# Mathematical Modeling of Steady Flow Distribution in Water Supply Networks with Pumping Stations and Regulating Capacitances

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**Abstract**

The article presents a mathematical model of the water supply system together with pumping stations and regulating capacitances. This mathematical model is necessary to analyze the quality of functioning of water supply systems when implementing control actions at pumping stations, to evaluate the effectiveness of solving the problem of operational planning of operating modes of water supply systems at a given control interval, as well as to verify the correctness of decisions made on the control of technological processes of water supply and distribution. The mathematical model provides the possibility of parallel connection of an arbitrary number of pumping station units without the need for preliminary equivalenting of their characteristics. Also, the developed mathematical model of the water supply system, together with pumping stations and regulating capacitances, is extremely important in simulation modeling of operational control systems for its operation modes. Researchs was carried out using the method of imitation modeling of water supply systems functioning of many major cities.

**Keywords 1**

mathematical modeling, water supply system, functioning, steady flow distribution, pumping station, regulating capacitance, control.



## 1. Introduction

The purpose of this article is to develop a mathematical model of the water supply system together with pumping stations and regulating capacitances. In the control of water supply systems (WSS) and simulation modeling of operational control systems for their operation modes, an important role is played by the task of analyzing the steady distribution of flows in networks. To solve this problem, it is necessary to have a mathematical model of the water supply system, including pumping stations (PS) operating for it and regulating capacitances, which would take into account the possibility of parallel connection of a different number of pumping station units without the need of preliminary equivalenting of their characteristics. Such a model is necessary to analyze the quality of functioning of water supply systems when implementing control actions at pumping stations, to evaluate the effectiveness of solving the problem of operational planning of operating modes of water supply systems at a given control interval, as well as for verify the correctness of decisions made on the control of water supply and distribution technological processes. Also, the mathematical model of the water supply system, together with pumping stations and regulating capacitances, is extremely important in simulation modeling of operational control systems for its operation modes.

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## 2. Analysis of Literature Data and Statement of the Problem

Many fundamental scientific works of Abramov N.N., Evdokimov A.G., Tevyashev A.D., Hasilev V.Y., Merenkov V.P., Novitsky N.N. and other scientists [3] in the 70s of the last century are devoted to the development of mathematical models of water supply networks. Basically, they were considered to solve the problem of hydraulic calculation of their operating modes, which is very important in the design of water supply networks.

The purpose of this task was to calculate the values of flow rates in all arcs of the water supply network graph and pressures in all its nodes with a certain combination of initial data that would provide pressures in the nodes with consumers connected to them not lower than some given minimum allowable value determined by the height of buildings and geodetic relief terrain.

In the works of Evdokimov A.G. [1, 2] also considered the problem inverse to this one - the problem of analyzing the steady flow distribution in the network, i.e. determining the values of flow rates in all arcs of the water supply network graph and pressures in all its nodes for fixed values of flow rates and pressures at the network inlets, i.e. at the outlets of pumping stations. As a result of solving this problem, the water in the network seemed to be distributed by gravity, and in the nodes with consumers connected to them, the pressure could be either above the minimum allowable value or below it. That is, in the second case, some or all consumers connected to such nodes received less water (on the upper floors of houses or high-lying areas), or did not receive it at all.

This task is extremely important in the operation of water supply networks, for dispatchers, i.e. persons making decisions on the control of the technological process of water supply. For dispatchers of water supply systems this were not very comfortable and did not manage to quickly provide the necessary values of flow rates and pressures at the outlets of all pumping stations, i.e. at the inlets of the water supply networks. Talking with the chief dispatchers and directors of the water supply companies of many large cities, it became clear their desire to see a complete picture of the constantly changing situations in the city's water supply network. And for this, it is necessary to know every time how the water flows will be distributed, and how consumers will be provided in the network, with various combinations of switched on pumping units, with known water levels in the reservoirs of pumping stations, and with smooth regulation of the supply of pumping units using asynchronous valve cascades.

Therefore, a mathematical model of the water supply system was developed together with pumping stations and regulating capacitances. This mathematical model provides the possibility of parallel connection of an arbitrary number of pumping station units without the need for preliminary equivalenting of their characteristics. Such a mathematical model is necessary to analyze the quality of functioning of water supply systems when implementing control actions at pumping stations, to evaluate the effectiveness of solving the problem of operational planning of operating modes of water supply systems at a given control interval, as well as to verify the correctness of decisions made on the control of technological processes of supply and distribution of water.

Also, the developed mathematical model of the water supply system, together with pumping stations and regulating capacitances, is extremely important in simulation modeling of operational control systems for its operation modes.

This mathematical model is presented in this article.

## 3. Research results

Let $G(V,E)$ is a graph of the WSS that models its structure and displays the relationships between individual elements. Here V is the set of all WSS nodes; E is the set of all arcs of the WSS. Let's connect all the inputs and outputs of the graph $G(V,E)$, through which water enters the network and is taken from it, respectively, with a zero fictitious point.

Let L be the set of pumping stations and regulating capacitances, then the elements of this set will be fictitious arcs connecting the zero point with the inputs of all PS and regulating capacitances; M is the set of passive elements, i.e. main sections (pipelines) of the water supply network, which are real arcs of the graph; N is the set of WSS nodes with consumers connected to them, i.e. the set of fictitious arcs of the WSS model. At the same time, the set of nodes of the WSS graph consists of two non-intersecting subsets $V = N \bigcup N'$, where $N'$ is a subset of intermediate nodes of the WSS model, i.e. such nodes in which there is no water intake, however they need to be reflected in the WSS model. In addition, we introduce a set $K = \bigcup_{j \in L} L_j$ which characterizing the total number of arcs with pumping units at all pumping stations of the WSS. Thus, the elements of the set are the links of all pumping stations, each of which directly includes the active element itself (pump), as well as sections adjacent to it with adjustable and unregulated valves. At the same time, as is known [3], each link of the pumping station corresponds to an equation that describes the process of water movement through this pumping unit:

$$\begin{gathered} H_{IN} - r_{i1} q_i^2 + \left(\Psi_{0i} + \Psi_{1i} q_i + \Psi_{2i} q_i^2\right)\left(D_i / D_i'\right)^2 \left(n_i / n_i'\right)^2 - \\ - r_{i3}(\lambda_i) q_i^2 - H_{OUT} = 0, \quad i \in L_j. \end{gathered} \tag{1}$$

Here $H_{IN}, H_{OUT}$ is the pressure at the inlet and outlet of the PS, respectively; $q_i$ - water flow through the i-th link; $\Psi_{0i}, \Psi_{1i}, \Psi_{2i}$ - coefficients of approximation of the load characteristics $H(q)$ of the i-th pumping unit; $r_{i1}, r_{i3}$ - resistance of sections with unregulated and adjustable gate valves located in the suction and pressure lines of the i-th pump, respectively; $\lambda_i$ - the degree of opening of the i-th adjustable gate valve; $D_i, D_i'$ - respectively, the normal and cut impeller diameter of the i-th pump; $n_i, n_i'$ - respectively, the actual and nominal speed of the impeller of the i-th pumping unit (if it has an adjustable drive).

For the unambiguity of the direction of the flow of water pumped by the i-th switched on WSS unit, we represent (1) in a slightly different form:

$$\begin{gathered} H_{IN} - r_{i1} q_i |q_i| + \left(\Psi_{0i} + \Psi_{1i} |q_i| + \Psi_{2i} q_i |q_i|\right)\left(D_i / D_i'\right)^2 \left(n_i / n_i'\right)^2 - \\ - r_{i3}(\lambda_i) q_i |q_i| - H_{OUT} = 0, \quad i \in L_j. \end{gathered} \tag{2}$$

The pressure loss for an active element, whether it is a PS or a separately operating pumping unit, is always a negative value, so in this case we can talk about the “acquisition” of pressure.

Let's choose a tree of the WSS graph in such a way that it includes the magistral sections of the network and sections with pumps (belonging to different PS of the WSS), as well as one fictitious branch connecting the zero point with the input of some of the PS.

Let us assign number 1 to it. In this case, all fictitious sections incident to the WSS nodes, which are the inputs of the PS (except for the first one) and regulating capacitances, as well as sections with consumers, will be assigned to chords, while the magistral sections and sections with pumps will partially become chords, and partially - the branches of a tree. We believe that index 1, assigned by sets, respectively $L, M, N, K, E$, characterizes the belonging of their elements to the branches of the tree, and index 2, to chords. As a result of such a choice, the set of all arcs of the WSS graph can be represented as $E = E_1 \bigcup E_2$, where $E_1 = M_1 \bigcup K_1 \bigcup L_1$, $E_2 = M_2 \bigcup K_2 \bigcup L_2 \bigcup N_2$, $L_1 = \{1\}$, $N_1 = \varnothing$, $N_2 = N$.

The set $L_2$ is divided into two non-intersecting subsets $L_2 = L_2^{(a)} \bigcup L_2^{(p)}$, where $L_2^{(a)}$ is the set of chords with active elements (PS); $L_2^{(p)}$ - a set of chords with regulating capacitances (water towers, columns, reservoirs, which are passive-active control elements of the network).

In addition, we denote $q_i$ - the water flow in the i-th section of the network, $i \in M$; $r_i$ - the hydraulic resistance of the i-th section of the network, $i \in M$; $h_i^{(r)}$ - the difference in geodetic marks of the beginning and end of the i-th section of the network,; $h_i$ - pressure loss in the i-th section of the network, $i \in M$.

Taking into account the choice of the tree of the graph of the WSS, as well as the fact that the sum of the differences in geodetic heights for any closed cycle containing the backbone sections of the network is equal to zero, i.e.

$$h_i^{(r)} + \sum_{r \in M_1} b_{1ri} h_r^{(r)} = 0, \quad i \in M_2, \tag{3}$$

the mathematical model of the steady flow distribution in the water supply network, together with active sources and regulating capacitances, will take the form

$$f_r = \operatorname{sign} q_r r_r |q_r|^2 + \sum_{i \in M_1} b_{1ri} \operatorname{sign} q_i r_i |q_i|^2 = 0, \quad r \in M_2 ; \tag{4}$$

$$f_r = H_{IN1} + \Psi_{0k} + \Psi_{1k}|q_k| + \Psi'_{2k} q_k |q_k| - h_r - \sum_{i \in M_1} b_{1ri}\left(\operatorname{sign} q_i r_i |q_i|^2 + h_i^{(r)}\right) = 0, \quad r \in N, \quad k \in K_1; \tag{5}$$

$$f_k = \Psi_{0k} + \Psi_{1k}|q_k| + \Psi'_{2k} q_k |q_k| + x_i\left(\Psi_{0i} + \Psi_{1i}|q_i| + \Psi'_{2i} q_i |q_i|\right) = 0, \quad k \in K_2, \quad i \in K_1; \tag{6}$$

$$f_r = \operatorname{sign} q_r r_r |q_r|^2 + h_r^{(r)} + h_r^{(p)} + \sum_{i \in M_1} b_{1ri}\left(\operatorname{sign} q_i r_i |q_i|^2 + h_i^{(r)}\right) + \Psi_{0k} + \Psi_{1k}|q_k| + \Psi'_{2k} q_k |q_k| = 0, \quad r \in L_2^{(p)}, \quad k \in K_1; \tag{7}$$

$$f_r = H_{IN1} - H_{INr} + \sum_{k \in K_1}\left[\operatorname{sign} q_k \left(\Psi_{0k} + \Psi_{1k}|q_k| + \Psi'_{2k} q_k |q_k|\right)\right] - \sum_{i \in M_1} b_{1ri}\left(\operatorname{sign} q_i r_i |q_i|^2 + h_i^{(r)}\right) = 0, \quad r \in L_2^{(a)}; \tag{8}$$

$$q_i = \sum_{r \in M_2} b_{1ri} q_r + \sum_{k \in K_2} x_k q_k + Q_i^+, \quad i \in M_1 \cup L_1 \cup K_1, \tag{9}$$

where $\Psi'_{2k} = \Psi_{2k}\left(D_k / D'_k\right)^2 \left(n_k / n'_k\right)^2 - r_{k1} - r_{k3}(\lambda_k), \quad k \in K_1 \cup K_2,$

$$x_i = \begin{cases} 1, \text{ if the i-th pump is on,} \\ 0, \text{ if the i-th pump is off, } \; i \in K_1 \cup K_1; \end{cases}$$

$$Q_i^+ = \sum_{k \in N \cup L_2} b_{1ki} q_k = const;$$

$H_{IN1}$, $H_{INk}$ are the inlet pressure of the 1st and k-th PS, respectively;

$b_{1ri}$ is an element of the cyclomatic matrix $B_1$ [1-2].

The value marked with the index "+" is considered to be set.

In the above mathematical model of the WSS, it is assumed that the fictitious sections with consumers are directed from the network to the zero fictitious point, and the sections with pumps and the fictitious sections corresponding to them are vice versa.

Let's analyze the conditions for the solvability of the system of equations of the mathematical model of the water supply network together with active sources and regulating capacitances. It can be solved if the boundary conditions for the functioning of the WSS are given in the form of a combination of values of variable flow rates and pressures at its inlets and outlets.

## 4. Conclusions

A mathematical model of the steady flow distribution in water supply systems containing pumping stations and regulating capacitances is necessary to analyze the quality of functioning of water supply systems when implementing control actions at pumping stations [2; 8; 9], to evaluate the effectiveness of solving the problem of operational planning of operating modes of water supply systems at a given control interval [2; 4; 5, 6; 7], as well as for verify the correctness of decisions made on the control of water supply and distribution technological processes [2; 4; 5, 6; 7].

A simulation model of the technological processes of water supply systems functioning together with pumping stations and regulating capacitances on any given time interval has been developed.

It is based on the solution of the problem of analyzing the steady flow distribution in the water supply network together with pumping stations and control tanks, which is determined by solving the system of equations [1, 2, 6, 7] of the corresponding mathematical model when setting a combination of flow rates and pressures as boundary conditions at the inlets and outlets of the water supply systems.

Researchs was carried out using the method of imitation modeling of water supply systems functioning of many major cities. Currently, all software is implemented in C++. In some cases, in addition to own software, individual EPANET functions can be used [10]. The developed databases were implemented with MS Access DBMS.

The use and widespread adoption of information technologies of optimal control operation of WSS allows in practice to improve the quality and effectiveness of their functioning by reducing the excess pressure in the networks (and, consequently, reducing unproductive costs of water), reducing electricity costs, reducing the probability of occurrence of emergency situations in networks.

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