Method of Constructing a Three-Dimensional Wireless Coverage Map

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Abstract. In recent years, wireless networks have become widespread. Devices connected to Wi-Fi and mobile networks generate a significant share of Internet traffic. Analysts predict that devices connected to Wi-Fi and mobile networks will generate 73% of Internet traffic by 2021. Due to the growing need for data exchange not only in two-dimensional space, there is a need for three-dimensional analysis of wireless network coverage. The use of modern algorithms of simultaneous localization and mapping allow to automate the acquisition of information about the level of the received signal from Wi-Fi access points simultaneously with the acquisition of sufficiently accurate data on the location of the receiver. The method under development will help network engineers and network administrators to build wireless coverage maps that will most representatively show existing network coverage. In this work, we develop and estimate the quality of a method for constructing a three-dimensional map of a network coverage by anchor points using approximation and interpolation methods.

Keywords: wireless networks, coverage analysis, interpolation methods.

1 Introduction

Over the past 10-15 years, wireless networks (especially the 802.11*) have become widespread. Wi-Fi is used in lecture halls, stadiums, train stations, exhibition complexes, meeting rooms in office complexes, etc. Analysts from CISCO predict that devices connected to Wi-Fi and mobile networks will generate 73% of Internet traffic by 2021 [1].

The high density of the location of WLAN users makes it more rational to use access points, more competently calculate their location.

Heatmaps are currently used to show representative network coverage. Sample of WLAN heatmap is shown in fig 1.

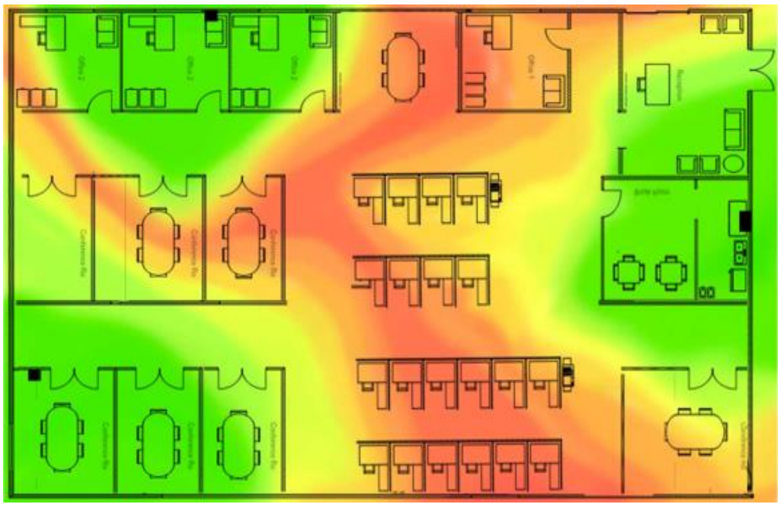

Fig. 1. Sample of wireless coverage heatmap (created with Ekahau HeatMapper).

In recent years, the use of wireless networks has ceased to be limited to two-dimensional space. Small unmanned vehicles, such as quadcopters, have been used in various areas of human activity. Also, IoT class devices are widely used, which also actively use wireless connections and are often located in hard-to-reach places.

All existing software for building heatmaps can be divided into 2 categories: based on model data[6-7] and based on experimental data[8-13].

For example, D-Link Wi-Fi Planner Pro [6] allows to simulate signal propagation taking into account the characteristics of network equipment (antenna patterns) and a given environment (dimensions and material from which surrounding objects and partitions are made). Most often, the simulation results do not correspond to real data, since the simulation does not take into account a number of parameters that significantly affect the signal propagation. In support of the above, the creators of this software say that after the actual deployment of the network, this model should not be used to replace the existing one [6]. Also, Wi-Fi Planner Pro does not allow to simulate a heat map for open spaces (street quarters, stadiums, etc.).

Due to the growing need for data exchange not only in two-dimensional space, there is a need for three-dimensional analysis of wireless network coverage [2].

They do not allow to accurately correlate the RSSI value and its corresponding coordinate in space, since the user has to “guess” his location on the floor plan.

The developed method will help network engineers, network administrators in constructing wireless coverage maps that will show the current status of network coverage.

The task of this type of analysis will be facilitated by combining algorithms for constructing a function by anchor points and algorithms for simultaneous localization and mapping – SLAM [4].

This method will allow you to receive the signal strength from Wi-Fi access points at the same time obtaining accurate data on the location of the receiver.

The result of data collection will be a built-up cloud of points of the surrounding space, the trajectory of the camera and the corresponding values of the received signal strength indicator – RSSI. An example of the obtained trajectory using the SLAM algorithm is shown in Fig. 2.

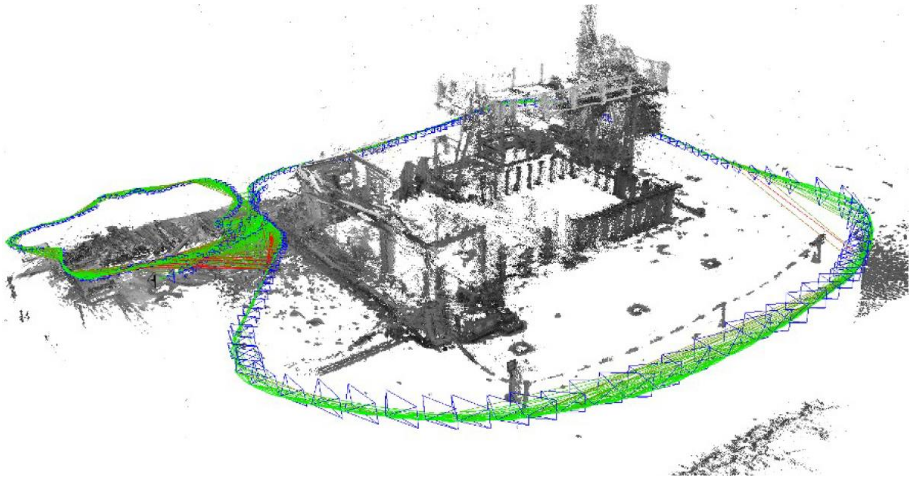

Fig. 2. Example of calculated point cloud and camera track [4]

In this paper, we propose a method for constructing a three-dimensional map of the coverage by anchor points using approximation and interpolation methods.

2 Related work

2.1 Development of mathematical model

The camera path and RSSI values will be obtained as $\{x_i, y_i, z_i, RSSI_i\}$, where $i \in 1 \dots N$ – is the number of collected points.

Next, the user sets the step of the spatial grid – D and the radius of the search for anchor points R. Figure 3 shows a schematic representation of the spatial grid: yellow points is known values; purple points will be used for RSSI calculations.

RSSI can be represented as a function of $RSSI(x, y, z)$. RSSI calculation should be based on known values using approximation and interpolation methods. For this, the authors propose using the Lagrange polynomial of the second degree.

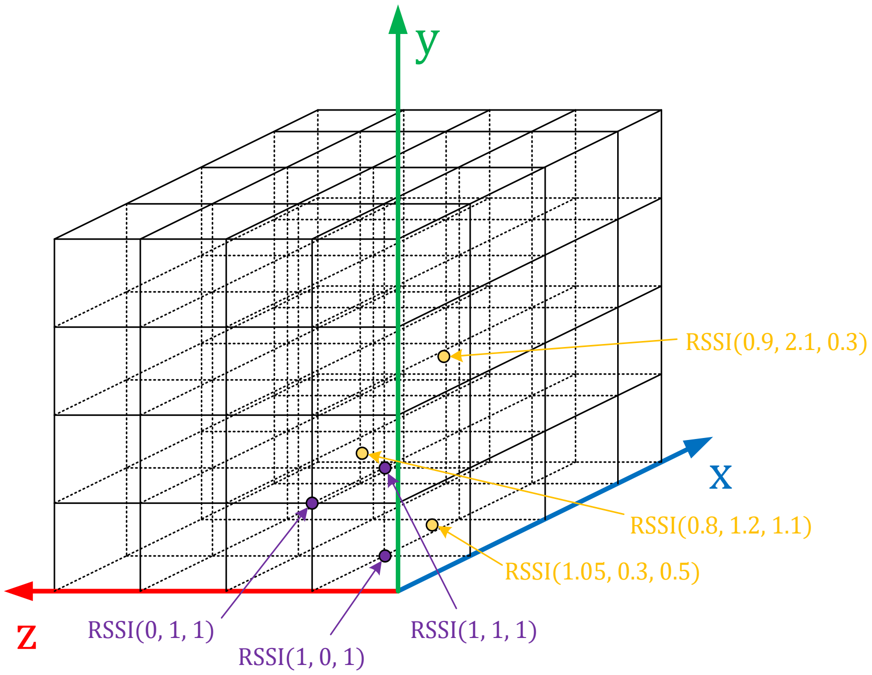

Fig. 3. Schematic representation of a spatial grid

The Lagrange interpolation polynomial is usually used to obtain a function of one variable in the form of a polynomial at given points (interpolation nodes), the values of which are known [11].

The single-degree polynomial has the following form [11]:

$$L_{(x)} = \sum_{i=1}^{n}\left(f_{(x_i)} \times \prod_{j=1,j\neq i}^{n} \frac{\begin{vmatrix} x & 1 \\ x_j & 1 \end{vmatrix}}{\begin{vmatrix} x_i & 1 \\ x_j & 1 \end{vmatrix}}\right), \tag{1}$$

where: n – is the number of interpolation nodes, i – is the index from 1 to n ($n \geq 2$) according to the node (anchor point) numbers, $f_{(x_i)}$ – values in the nodes.

The work [3] describes the application of the Lagrange polynomial for functions of several variables, and not of one. The formula for constructing a polynomial function of m is presented below:

$$L_{(x^{(1)},\dots,x^{(m)})} = \sum_{i\in\{1,\dots,n\}}\left(f_{(x_i^{(1)},\dots,x_i^{(m)})} \times \prod_{\substack{k_1\in\{\{1,\dots,n\}\setminus\{i\}\} \\ \vdots \\ k_m\in\{\{1,\dots,n\}\setminus\{i,k_1,\dots,k_{m-1}\}\}}} \frac{\begin{vmatrix} x^{(1)} & \cdots & x^{(m)} & 1 \\ x_{k_1}^{(1)} & \cdots & x_{k_1}^{(m)} & 1 \\ \vdots & \ddots & \vdots & \vdots \\ x_{k_m}^{(1)} & \cdots & x_{k_m}^{(m)} & 1 \end{vmatrix}}{\begin{vmatrix} x_i^{(1)} & \cdots & x_i^{(m)} & 1 \\ x_{k_1}^{(1)} & \cdots & x_{k_1}^{(m)} & 1 \\ \vdots & \ddots & \vdots & \vdots \\ x_{k_m}^{(1)} & \cdots & x_{k_m}^{(m)} & 1 \end{vmatrix}}\right), \tag{2}$$

where: m – is the number of given points ($m \leq n-1$); $k_1 \dots k_m$ – all possible linear combinations from 1 to m, excluding i; other variables have the same meaning as in formula (1) [2].

In the problem under consideration, the dimension of space is 4 $(x, y, z, RSSI)$, therefore, to compose a polynomial of the second degree, it suffices to have $m = 5$ anchor points.

Resultant formula for calculating RSSI at a random spatial point $\{x, y, z\}$ takes the form of the following linear combination:

$$RSSI_{(x,y,z)} = l^{(1)}_{(x,y,z)} + l^{(2)}_{(x,y,z)} + l^{(3)}_{(x,y,z)} + l^{(4)}_{(x,y,z)} + l^{(5)}_{(x,y,z)}, \tag{3}$$

Each of the elements of this linear combination has the following form:

$$l^{(\mathrm{i})}_{(x,y,z)} = RSSI_{(x_i,y_i,z_i)} \times \prod_{\substack{k_1 \in \{\{1,\dots,5\}\setminus\{i\}\} \\ \vdots \\ k_5 \in \{\{1,\dots,5\}\setminus\{i,k_1,\dots,k_5\}\}}} \frac{\begin{vmatrix} x & y & z & 1 \\ x_{k_1} & y_{k_1} & z_{k_1} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ x_{k_5} & y_{k_5} & z_{k_5} & 1 \end{vmatrix}}{\begin{vmatrix} x_i & y_i & z_i & 1 \\ x_{k_1} & y_{k_1} & z_{k_1} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ x_{k_5} & y_{k_5} & z_{k_5} & 1 \end{vmatrix}}, \tag{4}$$

Full formula for each element shown in (5-10)

$$l^{(1)}_{(x,y,z)} = RSSI_{(x_1,y_1,z_1)} \times \frac{\begin{vmatrix} x & y & z & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}} \times \frac{\begin{vmatrix} x & y & z & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}} \times \frac{\begin{vmatrix} x & y & z & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_4 & y_4 & z_4 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_4 & y_4 & z_4 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}} \times \frac{\begin{vmatrix} x & y & z & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}} \tag{5}$$

$$l^{(2)}_{(x,y,z)} = RSSI_{(x_2,y_2,z_2)} \times \frac{\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}}{\begin{vmatrix} x_2 & y_2 & z_2 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}} \times \frac{\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}}{\begin{vmatrix} x_2 & y_2 & z_2 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}} \times \frac{\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_4 & y_4 & z_4 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}}{\begin{vmatrix} x_2 & y_2 & z_2 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_4 & y_4 & z_4 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}} \times \frac{\begin{vmatrix} x & y & z & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}}{\begin{vmatrix} x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}} \tag{6}$$

$$l^{(3)}_{(x,y,z)} = RSSI_{(x_3,y_3,z_3)} \times \frac{\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}}{\begin{vmatrix} x_3 & y_3 & z_3 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}} \times \frac{\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}}{\begin{vmatrix} x_3 & y_3 & z_3 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}} \times \frac{\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_4 & y_4 & z_4 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}}{\begin{vmatrix} x_3 & y_3 & z_3 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_4 & y_4 & z_4 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}} \times \frac{\begin{vmatrix} x & y & z & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_4 & y_4 & z_4 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}}{\begin{vmatrix} x_3 & y_3 & z_3 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_4 & y_4 & z_4 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}} \tag{7}$$

$$l_{(x,y,z)}^{(4)} = RSSI_{(x_4,y_4,z_4)} \times \frac{\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix}}{\begin{vmatrix} x_4 & y_4 & z_4 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix}} \times \frac{\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix}}{\begin{vmatrix} x_4 & y_4 & z_4 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}} \times \frac{\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}}{\begin{vmatrix} x_4 & y_4 & z_4 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}} \times \frac{\begin{vmatrix} x & y & z & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}}{\begin{vmatrix} x_4 & y_4 & z_4 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_5 & y_5 & z_5 & 1 \end{vmatrix}} \quad (8)$$

$$l_{(x,y,z)}^{(5)} = RSSI_{(x_5,y_5,z_5)} \times \frac{\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix}}{\begin{vmatrix} x_5 & y_5 & z_5 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix}} \times \frac{\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix}}{\begin{vmatrix} x_5 & y_5 & z_5 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}} \times \frac{\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}}{\begin{vmatrix} x_5 & y_5 & z_5 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}} \times \frac{\begin{vmatrix} x & y & z & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}}{\begin{vmatrix} x_5 & y_5 & z_5 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}} \quad (9)$$

2.2 Modeling “Ideal” coverage

Path-Loss model

According to research [2] RSSI is measured by a receiver on a logarithmic scale in dBm and can be described by the following equation:

$$P_d = P_0 - 10 * n * \lg\left(\frac{d}{d_0}\right) \quad (10)$$

where:
d [м] is the distance from the device to the transmitter;
d_0 [м] is the distance from the device to the access point at which the signal power P_0 was measured;
P_0 $[dBm]$ is the signal power of the device, measured at a unit distance d_0 from the device;
n is the coefficient of signal power loss during propagation in the medium, dimensionless quantity ($n = 2$ for air, increases increase with obstacles);
P_d $[dBm]$ – RSSI.

In the paper [5], the dependence indicated above is confirmed experimentally. Figure 4 graphically displays the dependence. The black line indicates RSSI values indoors, the red line indicates outdoor RSSIs at a distance of up to 300 m.

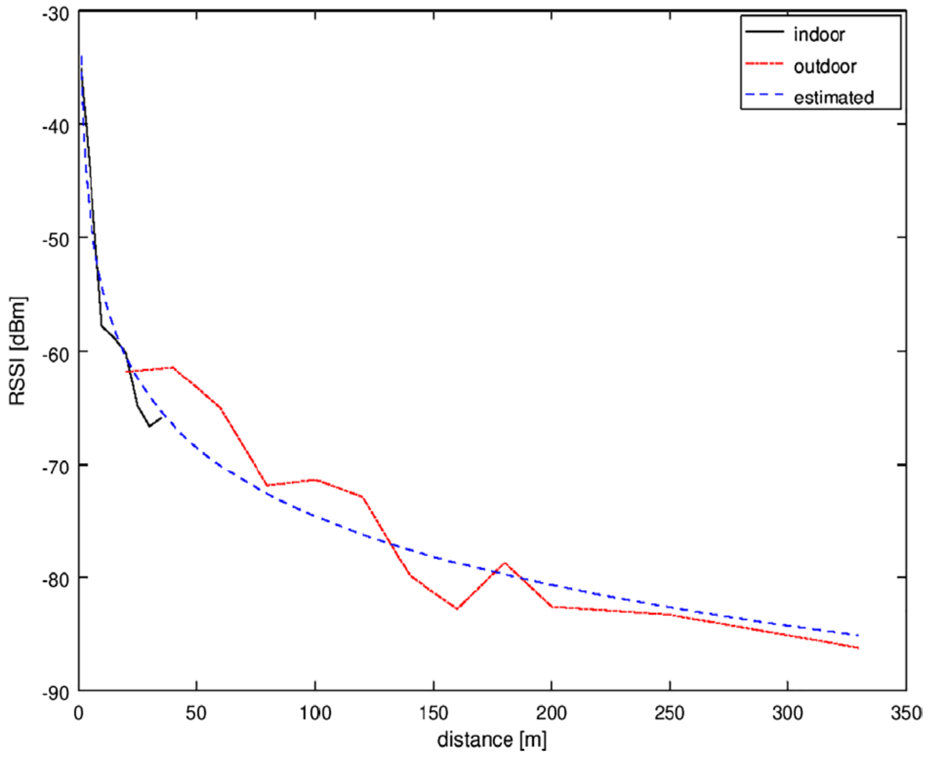

Fig. 4. Combined indoor and outdoor RSSI dependence from distance [5]

Application of Path-Loss model

To evaluate the quality of the developed method, the following steps are proposed:

1. simulating the ideal propagation of the signal in the room (result – dataset 1);
2. simulating the movement of the “receiver” in this room with the registration of RSSI values (result – dataset 2);
3. applying the proposed mathematical model to the dataset collected in the previous step (result – dataset 3);
4. compare the dataset 1 and dataset 2.

Room parameters:

- size: 30m * 30m * 15m;
- step of spatial grid (D): 1,35m;
- two access points with coordinates: {15m, 5m, 15m} and {25m, 20m, 7m}.

Calculated dataset 1 is shown in the fig. 5.

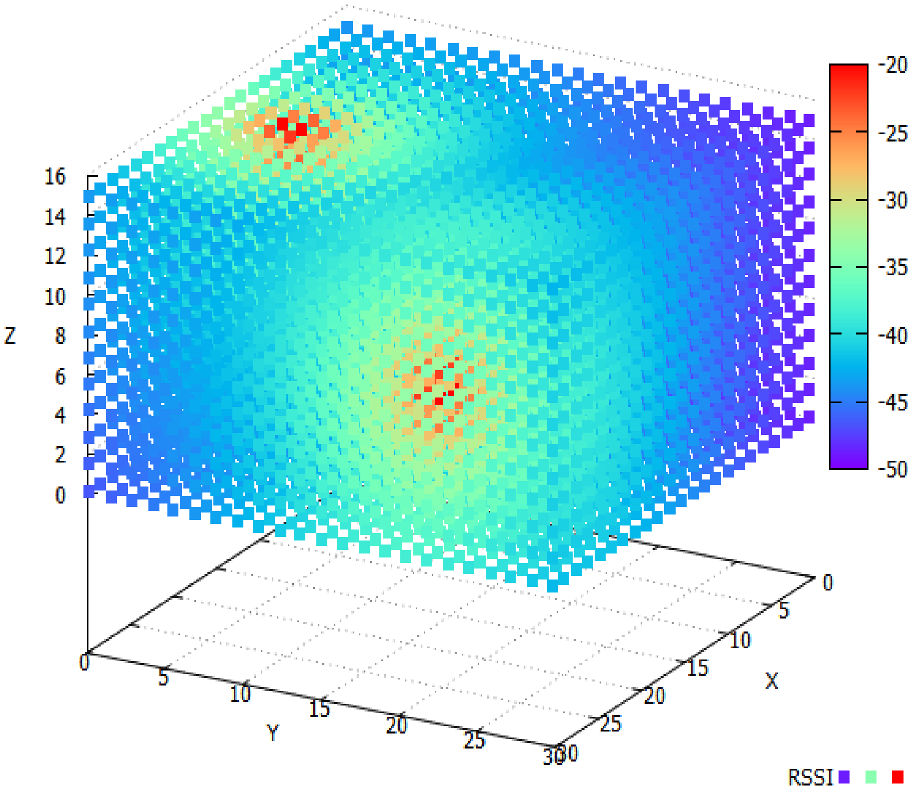

Fig. 5. Visual representation of dataset 1

To simulate the movement of the "receiver" in space, the Path-Loss model was applied to a certain number of points that have the shape of a spiral segment, resulting in a new dataset, visually displayed in fig. 6.

2.3 Application of developed mathematic model

As a result of applying formulas (3, 4) to dataset 2, a spatial grid is obtained, shown in fig. 7.

As we can be seen from fig. 7, in general, dataset 1 and dataset 3 looks similar, but only in a first approximation. Obviously, each of the calculated RSSI values has its own confidence level depending on the distance to the anchor points from dataset 2.

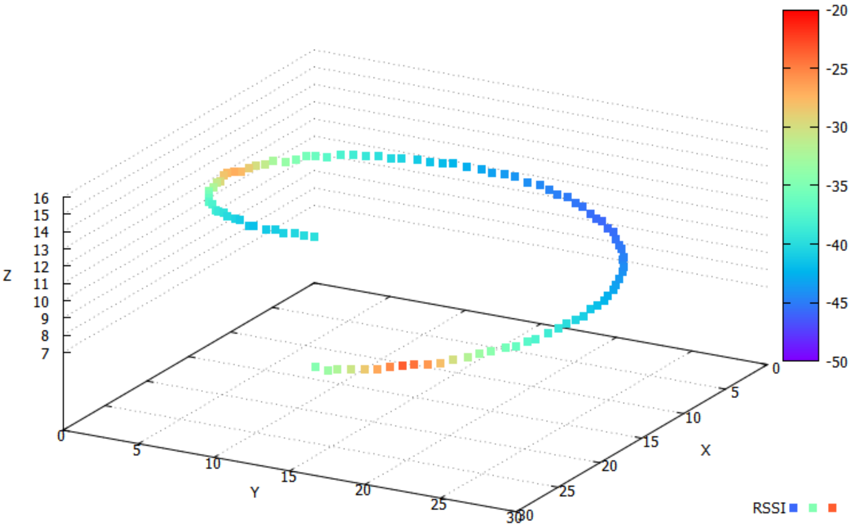

Fig. 6. Visual representation of dataset 2

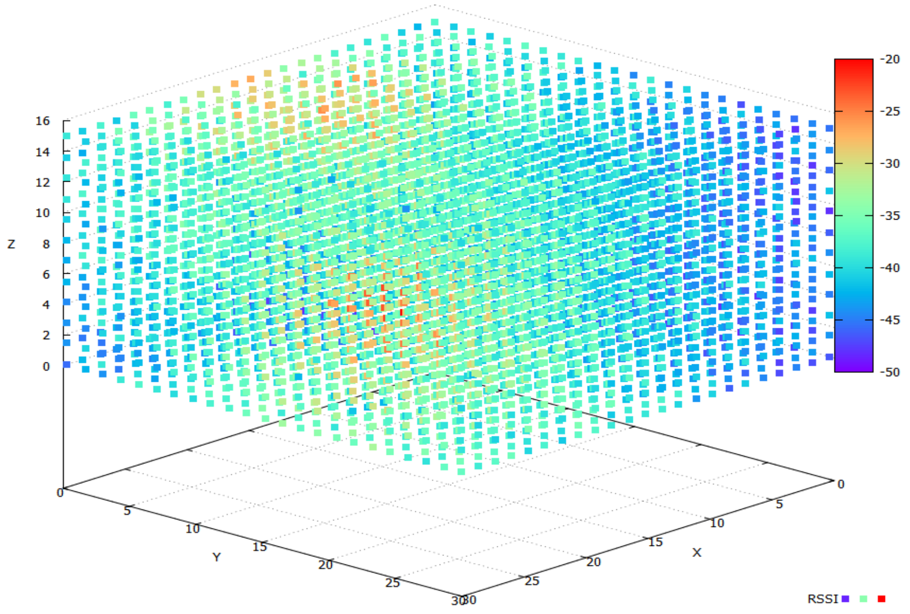

Fig. 7. Visual representation of dataset 3

Authors propose dividing all the calculated points from dataset 3 into several groups. Then, for each group, calculate the standard deviation of RSSI from ideal propagation (dataset 1). The results are presented in fig. 8.

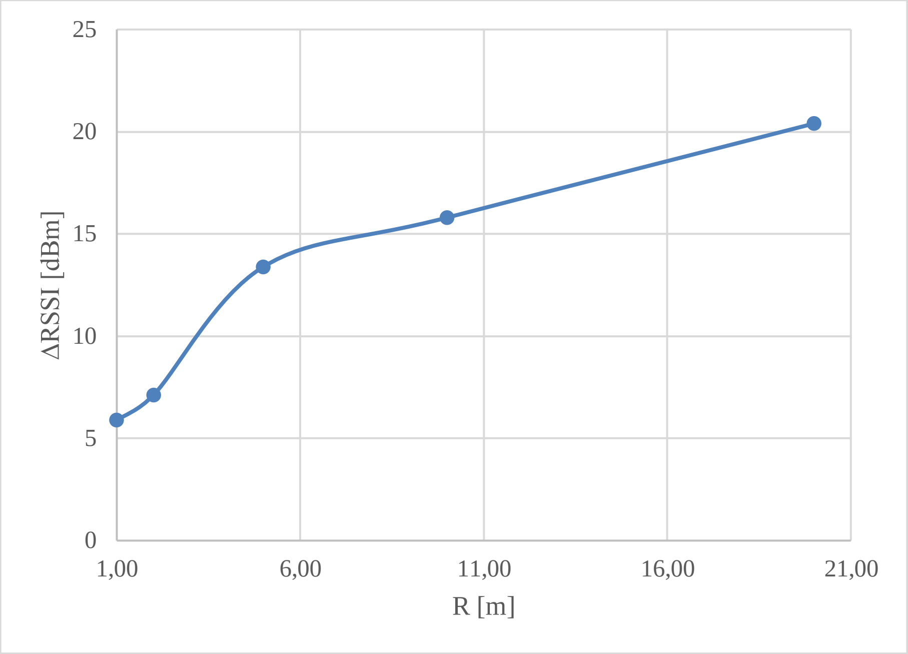

Fig. 8. Dependence of the standard deviation of the error (ΔRSSI, Y-axis) on the distance to the anchor points (R, X-axis)

The highest deviations at a distance of more than 5 meters from anchor points. These values are not acceptable for predicting signal propagation at these points. But they indicate that further research and refinement of the method, the use of more advanced approximation algorithms will reduce deviations from "ideal" model data.

3 CONCLUSIONS

The subject of this study has a weak theoretical and practical development. In this paper, the first steps in the creation of a new method of constructing maps of wireless propagation.

Due to the analysis of errors, directions for further developments were identified, hypothetical methods for increasing the accuracy of the method were found, and further development of the researches was determined.

With further development, this method will help network engineers and administrators to build three-dimensional maps of wireless signal propagation.

Also, when combining the developed method with SLAM algorithm, there will be no need for manual comparison of odometric data when measuring the level of the wireless signal in space, which will have a positive impact on the accuracy of the maps.

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