Online Parameter Estimation and Optimal Input Design * **

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Abstract. In flight system identification, flight test data is used to estimate model parameters. The aircraft is excited during the flight with a set of inputs (maneuvers) to generate the data for parameter estimation. Different approaches exist to design the inputs which result in an adequate information content in the acquired data during the flight tests. In most cases, inputs are applied to the aircraft which are designed based on a priori knowledge about the system. In this paper we investigate a method to update the model parameters during the flight and update the inputs onboard the aircraft. This method can increase the achieved parameter accuracy and is expected to reduce the required time for the flight test program, and therefore, also the program cost.

Keywords: Parameter estimation · Optimal input design · Optimal control · System identification.

1 Introduction

In flight vehicle system identification, a mathematical model is deducted from recorded flight data using a series of test maneuvers. If a fixed model structure is assumed, the system identification task narrows down to a parameter estimation problem. It has been shown that the type of inputs applied to an aircraft has a significant influence on the accuracy of the parameter estimates [11, 12]. In this context, it is desirable to apply inputs to the dynamic system which are expected to maximize the information content in the flight test data. As such, optimal inputs can help to reduce the flight test time while resulting in more accurate parameter estimates. This reduces the overall cost of a system identification project, specially when flight tests are expensive to perform and flight time is limited on the flight vehicle under investigation (e.g. cruise missiles). This problem can be formulated as an optimal control problem with a special cost

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function. However, an initial model (with initial values for the parameters) is required to pose and solve the optimal control problem. Since determining the parameter values in the model is the primary goal of the flight tests and the subsequent parameter estimation, accurate parameter values often do not exist when designing the experiments. Different approaches have been suggested to solve this problem, such as a robust approach to design the optimal inputs, in which the uncertainty of the optimal inputs can be considered in the input design process [5]. In this paper, we investigate the application of online parameter estimation to adapt the optimal inputs during the runtime of the experiment.

Besides flight testing for system identification and parameter estimation, indirect adaptive control is also an important use case for onboard adaption of optimal input design for parameter estimation. In such cases, the controller is adjusted based on the change in the system parameters. Therefore, maneuvers are pursued which lead to more accurate parameter estimates and therefore a better performing controller. Many previous studies such as [8] have investigated the topic of onboard optimal input design in connection with indirect adaptive control.

The problem discussed in this paper can be formulated as a Dual-Mode Batch-to-Batch Optimization as presented in [4]. Based on this approach, it is possible to assess our solution method and quantify the possible loss of optimality. This is however beyond the scope of the present study and will be investigated in future work.

2 Problem Definition

The problem in parameter estimation is to find the parameter values in a fixed model structure

$$\hat{\boldsymbol{\theta}} = \tilde{\boldsymbol{\theta}}(\boldsymbol{Z}, \boldsymbol{u}) \,. \tag{1}$$

In the above equation, $\hat{\boldsymbol{\theta}}$ is the vector of the parameter estimates, $\tilde{\boldsymbol{\theta}}$ is the estimator and $\boldsymbol{Z}$ and $\boldsymbol{u}$ are the measured outputs and inputs. In this paper, we limit our study to linear systems of the following form

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}, \tag{2}$$

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u}, \tag{3}$$

$$\boldsymbol{z}(t_i) = \boldsymbol{y}(t_i) + \boldsymbol{v}(t_i), \quad i = 1, 2, \ldots, N, \tag{4}$$

where $\boldsymbol{x}$ is the vector of the system states, $\boldsymbol{y}$ is the output vector, $\boldsymbol{u}$ is the input vector and $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ are the system-, input-, output- and feed-through matrices. Moreover, $\boldsymbol{v}$ is a zero-mean Gaussian white noise sequence with the covariance matrix $\boldsymbol{R}$.

It can be shown that the Fisher information matrix $\boldsymbol{M}$ provides an indication about the information content in the gathered flight test data during the experiment

$$\boldsymbol{M} = \sum_{i=1}^{N} \left[\frac{\partial \boldsymbol{y}(t_i)}{\partial \boldsymbol{\theta}} \right]^T \boldsymbol{R}^{-1} \left[\frac{\partial \boldsymbol{y}(t_i)}{\partial \boldsymbol{\theta}} \right]. \tag{5}$$

The inverse of the Fisher information matrix, the Dispersion matrix $\boldsymbol{D} = \boldsymbol{M}^{-1}$ is the theoretical lower bound for the covariance matrix of the parameter estimates if an unbiased estimator is used [7, 12]. The square roots of the diagonal elements of the Dispersion matrix $\boldsymbol{D}$ are called Cramér-Rao bounds and are the theoretical lower bounds for parameter estimate standard deviations. If an asymptotically efficient estimator is used, the Cramér-Rao bounds are an estimate for the parameter estimate standard deviations.

Since the Cramér-Rao bounds and the Dispersion matrix do not depend on the measurements, an initial model and initial values for parameters can be used to design inputs that minimize the remaining uncertainty in the parameters. In this study, we choose to minimize the trace of the Dispersion matrix which is the sum of the estimate for the parameter variances under consideration of practical state and input constraints. The optimal control problem (OCP) can be written as

$$\begin{aligned} \underset{\boldsymbol{u}}{\text{minimize}} \quad & \operatorname{tr}\left[\boldsymbol{M}^{-1}\right] \\ \text{subject to} \quad & \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{u}, \boldsymbol{x}, \boldsymbol{\theta}), \\ & \boldsymbol{x} \in [\boldsymbol{x}_{lb}, \boldsymbol{x}_{ub}], \\ & \boldsymbol{u} \in [\boldsymbol{u}_{lb}, \boldsymbol{u}_{ub}], \\ & \boldsymbol{\theta} = \boldsymbol{\theta}_0, \\ & t \in [0, T], \end{aligned} \tag{6}$$

where T is the duration of the maneuver.

Since the estimation of the parameter values is the ultimate goal of the flight test campaign, no precise values of the parameter estimates are available before the flight tests. A common approach is to perform flight tests with suboptimal inputs or optimal inputs based on inaccurate parameters, record the data and perform offline parameter estimation. The new parameter values are used to design new inputs which are used for subsequent flight tests. This becomes a costly iterative process. Robust optimal control methods have been used in [5] to solve this uncertain optimal control problem. However, as the aircraft is excited over time, more information becomes available about the aircraft dynamics which can then be used to estimate more accurate parameter values. The new parameter values can be used to adjust the maneuvers onboard the aircraft. To reduce the required time between the maneuvers, we suggest estimating the parameters online while new measurements become available.

3 Solution Method

The solution method applied in this study consists of an Extended Kalman Filter (EKF) for the online parameter estimation. The approaches for designing the optimal inputs are based on [6].

3.1 Online Parameter Estimation

Different methods exist for online flight vehicle parameter estimation. These methods allow to update the parameter values onboard the aircraft as new measurements become available rather than recording the measurements over a time period and analyzing them in batches. Online parameter estimation methods can be formulated in time and frequency domain as described in [10, 12]. In this study we use an EKF for estimating the states and parameters in realtime. The implementation in this study is based on [12]. The state vector of the dynamic system (2) is augmented with the system parameters as follows

$$\boldsymbol{x}_a = \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{\theta} \end{bmatrix}. \tag{7}$$

The dynamics of the original system states is governed by (2). Since we assume the parameters to be constant, their dynamics is governed by

$$\dot{\boldsymbol{\theta}} = \mathbf{0}. \tag{8}$$

The governing equations for the augmented system become

$$\begin{aligned} \dot{\boldsymbol{x}}_a &= \boldsymbol{A}_a \boldsymbol{x}_a + \boldsymbol{B}_a \boldsymbol{u}, && (9) \\ \boldsymbol{y} &= \boldsymbol{C}_a \boldsymbol{x}_a + \boldsymbol{D}\boldsymbol{u}, && (10) \\ \boldsymbol{z}(t_i) &= \boldsymbol{y}(t_i) + \boldsymbol{v}(t_i), \quad i = 1, 2, \ldots, N, && (11) \\ & && (12) \end{aligned}$$

with the initial conditions

$$\begin{aligned} E[\boldsymbol{x}_a(0)] &= \bar{\boldsymbol{x}}_{a,0}, && (13) \\ E\{[\boldsymbol{x}_a(0) - \bar{\boldsymbol{x}}_{a,0}][\boldsymbol{x}_a(0) - \bar{\boldsymbol{x}}_{a,0}]\} &= \boldsymbol{P}_{a,0}, && (14) \end{aligned}$$

and

$$\boldsymbol{A}_a = \begin{bmatrix} \boldsymbol{A}(\boldsymbol{\theta}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \ \boldsymbol{B}_a = \begin{bmatrix} \boldsymbol{B}(\boldsymbol{\theta}) \\ \mathbf{0} \end{bmatrix}, \ \boldsymbol{C}_a = \begin{bmatrix} \boldsymbol{B}(\boldsymbol{\theta}) & \mathbf{0} \end{bmatrix}, \ \bar{\boldsymbol{x}}_{a,0} = \begin{bmatrix} \bar{\boldsymbol{x}}_0 \\ \boldsymbol{\theta} \end{bmatrix}, \tag{15}$$

where $\bar{\boldsymbol{x}}_{a,0}$ is the mean value of the augmented states at the initial time point and $\boldsymbol{P}_{a,0}$ is its respective covariance matrix.

The dynamic system described above is not linear anymore since the new augmented states which include the parameters are elements of the system matrix $\boldsymbol{A}$ and are multiplied with the state vector $\boldsymbol{x}$. Therefore, the equations governing the nonlinear system (9) can be written as

$$\dot{\boldsymbol{x}}_a = \boldsymbol{f}(\boldsymbol{x}_a, \boldsymbol{u}) \,, \tag{16}$$

$$\boldsymbol{y} = \boldsymbol{g}(\boldsymbol{x}_a) \,, \tag{17}$$

$$\boldsymbol{z}(t_i) = \boldsymbol{y}(t_i) + \boldsymbol{v}(t_i), \quad i = 1, 2, \ldots, N. \tag{18}$$

The parameters of the system (2) are now states of the augmented system (16) and can be estimated via an EKF as described in [12]. In addition to the parameter values, an estimate for the parameter covariance matrix is also computed by the Kalman filter.

3.2 Optimal Input Design

As mentioned above, optimal inputs for parameter estimation are determined by solving the optimal control problem described in (6). The cost function of the mentioned problem is non-convex and previous studies in [6] have shown that an initial guess near the optimal point is required to achieve a solution via the direct method for optimal control (using gradient based optimization). The two stage optimization method of [6] is applied to solve the optimal control problem stated in (6). First, a dynamic programming method with a relatively rough discretization is applied to generate an initial guess for the optimal control and the respective system response. In a second step the direct method for optimal control (see [1]) is initialized with the results of the first step and applied to find the final solution. In this case the original optimal control problem (6) is transcribed into a nonlinear programming problem and solved with a gradient based solver. We use an optimal control and parameter estimation toolbox, Falcon.m [13], developed at the Institute of Flight System Dynamics of TU Munich for the second optimization stage. The details of the solution methodology and implementation for designing optimal inputs based on an initial model can be found in [6].

3.3 Onboard Adjustment of the Inputs

As stated before, the optimal inputs computed offline are based on an initial model with an initial set of parameters. As shown in Fig. 1 an optimal maneuver is designed offline with the initial values of the parameters. This maneuver is injected during the flight. An EKF updates the parameter values during the optimization as new data becomes available.

After each maneuver, the gradient based optimization is repeated with the updated parameter values to adapt the optimal inputs. Thereby, the optimal input generated with the last set of parameters and the corresponding system states are used to initialize the optimization with the direct method for optimal control. Similarly, each EKF run for online parameter estimation is initialized by previous estimates for the parameters, their covariance matrix and the state parameter correlations. In the first parameter estimation run, we initialize the covariance matrix of the parameters by a diagonal matrix with a large value

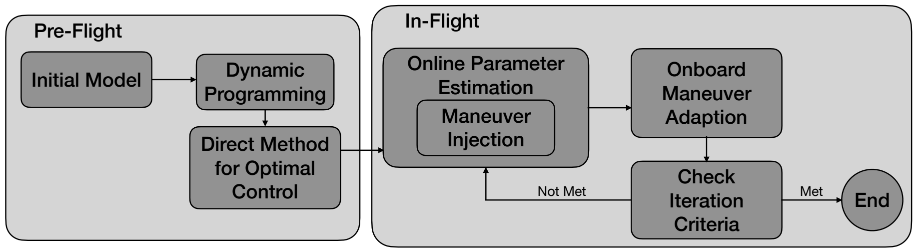


Fig. 1. Algorithmic steps of the pre-flight and in-flight maneuver optimization.

(here 10^5) on its diagonal. No correlation is assumed between the parameters and the states of the system in the initial covariance matrix of the augmented states (for the first maneuver). The initial state covariance matrix is considered as a tuning parameter and should be chosen considering the accuracy of the aircraft's navigation system and controller.

4 Numerical Results

A short period model of an aircraft is used to illustrate the proposed method in this study. The linear model was featured in multiple previous studies such as [2, 6, 9, 11]. The model is described as

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_\alpha & 1 \\ M_\alpha & M_q \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} Z_\eta \\ M_\eta \end{bmatrix} \eta, \tag{19}$$

where α is the angle of attack, q the pitch rate, and η the elevator deflection. The output equation is

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha(t) \\ q(t) \end{bmatrix}. \tag{20}$$

The measurement equations of the system are:

$$\begin{bmatrix} y_{m_1}(i) \\ y_{m_2}(i) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1(t_i) \\ y_2(t_i) \end{bmatrix} + \begin{bmatrix} v_1(t_i) \\ v_2(t_i) \end{bmatrix}, \quad i = 1, 2, \ldots, N, \tag{21}$$

$$\boldsymbol{v}(i) = \begin{bmatrix} v_1(t_i) \\ v_2(t_i) \end{bmatrix}. \tag{22}$$

In the above equation, $\boldsymbol{v}(t_i)$ is the i-th realization of a zero mean white Gaussian random process with the following covariance matrix

$$\boldsymbol{R} = \begin{bmatrix} 2.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}. \tag{23}$$

The model parameter values are provided in Table 1. The parameters M_α, M_q, and M_η are unknown. We design the inputs such that the information content with respect to these parameter is maximized (see (6)). Other parameters are considered to be known in this example. Significantly different initial parameter values (provided in Table 1) have been used to design the first optimal input signal (prior to the experiment). Four parameter estimation and input design iterations (described in Section 3.3) are performed during the test run-time. After each maneuver optimization, new parameter values are provided by the EKF based on virtual measurement data generated from the model described in (19)–(23). A sampling rate of 100 Hz is assumed and the run-time of each maneuver is 4 s. The discrete time step for the solution of the optimal control problem (6) with the direct method for optimal control is set to 0.05 s. We perform 4 iterations of parameter estimation and input optimization during the flight. The following input and state bounds are considered to design the inputs

$$\eta \in [-12.5^\circ, 12.5^\circ]\,, \tag{24}$$

$$\alpha \in [-10^\circ, 10^\circ]\,, \tag{25}$$

$$q \in \left[-12^\circ \mathrm{s}^{-1}, 12^\circ \mathrm{s}^{-1}\right]. \tag{26}$$

It can be seen in Fig. 2 that the structure of the optimal inputs remain roughly consistent as the parameter values get updated. This is in agreement with using the optimal inputs from the previous set of parameters as an initial guess for the maneuver optimization in the next step. Figure 3 shows the outputs of the dynamic system corresponding to the inputs of Fig. 2 at each iteration. The change of the parameters and the respective estimate for their standard deviations after each iteration are shown in Fig. 4. It can be seen that the final values of the standard deviations in Fig. 4 are consistently lower than the Cramér-Rao bounds in Table 1. This is due to the fact that the Kalman filter uses the available data in all of the five maneuvers, whereas only one maneuver run has been considered in Table 1. It is also worth mentioning that no process noise has been considered in this study. The initial state covariance matrix for each EKF run is set to

$$\boldsymbol{P}_0 = \begin{bmatrix} 0.2 & 0.0 \\ 0.0 & 0.1 \end{bmatrix}. \tag{27}$$

The achieved parameter Cramér-Rao bounds are consistently lower than the values determined for the same model in previous studies such as [5, 11]. This is mostly due to the higher sampling rate in this study and the fact that only three (main) model parameters have been selected for parameter estimation and therefore also optimal input design. The results show that the proposed method in this study achieves a similar accuracy in the parameter estimates to methods that assume a more accurate initial guess for designing the optimal inputs.

Table 1. Parameter values and optimization results for the linear short period example.

Parameter	True parameter value	Initial parameter value	Final parameter value	Cramér-Rao bounds
Z_α	−0.737	−0.737	−0.737	–
M_α	−0.562	−0.112	−0.543	0.0244
M_q	−1.588	−0.318	−1.621	0.0335
Z_η	0.005	0.005	0.005	–
M_η	−1.66	−2.324	−1.687	0.0194

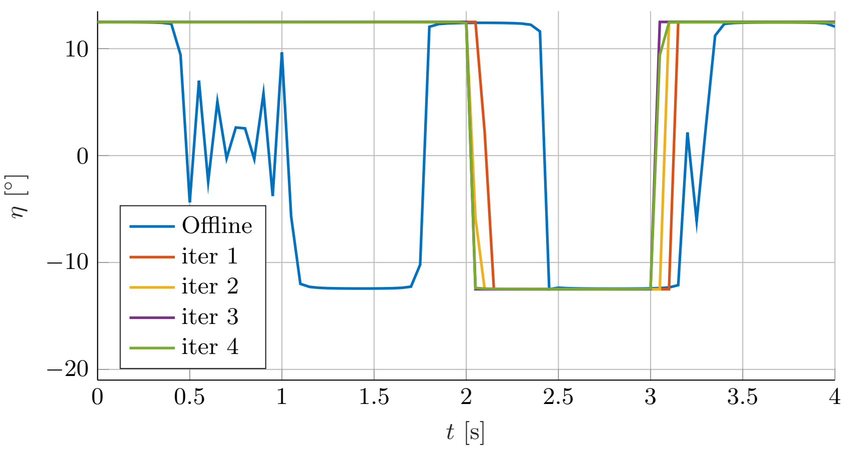


Fig. 2. Optimal elevator deflection.

4.1 Parameter Estimation

Two parameter estimation experiments are performed to investigate the improvement in the accuracy of the parameter estimates if the optimal inputs are adjusted during the experiment run-time, as suggested in this study. The parameter estimation experiments are performed using a special implementation of the Maximum Likelihood estimation method in the output error form as described in [3]. The same initial guesses are set for parameters as in Table 1. The results of the parameter estimation experiment are provided in Table 2. It can be seen that the optimal input adapted during the runtime of the experiment results in a significant improvement in parameter accuracy. Figure 5 visualizes the absolute value of the correlation matrices after each of the parameter estimation experiments. The correlation between the parameters M_q and M_η is higher when the adjusted inputs after optimization are in use. This may be contributed by the fact that only the diagonal elements of the dispersion matrix $\boldsymbol{D}$ are explicitly considered in the cost function.

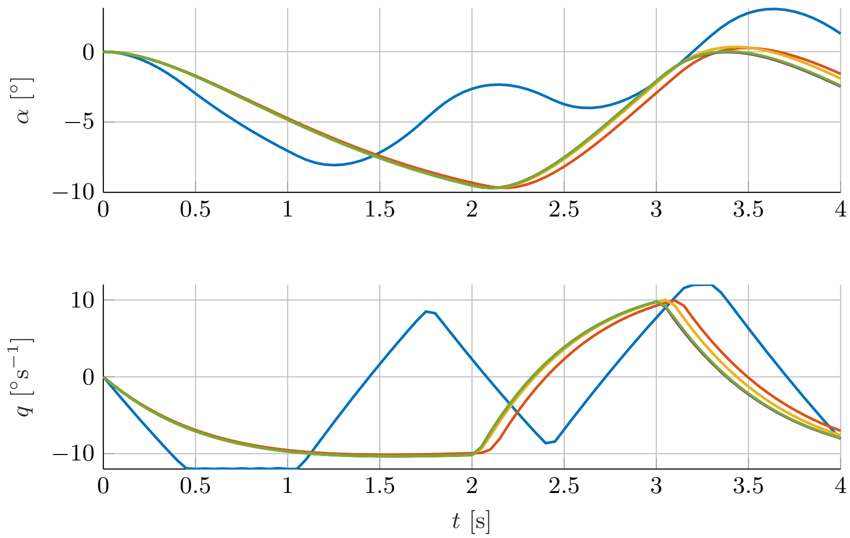


Fig. 3. Response of the linear short period model to the optimal inputs with the available set of parameters at each iteration.

Table 2. Estimated parameter values and their standard deviations (Std Devs) for the linear short period example.

Parameter	Estimated parameter with the pre-flight optimal input	Parameter std devs with the pre-flight optimal input	Estimated parameter with the adapted input	Parameter std devs with the adapted input
M_α (−0.562)	−0.61	0.0947	−0.562	0.0255
M_q (−1.588)	−1.585	0.0585	−1.603	0.0347
M_η (−1.66)	−1.688	0.0273	−1.668	0.02

5 Conclusion and Outlook

In this study we show that adaption of optimal inputs for parameter estimation during the experiment run-time significantly improves the results of a subsequent parameter estimation. However, this is a very complex process and can be difficult to successfully implement and apply to a real aircraft. First and foremost, the application of the proposed method in real-world systems is limited by the existence of process noise and deficiencies regarding the model structure, such as unmodeled nonlinearities. Among the implementation aspects, the low computing power and memory capabilities of common flight control computers can be considered challenges associated with this approach.

Furthermore, since the parameter values used to design the optimal inputs are not accurate, state bounds are not guaranteed to be satisfied during the flight.

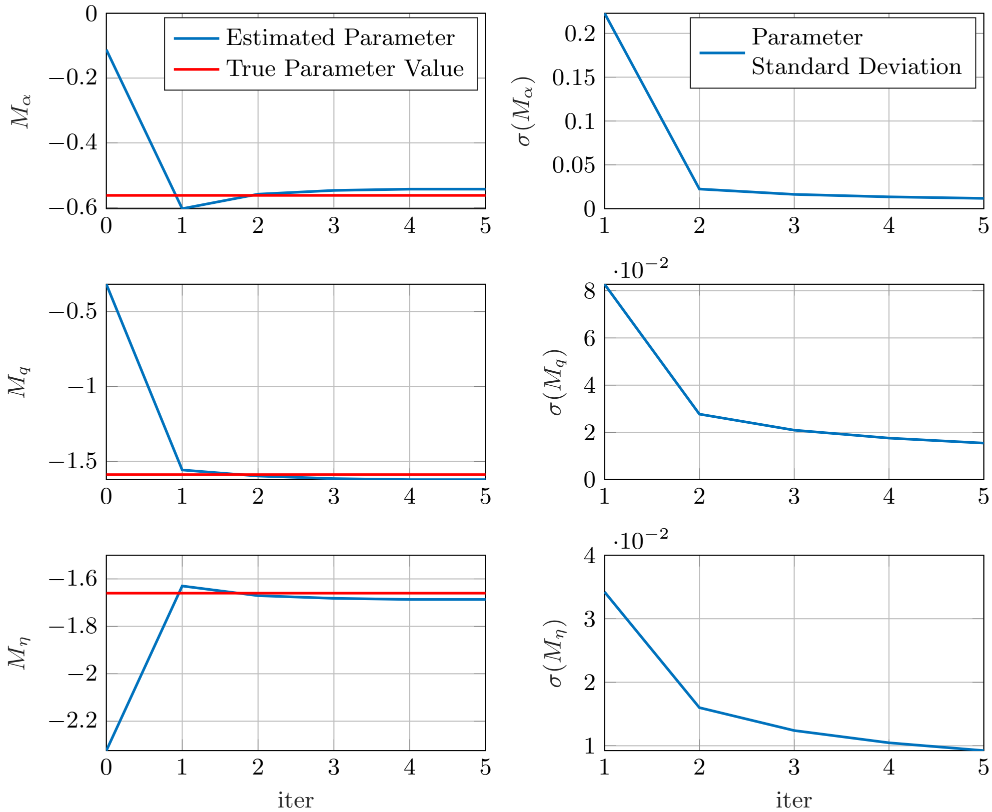
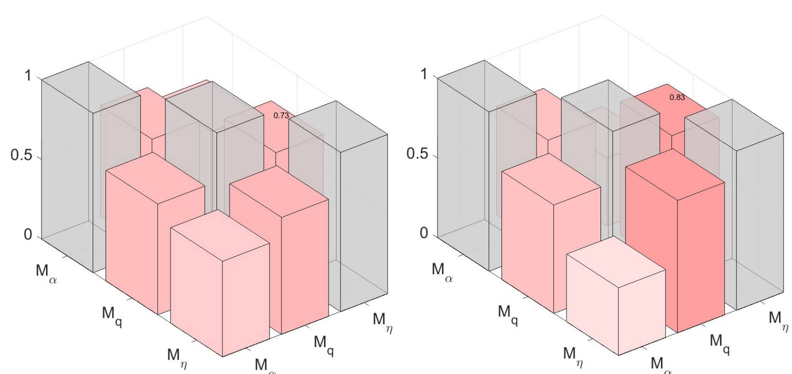


Fig. 4. Parameter values and their standard deviations at each iteration.



(a) Offline Optimization **(b)** Onboard Optimization

Fig. 5. Absolute value of the correlation matrices.

To address this issue, more conservative state bounds can be considered for the first maneuvers and the bounds can be expanded as more accurate parameter estimates become available.

The additional efforts required to implement this method can however be advantageous in use cases where flight test time is very expensive and no good initial guess for the system model exists. In the case of linear system identification, changing the trim point of the aircraft during the flight also changes the values of the linear system's matrix coefficients. This method can be applied to adapt the optimal inputs of the aircraft to the new trim point during the flight.

It might be possible, especially in the case of nonlinear systems, that significantly different parameter values result in significantly different maneuvers and therefore cause convergence problems in the short time between two maneuvers. This can be solved by updating the maneuvers online, as new parameter values become available. Online methods for optimal input design are currently being investigated by authors. Furthermore, it is planned to test the algorithm and its capabilities in a real-world experiment using an unmanned aircraft. Utilization of other online parameter estimation algorithms (e.g. the frequency domain method as discussed in [10]) will also be investigated in future work.

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