The Complexity of Theory Revision

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Abstract

A knowledge-based system uses its database (a k a its "theory") to produce answers to the queries it receives Unfortunately, these an swere may be incorrect if the underlying theory is faulty Standard "theory revision" systems use a given set of "labeled queries" (each a query paired with its correct answer) to transform the given theory, by adding and/or deleting either rules and/or antecedents, into a related theory that is as accurate as possible After formally denning the theory revision task and bounding its sample complexity, this paper addresses the task's computational complexity It first proves that, unless *P = NP,* no polynomial time algorithm can identify the opti mal theory, even given the exact distribution of queries, except in the most trivial of situations It also shows that, except in such trivial situations, no polynomial-time algorithm can produce a theory whose inaccuracy is even close (j e , within a particular polynomial factor) to optimal These results justify the standard practice of hill-chmbing to a locally-optimal theory, based on a given set of labeled samples

1 Introduction

There are many fielded knowledge-based systems, ranging from expert systems and logic programs to production systems and database management systems [Lev84] Each such system uses its database of general informa tion (a k a its "theory") to produce an answer to each given query, this can correspond to retrieving information from a database *(eg,* finding X such that "(mak«a acne X) & (color X red)") or to providing a diagnosis or repair, based on a given set of symptoms Unfortunately, these responses may be incorrect if the underly ing theory includes erroneous information If we observe that some answers are incorrect *(e g* , if the patient does not get better, or the proposed repair does not correct the device's faults), we can then ask a human expert to supply the correct answer *Theory revision* is the process of using such correctly-answered queries to modify the initial theory, to produce a new theory that is more accurate, i e , which will not make those mistakes again, while remaining correct on the other queries

Most theory revision algorithms use a set of transformations to hill-chmb through successive theories, until reaching a theory whose accuracy is (locally) optimal, based on a set of correctly-answered queries, c/, [Pol85, MB88, Coh90, OM94, WP93, CS90, LDRG94] This report addresses the obvious question Is there a better approach, which will directly yield the *globally* optimal theory[7]

Section 2 first states the theory revision objective more precisely as finding the theory with the highest accuracy from the space of theories formed by applying a sequence of transformations to a given initial theory, here each transform involves either adding or deleting either a rule or an antecedent It also proves that a polynomial number of training "labeled queries" (each a specific query paired with its correct answer) is sufficient, i e, they provide the information needed to identify a transformation-sequence that will transform the given theory into a new theory whose accuracy is arbitrarily close to optimal, with arbitrarily high probability Section 3 then addresses the *computational* complexity of the task of finding the optimal (or even near-optimal) revised theory It first proves that the task of computing the optimal theory within this space of theories is intractable, even in trivial contexts — *eg,* even when dealing with propositional Horn theories, or when considering with only atomic queries, or when considenng onl> a bounded number of transformations, etc [1] We then show that this task cannot even be approximated i e , that no efficient algorithm can find a theory whose inaccuracy is even close to *(i* e , within a particular small polynomial of) optimum' We also prove that these negative results apply even when we are only generalizing, or only specializing, the initial theory We also discuss the efficiency of other restricted variants of theory revision,

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throughout, we will assume that *P=/NP* [GJ79], which implies that any NP-hard problem is intractable This also implies certain approximation claims, presented below

providing sharp boundaries that describe exactly when this task is, versus is not, tractable

We view these results as sanctioning the standard approach of using a set of transformations to hill-climb to a local optimum, based on a set of samples The labeled training samples are required to obtain the needed distribution information, and the realization that no tractable algorithm will be able to find the *global* optimum justifies hill-chmbing to a local optimum, within the space formed using specified transformations

We close this section by describing related research [2]

Related Research While most learning systems begin with an "empty theory" and attempt to learn a target function (perhaps a decision tree, or a logic program), theory revision processes work by modify mg a given initial theory There are several implemented theory revision systems Most use essentially the same set of transformations we describe — e g , AUDREY [WP93], FONTE [MB88], EITHER [OM94] and A [LDRG94] all consider adding or deleting antecedents or rules Our analysis, and results, can easily be applied to many other types of modifications — e g , specializing or generalizing antecedents [OM94], using "n-of-m rules" [BM93], or merging rules and removing chains paths of rules that produced incorrect results [Coh90, Coh92] [3] While these projects provide empirical evidence of the effectiveness of their specific algorithms, and deal with classification (J e , determining whether a given element is a member of some target class) rather than general derivation, our work formally addresses the complexities inherent in finding the best theory, for handling arbitrary queries

Finally, note that, in some cases, our task can require extracting the best consistent sub-theory from a given inconsistent theory From this perspective, our work is related to "Knowledge Representation" form of theory revision, a *la* Gardenfors [Gar88, AGM85], Katsuno and Mendelzon [KM9I] and many others Our work differs by using the notion of expected accuracy to dictate which of the "revisions" is best

2 Framework

We define a "theory" as a collection of (propositional or first order) Horn clauses, where each clause is a disjunction of literals, at most one of which is positive Borrowing from [Lev84, DP9I], we also view a theory T as a function that maps each query to its proposed answer, hence, T Q-> A, where *Q* is a (possibly infinite) set of Horn queries, and *A* = { No, Yes } is the set of possible answers [4] Hence, given

T_1 =

```
h  - a, b
h  - f, g
i  - g, j
f  - c, d
g  - e
q   c   d   e
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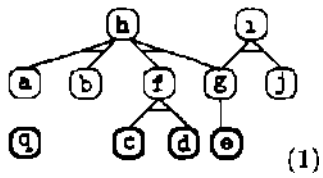


(1)

$T_1(h)$ = Yes, $T_1(i)$ = No and $T_1(i - e, j)$ = Yes Of course, different theories can return different answers to a given query For example, let T_2 be a theory that differs from T_1 only by excluding the "g - e" rule, then $T_2(h)$ = No

While the non atomic queries may seem unusual at first, they are actually quite common For example, a medical expert system typically collects relevant data $\{f_1(p), \ldots, f_n(p)\}$ about an individual patient p, then determines whether p has some specific disease $disease_i$, i e , if $T \cup \{f_1(p), \ldots, f_n(p)\} \models disease_i(p)$, where T is the expert system's initial theory that contains general information about diseases, etc Notice this entailment condition holds iff $T \models \neg f_1(p) \vee \ldots \vee \neg f_n(p) \vee disease_i(p)$, i e , iff the Horn query "$disease_i(p) - f_1(p), \ldots, f_n(p)$" follows from the initial theory (See also "entailment queries [FP93])

We assume there is a single correct answer to each question, and represent it using the real-world oracle $O \quad Q \mapsto A$ Here, perhaps, $O(h)$ = No, meaning that "h" should not hold We say an oracle is *consistent* if it produces the same answers as a Horn theory, over the set of queries $Q \quad N b$, we will sometimes deal with inconsistent oracles, e g , where $O(a)$ = Yes, $O(b - a)$ = Yes, and $O(b)$ = No

Our goal is to find a theory that is as close to $O(\ )$ as possible To quantify this, we first define the "accuracy function" $a(\ ,\ )$ where $a(T, q)$ is the accuracy of the answer the theory T returned for the query q

$$a(T, q) \stackrel{def}{=} \begin{cases} 1 & \text{if } T(q) = O(q) \\ 0 & \text{otherwise} \end{cases}$$

(Notice o(T,) implicitly depends on the oracle *0( ))* Hence, a s $O(h) = a(T_2, \text{"h"}) = 1$ s T 2 provides the correct answer while $a(T_1, \text{"h"}) = 0$ as T_1 returns the wrong answer

This a(T,) function measures T's accuracy for a sin gle query In general, our theories must deal with a range of queries We model this using a stationary, but unknown, probability function *Pr* $Q \mapsto [0, 1]$, "where *PT(Q)* IS the probability that the query *q* will be posed Given this distribution, we can compute the "expected accuracy" of a theory, T

$$A(T) = E[a(T, q)] = \sum_{q \in Q} Pr(q) \times a(T, q)$$

We will consider various sets of possible theories, $\mathcal{T} = \{T_i\}$, where each such T contains the set of theories

[2] The technical report [Gre95b] provides a more extensive literature survey, as well as proofs of the theorems

[3] The companion paper [Gre95a] considers yet other ways of modifying a t h e o ries, y rearranging its component rules or antecedents

[4] To simplify the presentation, the bulk of this paper will deal only with proportional logic, Section 2 3 below describes the extensions needed to deal with predicate calculus

formed by applying various sequences of transformations to a given initial theory, see Section 2 1 below Our challenge is to identify the theory $T_{opt} \in \mathcal{T}$ whose expected accuracy is optimal, i e ,

$$\forall T \in \mathcal{T} \quad A(T_{opt}) \geq A(T) \tag{2}$$

There are two challenges to finding such optimal theories The first is baaed on the observation that the expected accuracy of a theory depends on the distribution of queries, which means different theories will be optimal for different distributions While this distribution is not known initially, it can be estimated by observing a set of samples (each a query/answer pan), drawn from that distribution Section 2 2 below discusses the number of samples required to be confident of obtaining the information needed to identify a good $T^* \in \mathcal{T}$, with high probability

We are then left with the challenge of computing the best theory, once given this distributional estimate Section 3 addresses the computational complexity of this process, showing that the task is not just intractable,[5] but it is also not approximatable — *i e* , no efficient algorithm can even find a theory whose expected accuracy is even close (in a sense defined below) to the optimal value

We first close this section by describing the transformations we will use to define the various spaces of theories, then discussing the sample complexity of the implied learning process and finally providing the extensions needed to deal with predicate calculus

2 1 Standard Transformations

Standard theory revision algorithms implicitly explore the space of possible theories $\Sigma^{\infty}[T_0] = \{\sigma(T_0) \mid \sigma \in \Sigma^{\infty}\}$, which contains the theories formed by applying some sequence of theory-to-theory transformations $\sigma \in \Sigma^{\infty}$ to the given initial theory T_0 Each $\sigma = \tau_1 \circ \tau_2 \circ \ldots \circ \tau_k \in \Sigma^{\infty}$ sequence is formed from $\Sigma = \Sigma_{DR} \cup \Sigma_{AR} \cup \Sigma_{DA} \cup \Sigma_{AA}$, where each $\tau_{DR} \in \Sigma_{DR}$ deletes a rule from the theory, each $\tau_{AR} \in \Sigma_{AR}$ adds a new rule to the theory, each $\tau_{DA} \in \Sigma_{DA}$ deletes an existing antecedent from an existing rule, and each $\tau_{AA} \in \Sigma_{AA}$ adds a new antecedent to an existing rule In some situations, we will consider "K-bounded sequences"

$$\Sigma^{K} = \{\sigma = \tau_1 \circ \tau_2 \circ \ldots \circ \tau_k \mid \tau_i \in \Sigma \;\&\; c(\sigma) \leq K\}$$

whose members $\sigma = \tau_1 \circ \tau_2 \circ \ldots \circ \tau_k \in \Sigma^{K}$ are sequences of transformations whose total cost $c(\sigma) = c(\tau_1) + c(\tau_2) + \ldots + c(\tau_k)$ are at most K, where the cost $c(\tau)$ of the transformation τ is the number of symbols added to, or deleted from, T to form $\tau(T)$ In the propositional case, $c(\tau^{AA}) = c(\tau^{DA}) = 1$ for each transformation that either adds or deletes an antecedent, and $c(\tau^{AR}_{\rho}) = c(\tau^{DR}_{\rho}) = |\rho|$ for each add-rule (resp , delete-rule) transformation that adds (resp , deletes) the rule ρ, which has 1 conclusion and $|\rho| - 1$ antecedent literals

As an example, applying the 3-element sequence $\sigma = \tau^{AA}_{g\,-e\,+q} \circ \tau^{AR}_{b\,-f} \circ \tau^{DA}_{f\,-c\,d\,-c}$ with total cost $c(\sigma) = c(\tau^{AA}_{g\,-e\,+q}) + c(\tau^{AR}_{b\,-f}) + c(\tau^{DA}_{f\,-c\,d,\,-c}) = 1 + 2 + 1 = 4$, will transform T_1 into $\sigma(T_1) = \tau^{AA}_{g\,-e\,+q}(\tau^{AR}_{b\,-f}(\tau^{DA}_{f\,-c\,d\,-c}(T_1)))$ which is a theory with 8 clauses that differs from T_1 by including the clause "g -e,q" rather than "g -e", including the clause "f -d" rather than "f -c,d", and by including an extra clause "b -f"

Finally, we will also consider various other restricted spaces of transformation-sequences, which are formed from specified types of transformations, *e g* ,

$$\Sigma^{-R\,+A} = \{\sigma = \tau_1 \circ \tau_2 \circ \ldots \circ \tau_k \mid \tau_i \in \Sigma_{DR} \cup \Sigma_{AA}\}$$
$$\Sigma^{+R\,-A} = \{\sigma = \tau_1 \circ \tau_2 \circ \ldots \circ \tau_k \mid \tau_i \in \Sigma_{AR} \cup \Sigma_{DA}\}$$

correspond to (unbounded) transformation sequences that produce more specific (resp , more general) theories, as well as the bounded variants, *e g* ,$\Sigma^{-R\,+A\,(K)} = \{\sigma = \tau_1 \circ \ldots \circ \tau_k \mid \tau_i \in \Sigma_{DR} \cup \Sigma_{AA} \;\&\; c(\sigma) \leq K\}$ Note that the earlier $\Sigma^{\infty} = \Sigma^{+R\,-R\,+A\,-A}$ and $\Sigma^{K} = \Sigma^{+R,-R\,+A\,-A\,(K)}$

2 2 Sample Complexity

We can use following standard Computational Learning Theory theorem to bound the number of samples required to obtain the information needed to identify a good $T^* \in \mathcal{T}$ with high probability, showing in particular how this depends on the space of theories $\mathcal{T}$ being considered

Theorem 1 (from [Vap82, Theorem 6 2]) *Given a class of theories $\mathcal{T}$ and $\epsilon, \delta > 0$, let $T_* \in \mathcal{T}$ be the theory with the largest empirical accuracy after*

$$M_{upper}(\mathcal{T}, \epsilon, \delta) = \left\lceil \frac{2}{\epsilon^2} \ln\left(\frac{|\mathcal{T}|}{\delta}\right) \right\rceil$$

samples (each a labeled query), drawn from the stationary distribution, $Pr(\,)$ Then, with probability at least $1 - \delta$, the expected accuracy of T_* will be within ϵ of the optimal theory in $\mathcal{T}$, i e , $Pr[A(T_*) \geq A(T_{opt}) - \epsilon] \geq 1 - \delta$, using the T_{opt} from Equation 2

This means a polynomial number of samples is sufficient to identify an ϵ-good theory from $\mathcal{T}$ with probability at least $1 - \delta$, whenever $\ln(|\mathcal{T}|)$ is polynomial in the relevant parameters Notice this is true for most of the classes of theories being considered, *e g* , as $\Sigma^{-R}(T)$ is the power-set of the rules in T, $|\Sigma^{-R}(T)| = 2^{|\text{Rules}(T)|}$, which means $|\ln(\Sigma^{-R}(T))| = |\text{Rules}(T)| = O(|T|)$, which clearly is polynomial in the size of the initial theory This "$\ln(|\mathcal{T}|) = \text{poly}(|T|)$" claim is slightly problematic for transformations that can add symbols, notably for Σ^{+R} and Σ^{+A} But even here, the sample complexity remains polynomial *in the size of the revised theory*, which effectively means again that sample-efficient learning remains possible, *cf* , "nonuniform" pac-learning [Bl88]

[5] Afl a(T, *q)* requires computing T(g), which can require proving an arbitrary theorem, this computation alone can be computationally intractable, if not undecidable OUT results show that the task of finding the optimal theory is intractable even given *a poly time oracle for these* arbitrary *derivations* Of course, as we are considering only Horn theories, these computations are guaranteed to be poly time in the propositional; case [BCH90]

2 3 Dealing with Predicate Calculus

To handle predicate calculus expressions, we have to consider answers of the form $\{\texttt{Yes}[\ \{X_i/v_i\}\ ]\}$, where the expression within each `Yes[ ]` is a binding list of the free variables, corresponding to a single answer to the query For example, given the theory[6]

$$T_{pc} = \left\{ \begin{array}{ll} \texttt{tall(john)} & \texttt{short(fred)} \\ \texttt{rich(john)} & \texttt{rich(fred)} \\ \texttt{eligible(X)} & \texttt{- tall(X), rich(X)} \end{array} \right\}$$

the query `short(Y)` will return $T_{pc}(\texttt{short(Y)}) = \{\texttt{Yes}[\{Y/\texttt{fred}\}]\}$, the query `rich(Z)` will return the pair of answers $T_{pc}(\texttt{rich(Z)}) = \{\texttt{Yes}[\{Z/\texttt{john}\}], \texttt{Yes}[\{Z/\texttt{fred}\}]\}$, and $T_{pc}(\texttt{eligible(A)}) = \{\texttt{Yes}[\{A/\texttt{john}\}]\}$ As $\mathcal{O}(\ )$ and $T(\ )$ each returns a *set* of answers to each query, we therefore define T's accuracy score as $a(T, \sigma) = \frac{|\mathcal{O}(\sigma) \cap T(\sigma)|}{|\mathcal{O}(\sigma) \cup T(\sigma)|} \in [0,1]$ All of the theorems in this paper hold even when considering only non-recursive Datalog (*i e*, "function free") theories

3 Computational Complexity

Our basic challenge is to produce a theory T_{opt} whose accuracy is as large as possible The previous section supplied the number of samples needed to guarantee, with high probability, that the expected accuracy of the theory whose empirical accuracy is largest, T_n, will be within ϵ of the expected accuracy of this T_{opt} This section discusses the computational challenge of determining this T_n, given this distributional estimate We show first that this task is tractable in degenerate trivial situations when considering (1) only *atomic* queries posed to a (2) propositional theory and being allowed (3) an *arbitrarily large number of modifications* to the initial theory, to produce (4) a *perfect* theory (; e one that returns the correct answer to *every* query) This task becomes intractable, however, if we remove (essentially) any of these restrictions e *g*, if we seek optimal {rather than only seeking "perfect") propositions! theories and are allowed to pose Horn queries, or if we consider predicate *calculus* theories It also remains intractable even if we restrict the *number* of modifications allowed, which implies that the task of determining the *smallest* number of modifications required to find a perfect theory is intractable We next show that these tasks are not just intractable but worse, they are not even approximatable, except in the most trivial of situations

We also consider two special subtasks by restricting the allowed types of transformations, to consider revision processes that only specialize (respectively, only generalize) the initial theory We show that these tasks, also, are intractable and non-approximatable in essentially all situations, I e, except when all four of the above conditions hold[7] Figures 1 and 2 summarize the various cases

[6]Following PROLOG'S conventions, we will capitalize each variable, as in the "X" above

[7]Actually, there is one other tractable case in the generalization situation, see Figure 1

3 1 Basic Complexity Results

To formally state the problem

Definition 1 ($\mathrm{TR}[\Sigma^\dagger]$ **Decision Problem)**
INSTANCE
- *Initial theory* T,
- *Labeled training sample* $S = \{(q_i, \mathcal{O}(q_i))\}$ *containing a set of Horn queries and the correct answers, and*
- *Probability value* $p \in [0,1]$

QUESTION *Is there a theory* $T' \in \Sigma^\dagger[T]$ *such that* $A_S(T') = \frac{1}{|S|} \sum_{(q_i, \mathcal{O}(q_i)) \in S} a(T', q_i) \geq p$?

The $\Sigma^\dagger[\,]$ function maps a theory to a set of candidate revised theories, here, we will consider various $\Sigma^{\pm R \pm A}$ transformation sets To simplify our notation, we will write $A(T)$ for $A_S(T)$

We will also consider the following special cases $\mathrm{TR}_{Prf}[\Sigma^\dagger]$ requires that $p = 1$ (*i e*, seeking perfect theories, rather than 'optimal" theories $\mathrm{TR}_{(Opt)}[\Sigma^\dagger]$), $\mathrm{TR}_{Prop}[\Sigma^\dagger]$ deals with propositional logic (rather than predicate calculus, $\mathrm{TR}_{(PredCal)}[\Sigma^\dagger]$), and $\mathrm{TR}_{Atom}[\Sigma^\dagger]$ deals with only atomic queries (as opposed to Horn queries, $\mathrm{TR}_{(Horn)}[\Sigma^\dagger]$) We will also use $\mathrm{TR}_{Disj}[\Sigma^\dagger]$ to refer to the task when the queries can be arbitrary disjunctions, which need not be Horn (While the other subscripts are *restrictions* on $\mathrm{TR}[\Sigma^\dagger]$, this *Disj* case is more permissive) We will also combine subscripts, with the obvious meanings When $\mathrm{TR}_\chi[\Sigma^\dagger]$ is a special case of $\mathrm{TR}_\psi[\Sigma^\dagger]$, finding that $\mathrm{TR}_\chi[\Sigma^\dagger]$ is hard immediately implies that $\mathrm{TR}_\psi[\Sigma^\dagger]$ is hard Similarly, seeing that $\mathrm{TR}_\psi[\Sigma^\dagger]$ is easy immediately implies that each special case of $\mathrm{TR}_\psi[\Sigma^\dagger]$ is easy As a final note all of the hardness results presented in this paper hold even if we only consider "3-CNF Horn theories" — *i e*, rules whose antecedents contain at most 2 literals

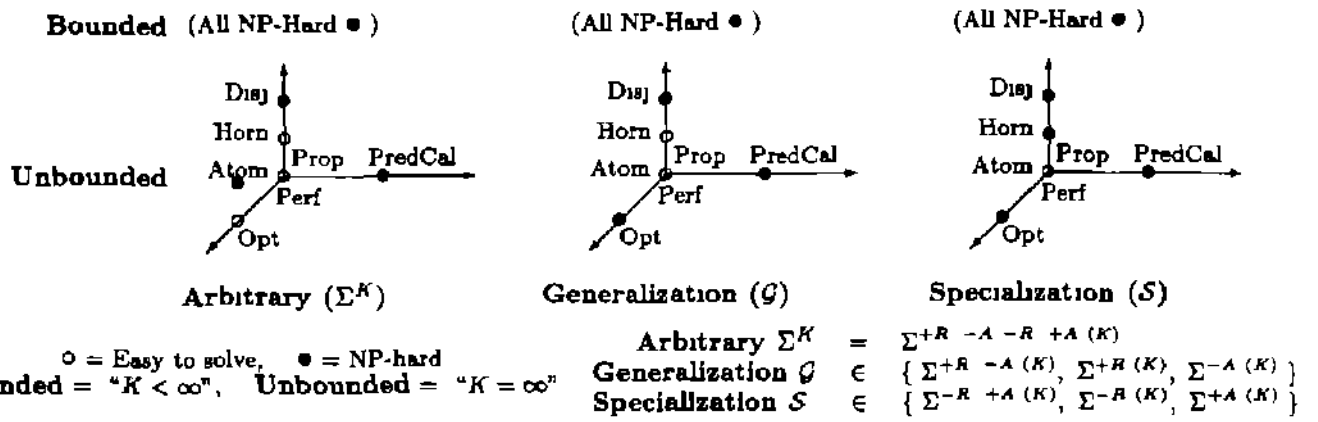
Here, it is easy to find the optimal theory in certain degenerate cases, where either the individual queries can be decoupled (*e g*, when using atomic propositional queries) or when our actions are forced (*e g*, when seeking perfect propositional theories) just throw away the original theory, then add in propositions corresponding to the "Yes-labeled queries" In every other case, however, the task is intractable

Theorem 2 *The* $\mathrm{TR}_{Prop\ Atom\ (Opt)}[\Sigma^\infty]$ *and* $\mathrm{TR}_{Prop\ (Horn)\ Perf}[\Sigma^\infty]$ *decision problems are easy, each other problem is NP-hard*

(This information is summarized in lower left "Unbounded, Arbitrary" graph of Figure 1)

The above theorem describes the complexity of computing the best theory when we are allowed to use an *arbitrarily expensive* sequence of transformations We get an even stronger negative results if we restrict the "expense" of the transformation sequence

Theorem 3
For some $K \in \mathcal{Z}^+$, *the* $\mathrm{TR}_{Prop\ Atom\ Perf}[\Sigma^K]$ *decision problem is NP-hard This is true even if we consider only labeled queries produced by a consistent oracle (i e,*



Any task that "projects" down to an NP-hard task, along any axis, is NP-hard Here, this means all of the "cross terms" are NP-hard (For example $\text{ThRev}_{PredCal\ Horn\ Perf}[\Sigma^\infty]$ is NP-hard, as its projection to the "Prop–PredCal x Perf–Opt" plane, $\text{ThRev}_{PredCal\ Atom\ Perf}[\Sigma^\infty]$ is NP-hard) The $\text{ThRev}_{Prop\ Horn\ Opt}[\Sigma^\infty]$ case is shown explicitly as each of its projections is easy, the figures omit all other cross-terms

Figure 1 Tractability of Theory Revision Tasks

even when there is a Horn theory that correctly labels all of the queries)

The observation that determining such "it-step perfect theories" is NP-hard leads immediately to

Corollary 3 1 *It is NP-hard to compute the minimal-cost transformation sequence required to produce a perfect theory (i e , to compute the smallest K for which there is a* $\mathbf{T}_{perfect} \in \Sigma^K[T]$ *such that* $A(\mathbf{T}_{perfect}) = 1$*), even m the propositional case when considering only atomic queries ft is also NP-hard to compute the "minimal-length" transformation, where the length of the transformation sequence* $\tau_1 \circ \tau_2 \circ \cdots \circ \tau_k$ *is simply k — j e , when each transformation has "unit cost"*

This negative result shows the intractability of the obvious proposal of using a breath-first transversal of the space of all possible theory revisions First test the initial theory To against the labeled queries, and return T_0 if it is 100% correct If not, then consider all theories formed by applying a single (unit-cost) transformation, and return any perfect $\mathbf{T}_1 \in \Sigma^1[T_0]$, and if not, consider all theories in $\Sigma^2[T_0]$ (formed by applying sequences of transformations with cost at most two), and return any perfect $\mathbf{T}_2 \in \Sigma^2[T_0]$, and so forth

3 2 Approximatability

Many decision problems correspond immediately to optimization problems, for example, the MINGRAPHCOLOR decision problem (given a gTaph $G = (N, E)$ and a positive integer A", can each node be labeled by one of K colors in such a way that no edge connects two nodes of the same color, see [GJ79, pl91(Chromatic Number)]) corresponds to the obvious minimization problem Find the minimal coloring of the given graph G We can simdarly view the $\text{TR}_\chi[\Sigma^\dagger]$ decision problem as either the maximization problem "Find the $\mathbf{T}' \in \Sigma^\dagger[T]$ whose accuracy is mammal" or the minimization problem "Find the $\mathbf{T}' \in \Sigma^\dagger[T]$ whose inaccuracy is minimal", where a theory's inaccuracy is obviously INA(T) = 1 - A(T)

(While the maximally accurate theory is also mini mally inaccurate, these two formulations can lead to different approximatability results) For notation, let "$\text{MAXTR}_\chi[\Sigma^\dagger]$" (resp , "$\text{MINTR}_\chi[\Sigma^\dagger]$") refer to the maximization (resp , minimization) problem

Now consider any algorithm B that, given any $\text{MINTR}_\chi[\Sigma^\dagger]$ instance x = (T, S) with initial theory T and labeled training sample S, computes a syntactically legal, but not necessarily optimal, revision $B(x) \in \Sigma^\dagger[T]$ Then B's "performance ratio for the instance x" is defined as

$$MinPerf[\text{MINTR}_\chi[\Sigma^\dagger]](B, x) = \begin{cases} \frac{InA(B(x))}{InA(opt(x))} & \text{if INA}(opt(x)) \neq 0 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

where $opt(x) = opt_{MinTR_\chi(\Sigma^\dagger)}(x)$ is the optimal solution for this instance, i e , $opt(x)$ is the theory $\mathbf{T}_{opt} \in \Sigma^\dagger[T]$ with minimal inaccuracy over S

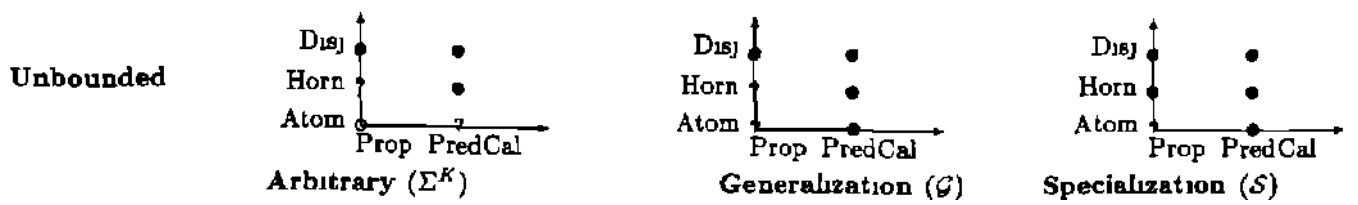
We say a function $g(\ )$ "bounds B's performance ratio (over $\text{MINTR}_\chi[\Sigma^\dagger]$)" iff

$$\forall x \in \text{MINTR}_\chi[\Sigma^\dagger], \quad MinPerf[\text{MINTR}_\chi[\Sigma^\dagger]](B, x) \leq g(|x|)$$

where $|x|$ is the size of the instance $x = \langle T, S \rangle$, which we define to be the number of symbols in T plus the number of symbols used in S Intuitively, this $g(\ )$ function indicates how closely the B algorithm comes to returning the best answer for x, over all $\text{MINTR}_\chi[\Sigma^\dagger]$ instances x

Now let $Poly(\text{MINTR}_\chi[\Sigma^\dagger])$ be the collection of all polytime algorithms that return legal answers to $\text{MINTR}_\chi[\Sigma^\dagger]$ instances It is natural to ask for the algorithm in $Poly(\text{MINTR}_\chi[\Sigma^\dagger])$ with the best performance ratio, this would indicate how close we can come to the optimal solution, using only a feasible computational time For example, if this function was the constant 1 for $\text{MINTR}_{Prop}[\Sigma^\infty]$, then a poly-time algorithm could produce the optimal solution to any $\text{MINTR}_{Prop}[\Sigma^\infty]$ instance, as $\text{TR}_{Prop}[\Sigma^\infty]$ is NP-complete,[8] this would

[8]While Theorem 2 only proves $\text{TR}_{Prop}[\Sigma^\infty]$ to be NP-



(• = NOTPOLYAPPROX, ○ = Easy (as poly time decision), ? = unknown approximatability class)

Figure 2 Approximatability of Theory Revision Tasks

mean $P = NP$, which is why we do not expect to obtain this result Or if this bound was some constant $c(x) = c \in \Re$, then we could efficiently obtain a solution within a factor of c of optimal, which may be good enough for some applications [9]

However, not all problems can be so approximated Following [CP91, Kan92], we define

Definition 2 *A minimization problem* MINP *is* NOTPOLYAPPROX *if there is a* $\gamma \in \Re^+$ *such that*

$$\forall B \in \text{Poly}(\text{MINP}),\ \exists x \in \text{MINP},\ \mathit{MinPerf}[\text{MINP}](B, x) \geq |x|^\gamma$$

Lund and Yannakakis [LY93] prove that the "MINGRAPHCOLOR minimization problem" is NOTPOLYAPPROX We can use that result to prove

Theorem 4 *Unless* $P = NP$, *each of* $\text{MINTR}_{Prop\ Disj}[\Sigma^\infty]$, $\text{MINTR}_{(PredCal)\ (Horn)}[\Sigma^\infty]$ *and* $\text{MINTR}_{Prop\ Atom}[\Sigma^K]$ *is* NOTPOLYAPPROX

While these results may at first seem trivial, given that it is NP-hard to determine if a perfect theory exists, notice from Equation 3 that $\mathit{MinPerf}[\text{MINTR}[\Sigma^\infty]]( )$ essentially ignores such perfect theories Note also that this result holds in the context based on an "inconsistent" oracle, in such situations, no theory can be perfect

As $|x|$ can get arbitrary large, this result means that these $\text{MINTR}_\chi[\Sigma^\dagger]$ tasks cannot be approximated by any constant, nor even by any logarithmic factor nor any sufficiently small polynomial, etc

3 3 Special Cases

If the theory is too general (*i e* , returns Yes too often), then we may want to consider "specializing" it by applying only the "delete rule" and "add antecedent" transformations In particular, recall that $\Sigma^{+A-R}[\text{T}]$ is the set of theories obtained using *an arbitrary number* of such transformations, and $\Sigma^{-R}[\text{T}]$ (resp , $\Sigma^{+A}[\text{T}]$), is the set of theories obtained by applying an arbitrary number of "delete rule" (respectively, "add antecedent") transformations Similarly, if the theory is too specific (*i e* , returns No too often), then we may want to consider "generalizing" it by applying only the "add rule" and "delete antecedent" transformations, here, we consider $\Sigma^{+R-A}[\text{T}]$, $\Sigma^{+R}[\text{T}]$ and $\Sigma^{-A}[\text{T}]$, which are the set of theories obtained by applying an arbitrary number of such transformations

Ever using only these transformations, almost all of these tasks remain intractable

Theorem 5 *It is easy to solve*

$\text{TR}_{Prop\ Perf}[\mathcal{G}]$ *for* $\mathcal{G} \in \{\Sigma^{+R-A}\ \Sigma^{+R}, \Sigma^{-A}\}$

$\text{TR}_{Prop\ Atom\ Perf}[\mathcal{S}]$ *for* $\mathcal{S} \in \{\Sigma^{-R+A}, \Sigma^{-R}, \Sigma^{+A}\}$

However, every other situation, *formed by any other combination of restrictions (read "subscripts") is NP hard (See middle and right of Figure 1)*

Worse,

Theorem 6 *Unless* $P = NP$, *each of the following is* NOTPOLYAPPROX

- $\text{MINTR}_{(PredCal)\ Atom}[\mathcal{S}]$, $\text{MINTR}_{Prop\ (Horn)}[\mathcal{S}]$ *for* $\mathcal{S} \in \{\Sigma^{-R+A}, \Sigma^{-R}, \Sigma^{+A}\}$
- $\text{MINTR}_{(PredCal)\ Atom}[\mathcal{G}]$, $\text{MINTR}_{Prop\ Disj}[\mathcal{G}]$ *for* $\mathcal{G} \in \{\Sigma^{+R-A}, \Sigma^{+R}, \Sigma^{-A}\}$
- $\text{MINTR}_{Prop\ Atom}[\Sigma^\dagger]$ *for* $\Sigma^\dagger \in \left\{ \begin{matrix} \Sigma^{+A-R(K)}, & \Sigma^{-R(K)}, & \Sigma^{+A(K)} \\ \Sigma^{-A+R(K)}, & \Sigma^{+R(K)}, & \Sigma^{-A(K)} \end{matrix} \right\}$

(See middle and right of Figure *2)*

In each of these cases, however, there is a trivial polynomial-time algorithm that can produce a theory whose *accuracy* (n *b* , not inaccuracy) is within a factor of 2 of optimal That is, using the ratio of an algorithm's *accuracy* to the optimal value,

$$\mathit{MaxPerf}[\text{MAXTR}_\chi[\Sigma^\dagger]](B, x) = \frac{A(opt(x))}{A(B(x))}$$

Theorem 7
For $\Psi \in \{\Sigma^{-R+A}, \Sigma^{-R}, \Sigma^{+A}, \Sigma^{+R-A}, \Sigma^{+R}, \Sigma^{-A}\}$,
$\exists B \in \text{Poly}(\text{MAXTR}[\Psi])$,
$\mathit{MaxPerf}[\text{MAXTR}[\Psi]](B, x) \leq 2$

The companion paper [Gre95a] considers other related cases, including the above special cases in the context where our underlying theories can use the not( ) operator to return Yes if the specified goal cannot be proven, *1 e* , using Negation-as-Failure [Cla78] It also considers the effect of re-ordenng the rules and the antecedents, in the context where such shufflings can affect the answers returned In most of these cases, we show that the corresponding maximization problem is not approximatable within a particular polynomial

hard, this problem is clearly in NP

[9] There are such constants for some other NP-hard minimization problems For example, there is a polynomial-time algorithm that computes a solution whose cost is within a factor of 1 5 for any TRAVELINGSALESMAN WITHTR!ANGLE_EQUALITY problem, see [GJ79, Theorem 6 5]

(The extended [Gre95b] explainB the asymmetry between $TR_{prop\ Perf[E}$rsus $TR_{Prop\ perf[E+^R]}$, and discusses how these results relate to both inductive logic programming, and to default theories)

4 Conclusion

A knowledge-based system can produce incorrect answers to quenes if its underlying theory is faulty A "theory revision" system attempts to transform a given theory into a related one that is as accurate as possible, using a given set of correctly answered "training quenes" This report describes both the sample and computational complexity of this task It first provides the number of samples required to obtain the statistics needed to identify a theory (from within a class of theories defined by applying various standard transformations to a given initial theory) whose accuracy will be within *e* of the optimal theory in this class, with probability at least 1 — *6* It then shows that, in general, the task of computing this globally optimal theory is intractable — and worse, that no polynomial time algorithm can be guaranteed to find a solution that is even close to optimal (given the standard *P =/ NP* assumption) We also present special cases of these tasks, which pin-point exactly when the task becomes tractable

References

[AGM85] Carlos E Alchourron, Peter Gardenfors, and David Makinson On the logic of theory change Partial meet contraction and revision functions *Journal of Symbolic Logic,* 50 510-30, 1985

[BCH90] E Boros, Y Crama, and P L Hammer Polynomial time inference of all valid implications for horn and related formulae *Annals of Mathematics and Artificial Intelligence,* 1 21-32, 1990

[BI88] G Benedek and A Itai Nonuniform learnability In *Proceedings ICALP 88,* pages 82-92, 1988

[BM93] Paul T Baffes and Raymond J Mooney Symbohc revision of theories with M-of N rules In *Proceedings of IJCAI 93,* August 1993

[Cla78] K Clark Negation as fadure In H Gallaire and J Minker, editors, *Logic and Data Bases,* pages 293-322 Plenum Press, New York, 1978

[Coh90] William W Cohen Learning from textbook knowledge A case study In *Proceeding of AAA] 90,* 1990

[Coh92] William W Cohen Abductive explanation based learning A solution to the multiple inconsistent explanation problems *Machine Learning,* 8(2) 167-219, March 1992

[CP91] P Crescenzi and A Panconesi Completeness in approximation classes *Information and Computation,* 93(2) 241-62, 1991

[CS90] Susan C raw and Derek Sleeman Automating the refinement of know ledge-based systems In L C Aiello, editor, *Proceedings of ECAI 90* Pitman, 1990

[DP9l] Jon Doyle and Ramesh Patil Two theses of knowledge representation Language restrict ions, taxonomic classification, and the utility of repre sentation service* *Artificial Intelligence,* 48(3) 1991

[FP93] Michael Frazier and Leonard Pitt Learning from entailment AD application to prepositional horn sentences In *Proceedings of IML 93,* pages 120-27 Morgan Kaufrnann, 1993

[Gar88] Peter Gardenfore *Knowledge i*n Flux *Modeling the Dynamics of the Epistemic States* Bradford Book, MIT Press, Cambridge MA, 1988

[GJ79] Michael R Garey and David S Johnson *Computers and Intractability A Guide to the Theory of NP Completeness* W H Freeman and Company, New York, 1979

[Gre95a] Russell Greiner The challenge of revising impure theories In *Proceedings of the Twelfth Internationol* Machine *Learning* Conference, 1995

[Gre95b] Russell Greiner The complexity of theory revision Technical Report SCR 95-TR-539 Siemens Corporate Research, 1995 ftp //scr Eiamani com/pub/learning/P»pera/greinar/comp-tr pE

[Kan92] Viggo Kann *On the Approximability of NP Complete Optimization Problems* PhD thesis, Royal Institute of Technology, Stockholm, 1992

[KM91] Hirofumi Katsuno and Alberto Mendelzon On the difference between updating a knowledge base and revising it In *Proceedings of KR 91,* pages 387-94, Boston, April 1991

[LDRG94] Pat Langley, George Drastal, R Bharat Rao, and Russell Greiner Theory revision in fault hierarchies In *Proceedings of The Fifth International Workshop on Principles of Diagnosis (DX 94)* New Paltz, NY, 1994

|Lev84] Hector J Levesque Foundations of a functional approach to knowledge representation *Artificial Intelligence,* 23 155-212, 1984

[LY93] Carsten Lund and Mihalis Yannakakis On the hardness of approximating minimization problems In *Proceeding of Twenty fifth Annual A CM Symposium on Theory of Computation (STOC 93),* pages 286-93, 1993

[MB88] S Muggleton and W Buntine Machine invention of first order predicates by inverting resolution In *Proceedings of IML 88,* pages 339—51 Morgan Kaufmans, 1988

[OM94] Dirk Ourston and Raymond J Mooney Theory refinement combining analytical and empirical methods *Artificial Intelligence,* 66(2) 273-310, 1994

[Pol85] PG Politakia *Empirical Analysis for Expert Systerns* Pitman Research Notes in Artificial Intelhgence, 1985

[Vap82] V N Vapnik *Estimation of Dependences Based on Empirical Data* Springer Verlag, New York, 1982

[WP93] James Wogulis and Michael J Pazzani A methodology for evaluating theory revision systems Results with Audrey II In *Proceedings of IJCAI 9S,* pages 1128-1134, 1993