarrow \{\neg Env(x,Back,turn(x))\} \quad (2)$$

In order to analyze the rules to be tested we choose the first representation which makes the tasks isomorphic. The rules in the tasks can be expressed as follows,

$$Card(x,y) \rightarrow \{Vowel(x) \rightarrow Even(y)\} \quad (3)$$
$$Card(x,y) \rightarrow \{Vowel(y) \rightarrow Even(x)\} \quad («)$$
$$Env(x,y) \rightarrow \{x = Sealed \rightarrow y = 5p\_stamp\} \quad (5)$$
$$Env(x,y) \rightarrow \{y = Sealed \rightarrow x = 5p\_stamp\} \quad (6)$$

The rules in (4) and (6) represent those cases in which the reasoner has to think of possible hidden values. A reasoner who only chooses to turn A might have excluded (4) from his/her model. If a reasoner chooses to introduce the following rule instead of (4) he/she will turn A and 4,

$$Card(x,y) \rightarrow \{Vowel(y) \leftarrow Even(x)\} \quad (7)$$

In the "envelope" task the relevant assumptions are more easily introduced because of the natural relation between the values of each object. In other words, knowledge about the domain facilitate the introduction of propositions. For example, an envelope can only be sealed or unsealed but not both. If there is a 5p.stamp it does not matter whether it is sealed or not. Thus, it is straight-forward to recognize that if it is not a 5p.stamp on it, then it must not be sealed,

$$Env(x,y) \rightarrow \{\neg(x = 5p\_stamp) \rightarrow \neg(y = Sealed)\} \quad (8)$$

This assumption is equivalent to (6) but expressed as a contrapositive and it is straight-forward to apply. In contrast, the contrapositive to (4) is not easy to recognize. It might be easier to introduce th converse premise as shown in (7).

In short, the postulated isomorphism is not straight-forward, In general, the ambiguity in a reasoning task can trigger the construction of *very* different models. Inferences that on the surface seem to be invalid can be completely valid if the particular mental model from which they were derived is taken into account.

Episode #1	
F1	did not play away against B
F2	yeah according to the first statement
F3	would A
F4	probably win then
F5	if they played at all
F6	if A plays at home against C
F7	then A will not lose
F8	that has nothing to with this
Episode #2	
F9	A did not play away against B
F10	but they did play against B
F11	one could ask
F12	or if they perhaps played at home against C
F13	then they played at home against B
F14	then they should win
Episode #3	
F15	if they don't play against B at all
F16	but at home against C
F17	then A will not lose
F18	are they playing as visitors against C
F19	then you don't know
F20	you can't say anything

Figure 1. A think-aloud protocol for a subject S

4. Towards the study of cognitive model construction

In this section we briefly present an attempt to study the reasoning process within the logic modelling framework. Also, we show how the two processes, construction and execution, interact in a *construct - run - refine - cycle.*

Consider the following reasoning task [9],

If A plays away against B, A will lose. If A plays at home against C, A will not lose. Now, A did not play away against B. What can then be said?

Figure 1 shows a complete protocol given by a subject who had never seen this task before and who had never studied logic.

The fragments are divided into three episodes corresponding to three reasoning attempts. Each episode terminates with an inference and the next episode starts with a smaller or larger revision of the earlier model.

In the first episode there are two inferences made,

Win(A,B)	F4
"that has nothing to do with this"	F8

Only one inference is made in #2, and in #3 there are two,

$Win(A,B)$	F14
$\neg Lose(A,C)$	F17
"then you don't know"	F19

In the first episode, S attends to the first and third premise (see F6-F8). The following propositional model is inferred from the protocol,

Episode #1

$\neg Play\_away(A,B)$	E11
$Play\_away(A,B) \rightarrow Lose(A,B)$	E12
$Play\_home(A,B) \rightarrow Win(A,B)$	E13
$\neg Play\_away(x,y) \rightarrow Play\_home(x,y)$	E14

S excludes the second premise and introduces two assumptions (E13 and E14). The inference in F4 follows naturally from this model if it is executed forwardly.

In #2, S concludes that "A should win if they played at home against B". This is the same inference as in the first episode. The model is not revised, but the fragments F10 and F11 indicate that S is beginning to revise it. In #3 the model is modified,

Episode #3

$\neg Play\_away(A,B)$	E31
$Play\_away(A,B) \rightarrow Lose(A,B)$	E32
$Play\_home(A,C) \rightarrow \neg Lose(A,C)$	E33
$\neg Play(x,y) \rightarrow Play(x,z)$	E34
$\neg Play\_away(x,y) \rightarrow \neg Play(x,y)$	E35
$Play(x,y) \rightarrow \{Play\_home(x,y) \vee Play\_away(x,y)\}$	E36

S has changed his understanding of the task so that he now attends to all the premises in the task. Also, he makes a case analysis (OR-elimination) in which two alternatives are tried. Let us give an example run of this model,

Asserted proposition	Step: reason
$\neg Play(A,B)$	1: E31, E35
$Play(A,C)$	2: E34, Step 1
$Play\_home(A,C) \vee Play\_away(A,C)$	3: E36, Step 2
$Play\_home(A,C)$	4: Case 1, Step 3
$\neg Lose(A,C)$	5: E33, Step 4
$Play\_away(A,C)$	6: Case 2, Step 3
"then you don't know"	7: No rule, Step 6

In step 5 and 7 we find the inferences that S makes in the third episode (F17 and F19).

This brief analysis shows that a reasoning process can be viewed as interactions of constructions and executions. It also shows how it is possible to study reasoning models [9].

5. Concluding remarks

In this paper we have presented elements of an approach to the study of human reasoning that is called logic modelling. We proposed that a reasoning process is composed of two processes, called the construction and execution process. Knowledge and attentional mechanisms directs the construction of a propositional model. An acceptable model is executed within a logical control structure. The basics of this structure are unification, forward and backward inferencing, and rules of reasoning. A few examples have been put forward to illustrate some aspects of logic modelling. The major arguments in the analyses are *(i)* that inferences should be evaluated against the models from which they were derived and *(ii)* that models seldom are one-to-one maps of external premises.

In short, we think that logic modelling is a framework which can contribute to the general study of human reasoning. However, as this paper has indicated, we need to study and analyze the processes of construction and execution in much more detail. For example, "when and why" are implicit assumptions introduced, "when and how" are a model modified, "what" inference rules are used, "how" are reasoning rules designed, and so on.

6. Acknowledgment!

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