

On the Relation between Argumentation and Non-monotonic Coherence-Based Entailment

Claudette Cayrol
Institut de Recherche en Informatique de Toulouse (I R I T)
University Paul Sabatter, 118 route de Narbonne
31062 Toulouse Cedex (FRANCE)

Abstract

The purpose of this paper is to discuss and compare two types of methods for reasoning with inconsistent belief bases: coherence based approaches to non monotonic entailment based on the selection and management of consistent subbases and argumentation systems based on the construction and selection of arguments in favor of a conclusion. We present several argumentation systems then we show that most of the associated inference relations can be also defined using well known principles for selecting consistent subbases. So we establish a formal correspondence between the so-called argumentation paradigm and recent work on nonmonotonic entailment. Lastly we propose several directions for further research concerning the integration of preference relations into argumentation systems.

1 Introduction

Reasoning from inconsistent beliefs is a significant problem in Artificial Intelligence. In this paper belief bases are considered "syntactically" as in [Nebel 1991] each belief is a distinct piece of information and only beliefs which are explicitly present in the base are taken into account. It departs from the logical point of view where a base is identified with the set of its models. Our purpose is to compare two main types of approaches to reasoning from inconsistency.

The approaches based on the notion of *maximal consistent subbase*¹ where non monotonic inference relations are defined by the combination of a mechanism for generating "preferred" belief subbases, a principle for selecting some of these subbases and the classical inference (See [Cayrol and Lagasque-Schiex, 1993, 1994] for a thorough presentation of these inference relations). As preferred subbases all the maximal consistent subbases may be considered or the consistent subbases of maximal cardinality [Benferhat et al 1993a]. Other meanings of "preferred" have been studied induced by the presence of an ordering on the belief base [Brewka 1989, Geffner 1992, Cayrol et al 1993]. As a selection principle (see [Pinkas

and Lorn 1992] for a taxonomy of selection principle;) we usually find the existential one selecting one of the preferred subbases or the universal one selecting all the preferred subbases. An intermediary principle called argumentative in [Benferhat et al 1993b] leads to the following consequence relations "I conclude ϕ from the belief base B if at least one preferred subbase classically entails ϕ and no preferred subbase classically entails $\neg \phi$ ".

- Independently argument based approaches to defeasible reasoning have been developed [Lin and Shoham 1989, Vreeswijk 1991, Pollock 1992, Stman and Ixui 1992, Dung 1993, Bvang Goransson et al 1993a & 1993b, Hunter 1994]. Argumentation is a general principle based on the construction and use of arguments. An argumentation system is defined by specifying the way of constructing and using arguments. The basic idea is to view reasoning as a process of first constructing arguments in favor of a conclusion and then selecting the most acceptable of them. In other words a statement will be inferred if the arguments supporting this statement can be successfully defended against the arguments supporting the opposite statement.

As recently discussed in [Benferhat et al 1995] these two types of approaches correspond to two attitudes in front of inconsistent beliefs: one attitude is to restore coherence by selecting consistent subbases, the other one is to accept inconsistency by providing arguments for each conclusion.

In this paper we establish a close correspondence between these two models of reasoning referred to as coherence based reasoning and argument-based reasoning in the following. First we present and discuss two completely formalized argumentation systems. Then we show that the associated inference relations can be exactly restated in the framework of coherence based entailment with an appropriate selection principle on maxima consistent subbases. Finally we propose directions for further research concerning the integration of preference relations in argumentation systems. Results are given without proofs due to space limitations. Proofs appear in [Cayrol 1995].

2 Argumentation Systems

Throughout this paper X is a propositional language, \models denotes classical entailment, K and E denote sets of formulas of L , K which may be empty represents a core of knowledge and is assumed consistent. Contrastedly formulas of E represent defeasible pieces of knowledge or

¹ maximal for set inclusion consistent subbases were first introduced by Rescher and Manor [1970]

beliefs. So $\mathcal{X} \cup \mathcal{E}$ may be inconsistent. \mathcal{E} will be referred to as the *belief base*.

2.1 Arguments Built from a Belief Base

We introduce the notion of argument in the framework $(\mathcal{X}, \mathcal{E})$. A similar definition appears for instance in [Simari and Low 1992].

Definition 1 An *argument* of \mathcal{E} is a pair (H, h) where h is a formula of \mathcal{L} and H is a subbase of \mathcal{E} satisfying (i) $\mathcal{X} \cup H$ is consistent (ii) $\mathcal{X} \cup H \vdash h$ (iii) H is minimal (no strict subset of H satisfies ii). H is called the *support* and h the *conclusion* of the argument. The set of all the arguments of \mathcal{E} will be denoted by $AR(\mathcal{E})$.

Remarks

All the arguments considered in this paper have a consistent support.

- In the work reported here, an argument is simply a pair of set of supporting sentences and conclusion. We do not consider structured arguments involving chains of reasons as in [Pollock 1992].

The minimality criterion (point iii) in Definition 1) might be enforced by other criteria such as specificity or any other preference criteria. That point will be discussed in Section 5.

According to [Dung 1993], an argumentation system is defined given any set of arguments \mathcal{A} equipped with a binary relation \mathcal{R} . The intended meaning of " $\mathcal{R}(A_1, A_2)$ " is "the argument A_1 defeats the argument A_2 ". We present below different definitions for that relation of "defeat" within the above context where arguments are built from a belief base.

Definition 2 [Elvang-Goransson *et al.* 1993a] Let (H_1, h_1) and (H_2, h_2) be two arguments of \mathcal{E} . (H_1, h_1) *rebuts* (H_2, h_2) iff $h_1 \equiv \neg h_2$ (\equiv means logical equivalence).

An argument is rebutted iff there exists an argument for the negated conclusion.

Definition 3 [Elvang-Goransson *et al.* 1993a] Let (H_1, h_1) and (H_2, h_2) be two arguments of \mathcal{E} . (H_1, h_1) *undercuts* (H_2, h_2) iff for some $h \in H_2$, $h \equiv \neg h_1$.

An argument is undercut iff there exists an argument for the negation of an element of its support. Obviously, an argument which undercuts another argument is rebutted. Another family of definitions was proposed in the work of Simari and Low [1992]. With the restriction to the above context, we obtain:

Definition 4 Let (H_1, h_1) and (H_2, h_2) be two arguments of \mathcal{E} . (H_1, h_1) and (H_2, h_2) *disagree* (or are in *disagreement*) iff $\mathcal{X} \cup \{h_1, h_2\}$ is inconsistent.

(H_1, h_1) is a *subargument* of (H_2, h_2) iff $H_1 \subseteq H_2$.

(H_1, h_1) *counterargues* (H_2, h_2) iff (H_1, h_1) is in disagreement with a subargument of (H_2, h_2) .

Note that the counterargument relation is a kind of refinement of the disagreement relation in the sense that it considers subarguments. In an analogous way, we propose a weaker version of definition 3:

Definition 5 (H_1, h_1) *weakly-undercuts* (H_2, h_2) iff (H_1, h_1) rebuts a subargument of (H_2, h_2) (iff $\mathcal{X} \cup H_2 \vdash \neg h_1$).

2.2 About the Various Meanings of "Defeat"

It seems interesting to compare these different relations defined on the set of arguments $AR(\mathcal{E})$. Proofs of the following results can be found in [Cayrol 1995].

Proposition 1

- Obviously, if (H_1, h_1) rebuts (H_2, h_2) , these arguments disagree. The converse is false as shown by $\mathcal{X} = \{a, b, x \rightarrow c\}$, $\mathcal{E} = \{a \rightarrow x, b \rightarrow \neg c\}$ and the arguments $((b \rightarrow \neg c), \neg c)$ and $((a \rightarrow x), x)$.

- If two arguments disagree, each one is rebutted by a subargument of the other one. The converse is false as shown by $\mathcal{X} = \emptyset$, $\mathcal{E} = \{a, b, x \rightarrow c, a \rightarrow x, b \rightarrow \neg c\}$ and the arguments $((b \rightarrow \neg c), x \rightarrow c)$, $\neg x$ and $((a \rightarrow x), x \rightarrow c)$.

- If (H_1, h_1) undercuts (H_2, h_2) , then (H_1, h_1) weakly-undercuts (H_2, h_2) and then (H_1, h_1) counterargues (H_2, h_2) .

- (H_1, h_1) counterargues (H_2, h_2) iff (H_1, h_1) is rebutted by a subargument of (H_2, h_2) . Or equivalently, (H_1, h_1) counterargues (H_2, h_2) iff (H_1, h_1) weakly-undercuts (H_2, h_2) .

To summarize, A_1 and A_2 denoting two arguments of \mathcal{E} , we have:

A_1 undercuts $A_2 \Rightarrow A_1$ counterargues $A_2 \Leftrightarrow A_1$ weakly-undercuts A_2

A_1 rebuts $A_2 \Rightarrow A_1$ and A_2 disagree

3 Argumentation-Based Inference

Due to the inconsistency of the available knowledge, arguments may be constructed in favor of a statement and other arguments may be constructed in favor of the opposite statement. The main approaches which have been developed for reasoning within an argumentation system rely on the idea of differentiating arguments with a notion of acceptability. In the proposal by [Elvang-Goransson *et al.* 1993a, 1993b], acceptability levels are assigned to arguments on the basis of other constructible arguments. Then, from a taxonomy of acceptability classes, consequence relations are defined. Quite independently, Dung [1993] formalized a kind of global acceptability. The set of all the arguments that a rational agent may accept must defend itself against all attacks on it. This leads to defining extensions of an argumentation system.

In this section, we apply both methodologies on the same argumentation system and establish some interesting results about argumentation-based inference, which will contribute to unify different approaches and will particularly enable us to recover coherence-based entailment relations in Section 4.

3.1 Acceptability Classes

Here we show that the consequence relations induced by the construction of acceptability classes are the same whether we consider the family of relations ("rebuts", "undercuts") or the family ("disagrees", "counterargues"). We still consider the framework $(\mathcal{X}, \mathcal{E})$ and the set of arguments $AR(\mathcal{E})$.

Definition 6 [Elvang-Goransson *et al* 1993a]

$AR^*(\mathcal{E})$ denotes the set of arguments of \mathcal{E} with an empty support $(\emptyset, h) \in AR^*(\mathcal{E})$ iff $\mathcal{X} \vdash h$

$AR+(\mathcal{E})$ denotes the set of arguments of \mathcal{E} which are not rebutted by some argument of \mathcal{E}

$AR++(\mathcal{E})$ denotes the set of arguments of $AR+(\mathcal{E})$ (or equivalently of \mathcal{E} as proved in [Elvang-Goransson *et al* 1993a]) which are not undercut by some argument of \mathcal{E}

The following inclusions hold between the so called acceptability classes $C4 = AR^*(\mathcal{E}) \subseteq C3 = AR++(\mathcal{E}) \subseteq C2 = AR+(\mathcal{E}) \subseteq C1 = AR(\mathcal{E})$. Inclusion is generally strict as shown by $\mathcal{X} = \emptyset$, $\mathcal{E} = \{a, a \rightarrow b, \neg a\}$ and the argument $(\{a, a \rightarrow b\}, b)$ which belongs to $AR+(\mathcal{E})$ but is undercut by $(\{\neg a\}, \neg a)$

The argument $(H1, h1)$ is said more acceptable than $(H2, h2)$ iff there exists a class Ci ($1 \leq i \leq 4$) containing $(H1, h1)$ but not containing $(H2, h2)$. Consequence relations are defined by

Definition 7 [Elvang-Goransson *et al* 1993a] Let ϕ be a formula of \mathcal{L}

ϕ is a *certain* consequence of \mathcal{E} ($\mathcal{E} \vdash_{ce} \phi$) iff $AR^*(\mathcal{E})$ contains an argument concluding ϕ

ϕ is a *confirmed* consequence of \mathcal{E} ($\mathcal{E} \vdash_{co} \phi$) iff there exists $H \subseteq \mathcal{E}$ with $(H, \phi) \in AR++(\mathcal{E})$

ϕ is a *probable* consequence of \mathcal{E} ($\mathcal{E} \vdash_{pr} \phi$) iff there exists $H \subseteq \mathcal{E}$ with $(H, \phi) \in AR+(\mathcal{E})$

ϕ is a *plausible* consequence of \mathcal{E} ($\mathcal{E} \vdash_{pl} \phi$) iff there exists $H \subseteq \mathcal{E}$ with $(H, \phi) \in AR(\mathcal{E})$

Remarks

The "probable consequence" has been considered independently in [Benferhat *et al* 1993b] where it was called "argumentative consequence"

When the belief base \mathcal{E} is a consistent confirmed consequence, probable consequence and plausible consequence coincide

The following implications hold $\mathcal{E} \vdash_{ce} \phi \Rightarrow \mathcal{E} \vdash_{co} \phi \Rightarrow \mathcal{E} \vdash_{pr} \phi \Rightarrow \mathcal{E} \vdash_{pl} \phi$, but the converse do not generally. Take $\mathcal{X} = \emptyset$, $\mathcal{E} = \{a, a \rightarrow b, \neg a\}$, $a \rightarrow b$ is a confirmed but not certain consequence, b is a probable but not confirmed consequence, a is a plausible but not probable consequence.

Following the above construction we consider now the relations of disagreement and counterargument

Definition 8

$AR'+(\mathcal{E})$ denotes the set of arguments of \mathcal{E} which disagree with no argument of \mathcal{E}

$AR'++(\mathcal{E})$ denotes the set of arguments of \mathcal{E} which are not counterargued by some argument of \mathcal{E} (or equivalently which are never weakly-undercut)

Proposition 2 $AR+(\mathcal{E}) = AR'+(\mathcal{E})$ and $AR++(\mathcal{E}) = AR'++(\mathcal{E})$

Proof from Proposition 1. See [Cayrol 1995]

Though the relations "rebutts" and "undercuts" of [Elvang-Goransson *et al* 1993a] are stronger than the relations

"disagrees" and "counterargues" of [Siman and Loui, 1992] we obtain exactly the same consequence relations

3.2 Extensions of an Argumentation System

First we recall the definitions of [Dung 1993] in a general framework. Let $(AR, \text{"defeats"})$ denote an argumentation system

Definition 9 [Dung 1993]

A subset S of AR is *conflict free* iff there are no two arguments $A1, A2$ in S such that $A1$ "defeats" $A2$

A subset S of AR is a *stable extension* iff

(i) S is conflict free

(ii) for each argument A not belonging to S there exists an argument B in S such that B "defeats" A . In other words S "defeats" each argument which does not belong to S

For instance $AR+(\mathcal{E})$ is conflict-free in the system $(AR(\mathcal{E}), \text{"rebutts"})$ and $AR++(\mathcal{E})$ is conflict-free in the system $(AR(\mathcal{E}), \text{"undercuts"})$. Intuitively a stable extension corresponds to a maximal set of acceptable or admissible arguments for a rational agent. Then a common proposal to handle multiple stable extensions would be to accept a formula as a consequence when it can be classically inferred from all the stable extensions (conservative point of view) or when it can be classically inferred from at least one stable extension (permissive point of view)

Now we focus on the particular case of the argumentation system $(AR(\mathcal{E}), \text{"undercuts"})$ defined in Section 2. Let S be a subset of $AR(\mathcal{E})$. S is conflict free means there are no two arguments $(H1, h1), (H2, h2)$ in S such that for some $h \in H2$, $h \equiv \neg h1$. S is a stable extension means moreover: If the argument (H, h) does not belong to S there exists (H', h') in S such that for some $h'' \in H$, $h'' \equiv \neg h'$

Let $For(S)$ denote the set of conclusions of arguments of S and $Supp(S)$ denote the union of the supports of arguments of S . T being a subbase of \mathcal{E} let $Arg(T)$ denote $\{(H, h) \in AR(\mathcal{E}) : H \subseteq T\}$

Remind that a "maximal for set inclusion \mathcal{X} -consistent" subbase of \mathcal{E} (or "maximal \mathcal{X} -consistent" subbase for short) is a \mathcal{X} -consistent subbase of \mathcal{E} which is maximal for set inclusion among all the \mathcal{X} -consistent subbases of \mathcal{E}

Proposition 3 Let S be a stable extension of the system $(AR(\mathcal{E}), \text{"undercuts"})$

$Supp(S)$ is a maximal \mathcal{X} -consistent subbase of \mathcal{E}

$Arg(Supp(S)) = S$

$\mathcal{X} \cup Supp(S) \vdash \phi$ iff $\phi \in For(S)$

Proposition 4 Let T be a maximal \mathcal{X} -consistent subbase of \mathcal{E} . $Arg(T)$ is a stable extension of the system $(AR(\mathcal{E}), \text{"undercuts"})$. $Supp(Arg(T)) = T$

As a direct consequence of these two propositions we obtain the following characterization

Proposition 5 In the argumentation system $(AR(\mathcal{E}), \text{"undercuts"})$ the stable extensions are exactly the $Arg(T)$ where T is a maximal \mathcal{X} -consistent subbase of \mathcal{E}

However no such characterization exists in the case of the argumentation system $(AR(\mathcal{E}), \text{"counterargues"})$. In that system S is conflict free means there are no two

arguments (H1 b1) (H2 h2) in S such that $\mathcal{X} \cup H2 \vdash \neg h1$ S is a *stable extension* means moreover If the argument (H b) does not belong to S there exists (H' b') in S such that $\mathcal{X} \cup H' \vdash \neg b'$

Proposition 6 Let \mathcal{E} be a maximal \mathcal{X} -consistent subbase of \mathcal{E} Arg(T) is a *stable extension* of the system (AR(\mathcal{E}) "counterargues")

However there are other *stable extensions* More particularly there are *stable extensions* S such that Supp(S) is \mathcal{X} -inconsistent

Example 1 $\mathcal{X} = \{(a \wedge b) \rightarrow c\}$ $\mathcal{E} = \{a, b, \neg c\}$ Let S be the set of arguments containing all the arguments of the form (\emptyset, g) where $\mathcal{X} \vdash g$ all the arguments of the form $(\{a\}, h)$ all the arguments of the form $(\{b\}, h)$ all the arguments of the form $(\{\neg c\}, h)$ and no other argument So S contains at least the set $\{(\emptyset, (a \wedge b) \rightarrow c), (\{a\}, a), (\{a\}, b \rightarrow c), (\{b\}, b), (\{b\}, a \rightarrow c), (\{\neg c\}, \neg c), (\{\neg c\}, \neg a \vee \neg b)\}$ Note that $\text{Supp}(S) = \mathcal{E}$ and then S is \mathcal{X} -inconsistent It can be proved [Cayrol 1995] that S is a *stable extension* of the system (AR(\mathcal{E}) "counterargues")

We conclude this section with a few words about related work A similar notion of *stable extension* was proposed in [Siman and Loui 1992] for modelling argumentation-based inference The construction relies upon a refinement of the counterargumentation which takes into account the specificity of arguments MoTe precisely an argument A "defeats" an argument B iff B contains a subargument less specific than A and which is in disagreement with A Geffner [1992] proposed a notion of *stable argument* to account for conditional entailment in the framework of default reasoning A *stable argument* is an argument which "defeats" each conflicting argument where A "defeats" B iff A contains a subargument preferred than B and conflicting with B The preference relation is extracted from the knowledge base and accounts for conditional aspects of defaults including specificity Most inferences authorized by conditional entailment are recovered with the concept of *stable argument* Specificity and more generally preference criteria will be briefly discussed in Section 5

4 Recovering Coherence-Based Non-monotonic Entailment

Throughout this section we consider the argumentation system (AR(\mathcal{E}) "undercuts") We recall that the belief base E may be K inconsistent The coherence-based approach for handling inconsistency involves a revision step (the selection of several consistent subbases (the so called preferred belief subbases) before applying classical entailment The term "syntax-based" has been first used to designate that kind of approach For instance syntax-based revision procedures were defined by Nebel [1991] and prioritized syntax-based entailment has been studied by [Benferhat et al 1993a] Here we consider syntax-based inference (from $3C \cup \mathcal{E}$) as defined through the management of maximal K-consistent subbases of E A maximal K-consistent subbase of E is called a *thesis* of (K, E) in the

following Three non-monotonic consequence relations may be defined using three main selection principles

Definition 10 [Cayrol and Lagasque Schiex 1993 1994]

According to the *skeptical or universal principle* ϕ is a *strong consequence* of \mathcal{E} ($\mathcal{E} \vdash^{\forall} \phi$) iff for each thesis T of (\mathcal{X}, \mathcal{E}) $\mathcal{X} \cup T \vdash \phi$

According to the *credulous or existential principle* ϕ is a *weak consequence* of \mathcal{E} ($\mathcal{E} \vdash^{\exists} \phi$) iff there exists at least one thesis T of (\mathcal{X}, \mathcal{E}) such that $\mathcal{X} \cup T \vdash \phi$

According to the *argumentative principle* ϕ is an *argumentative consequence* of \mathcal{E} ($\mathcal{E} \vdash^A \phi$) iff there exists at least one thesis T of (\mathcal{X}, \mathcal{E}) such that $\mathcal{X} \cup T \vdash \phi$ and no thesis T' of (\mathcal{X}, \mathcal{E}) such that $\mathcal{X} \cup T' \vdash \neg \phi$

Now we are able to restate the argumentation-based inference schemas presented in Section 3 in the framework of non monotonic syntax based entailment

Proposition 7 ϕ is a *plausible consequence* of \mathcal{E} ($\mathcal{E} \vdash_{pl} \phi$) iff ϕ is a *weak consequence* of \mathcal{E} ($\mathcal{E} \vdash^{\exists} \phi$) iff there exists at least one *stable extension* S of (AR(\mathcal{E}) "undercuts") such that $\phi \in \text{For}(S)$

Proof consequence of Proposition 4

Proposition 8 ϕ is a *probable consequence* of \mathcal{E} ($\mathcal{E} \vdash_{pr} \phi$) iff ϕ is an *argumentative consequence* of \mathcal{E} ($\mathcal{E} \vdash^A \phi$) iff there exists at least a *stable extension* S of (AR(\mathcal{E}) "undercuts") such that $\phi \in \text{For}(S)$ and no *stable extension* S' such that $\neg \phi \in \text{For}(S')$

Proof consequence of Proposition 7

Note that the first equivalence ($\mathcal{E} \vdash_{pr} \phi$ iff $\mathcal{E} \vdash^A \phi$) is also proved in [Benferhat et al 1993b]

Proposition 9 ϕ is a *confirmed consequence* of \mathcal{E} ($\mathcal{E} \vdash_{co} \phi$) iff there exists an argument (H, ϕ) such that H is included in *each* thesis of (\mathcal{X}, \mathcal{E}) iff there exists an argument (H, ϕ) which belongs to *each* *stable extension* of (AR(\mathcal{E}) "undercuts")

Proof consequence of Proposition 4

Proposition 10 ϕ is a *confirmed consequence* of \mathcal{E} ($\mathcal{E} \vdash_{co} \phi$) => ϕ is a *strong consequence* of \mathcal{E} ($\mathcal{E} \vdash^{\forall} \phi$)

Proof consequence of Proposition 9 The converse is false as shown by the following example

Example 2 $\mathcal{X} = \emptyset$, $\mathcal{E} = \{a, b, a \rightarrow \neg b, a \rightarrow c, b \rightarrow c\}$ We obtain three maximal \mathcal{X} -consistent subbases of \mathcal{E} T1 = $\{a, a \rightarrow \neg b, a \rightarrow c, b \rightarrow c\}$ T2 = $\{a, b, a \rightarrow c, b \rightarrow c\}$ T3 = $\{b, a \rightarrow \neg b, a \rightarrow c, b \rightarrow c\}$ So c is a *strong consequence* of \mathcal{E} Besides, we obtain two arguments concluding c ($\{a, a \rightarrow c\}$ c) and ($\{b, b \rightarrow c\}$, c) But neither of them belongs to AR++(\mathcal{E}) since there exists an argument concluding $\neg a$ and there exists an argument concluding $\neg b$ Hence c is not a *confirmed consequence* of \mathcal{E}

Another important consequence of Proposition 9 is

Proposition 11 ϕ is a *confirmed* consequence of \mathcal{E} ($\mathcal{E} \models_{\text{CD}} \phi$) iff ϕ is classically entailed by (the union of \mathcal{X} and the intersection of all the theses of \mathcal{X})

Note that in [Benferhat *et al.* 1993b] the intersection of all the theses of $(\mathcal{X}, \mathcal{E})$ is called the *Free* part of \mathcal{E} and is denoted by $\text{Free}(\mathcal{E})$. ϕ is said a *free* consequence of \mathcal{E} iff $\mathcal{X} \cup \text{Free}(\mathcal{E}) \vdash \phi$. Then Proposition 11 states that the *free* consequence defined from the theses of $(\mathcal{X}, \mathcal{E})$ is equivalent to the *confirmed* consequence defined from the argumentation system $(\text{AR}(\mathcal{E})$ "undercuts"). The notion of *free* consequence is also present but called *cautious* consequence in a work about inference with super normal defaults [Brass 1993].

Summary

A consequence relation R is said more conservative than a consequence relation R' iff for each formula ϕ of \mathcal{L} , $\mathcal{E} R \phi \Rightarrow \mathcal{E} R' \phi$. We obtain the hierarchy pictured in Table 1 where the consequence relations are less and less conservative. For each relation we find the definition in terms of theses and (if it exists) the associated acceptability class of arguments.

5 Conclusions and Perspectives

In the work reported here our purpose was to provide a better understanding of fundamental mechanisms of defeasible reasoning and not to propose one more formalism. Our most significant contribution to unifying numerous approaches to defeasible reasoning is the reconstruction of coherence based non-monotonic inference operations from various argument based approaches.

We have investigated the relations between argumentation based reasoning and non-monotonic coherence based reasoning through the study of two particular argumentation systems. Our best results have been obtained with the system defined by the binary relation "undercut" on the set of arguments built from a belief base \mathcal{E} .

However many other relations may be defined to compare arguments of \mathcal{E} : a specificity relation [Simari and Loui 1992], preference relations induced by a priority relation on the belief base [Liang-Goransson *et al.* 1993b]

[Geffner 1992], [Cayrol *et al.* 1993] or $b >$ certainty degrees [Benferhat *et al.* 1993b].

More works need to be done to integrate preference ordering between arguments into argumentation systems particularly as regards the definition of consequence relations. More precisely the following points deserve consideration:

comparison of arguments in favor of the same conclusion. Given a priority relation on the belief base we need appropriate aggregation modes to define a preference relation on the different supports of a statement. This kind of preference should respect the minimality for set inclusion. See [Cayrol *et al.* 1993] for preliminary work on the subject.

comparison of arguments for contradictory conclusions. A preference relation between "conflicting" arguments may lead to a new definition of the relation "defeats" [Simari and Loui 1992] [Geffner 1992]. For instance the argument $(H1, h1)$ "defeats" $(H2, h2)$ iff there exists a subargument of $(H2, h2)$ in disagreement with but less preferred than $(H1, h1)$. Thus a new argumentation system is obtained in which the construction of acceptability classes and associated consequence relations can be performed. A preference between conflicting arguments may be also directly used to define a consequence relation such as \circ is inferred if there exists an argument concluding \circ which is preferred to each argument concluding \circ . This definition was proposed in the framework of possibilistic logic by [Benferhat *et al.* 1993b].

definition of new acceptability classes. Given an argumentation system for instance $(\text{XR}(\mathcal{E})$ "undercuts") we may consider the arguments which are never "rebutted" (resp "undercut") by a preferred argument. Once again new acceptability classes will induce (a priori) new consequence relations.

Besides much work has been devoted to the integration of preference relations into non-monotonic inference schemes (see [Cayrol and Lngasquie-Schiex 1993] for a survey). When the belief base is equipped with a pre-ordering (namely a priority relation) a selection principle (universal, existential or argumentative) is coupled with a mechanism of generation of preferred subbases. These generation mechanisms usually respect maximality for set

free consequence \Leftrightarrow	$\mathcal{X} \cup (\bigcap \text{theses}) \vdash \phi$
confirmed consequence	$\exists (H, \phi) \in \text{AR}_{++}(\mathcal{E})$
strong consequence	$\forall \text{Thesis } \mathcal{X} \cup \text{Thesis} \vdash \phi$
argumentative consequence \Leftrightarrow	$\exists \text{Thesis } \mathcal{X} \cup \text{Thesis} \vdash \phi$ and $\forall \text{Thesis } \mathcal{X} \cup \text{Thesis} \text{ is } \phi \text{ consistent}$
probable consequence	$\exists (H, \phi) \in \text{AR}_+(\mathcal{E})$
weak consequence \Leftarrow	$\exists \text{Thesis } \mathcal{X} \cup \text{Thesis} \vdash \phi$
plausible consequence	$\exists (H, \phi) \in \text{AR}(\mathcal{E})$

Table 1

inclusion and select preferred subbases among the theses. For instance, inclusion-based preference combines priorities and maxima consistent subbases while lexicographic preference [Benferhat *et al.* 1993a] combines priorities and consistent subbases of maximal cardinality. As in the case of a flat belief base, it will be interesting to establish connections between the various inference schemas developed in the framework of argumentation systems with preference and the inference relations defined through selection principles of preferred theses. We are currently working on this topic.

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