Towards Greener Logistics: a Multi-Body Simulation to Test Pallet Stability *

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Abstract
Pallets are critical components in the logistics of food and beverage products transportation, and their stability is essential to ensure reliability of the handling system as well as safety during road and rail freight. This work aims to develop an innovative solution for evaluating pallet stability very early, ideally during the design phase of the pallet schema and the wrapping format. Differently from other investigations, the goal of our work is to analyse the dynamics of pallets wrapped in an envelope of paper material instead of plastic. By collecting raw video data from an acceleration test bench, and using computer vision and machine learning techniques, we develop a physically realistic multi-body simulation. The simulation is completely virtual and capable to evaluate the stability of the pallet under different configurations and loading conditions.

Keywords
Multi-body simulation, Machine learning, Industrial application of AI, Paper wrapping

1. Introduction

Nowadays, the use of elasto-plastic materials, like the LLDPE (Linear Low Density Polyethylene) stretch film, or the heat-shrink wrap, is still the favorite choice in the automatic wrapping of pallets stacked with food or beverage products. Such a *wrapping* is called tertiary packaging, a term which refers to the external envelope that surrounds both a wooden pallet and the products on it. The single product's actual container and their grouping into a larger set for handling purposes are called, respectively, primary and secondary packaging.

Despite institutional initiatives aimed at reducing pollution and plastic usage, like the European Strategy for Plastics in a Circular Economy [1], the current state of affairs in food and beverages industry made plastics the only feasible choice for tertiary packaging [2]. This is due to the unavailability of equipment and machinery that works with other, more sustainable, materials, as well as the lack of knowledge about how these materials behave in such situations. In fact, an ideal wrapping equipment should match the high standards of modern end-of-line packaging machines, at the same time being able to use other materials than plastic, and to maintain both the high speed of those production chains and the adequate safety and stability standards of the final load unit. Moreover, plastic materials as LLDPE have also side advantages like resistance to water and UV-rays, and their elasticity permits them to hold up well to considerable tensions. For these reasons, there have not been attempts in producing wrapping machines that work with more sustainable materials so far.

ACMI S.p.A.[1] is an Italian manufacturer of high-tech bottling and packaging lines, specialized for beverages and food, which has international relevance. The innovative contribution of ACMI to the transition from plastic to paper is the proposal of a novel prototype of wrapping machine that works with a recyclable and biodegradable paper with specific elastic strength: the Mondi's Advantage StretchWrap Kraft paper[2]. Nevertheless, to put in production such a machinery, there are a series of engineering and automation challenges to be explored.

One of the most important among these challenges is to ensure the safety of the handling and transport systems, with an envelope wrapped with paper instead of plastic. As a matter of fact, pallets are critical components in the logistics of food and beverage products transportation, and their stability during road or rail freight is essential for the reliability of the handling system, in order to

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[1]https://www.acmispa.it/en/
[2]https://www.mondigroup.com/en/products-and-solutions/speciality-kraft-paper/speciality-kraft-paper-products/advantage-stretchwrap/

avoid loss of products, and for the safety of truck workers. Rules to certify an adequate safety during transports are encoded in the European Road Worthiness Directive, and in particular in the European Standard EUMOS 40509 [3]. The stability analysis requires a deep understanding of the wrapping material behavior in different conditions. For example, it is crucial to realize how many layers of paper are needed for wrapping and how they should be stratified, and how much pulling tension has to be applied to paper while wrapping. Also, the design of the pallet loading schema considerably impacts on the dynamic of the system, therefore the analysis must also take into account that information. Finally, acquiring knowledge about the dynamic of a paper-wrapped envelope is of key importance to provide engineers hints for the actual development of the automatic wrapping machine and its controlling software.

Our work in collaboration with ACMI S.p.A. is aimed at searching methods to perform such a stability analysis on pallets wrapped with Kraft paper. The idea is to investigate pallet stability by means of a multi-body dynamics simulation, capable to virtually reproduce the behavior of the load unit when subject to external forces. This paper describes the set up of such a virtual simulation, starting with the collection of real-world data from the observation of actual physical tests. Such data are analysed with computer vision techniques. In order to setup the physics simulation, we need to learn parameters that controls the dynamics: in particular, the retrieval of (i) static and dynamic friction forces at work between each bundle of the test and (ii) elastic coefficient that model tensions at each point of a mesh representing the wrapping envelope. We design the resolution of two inverse problems, one for each kind of parameters.

2. Background

The European Road Worthiness Directive is the European norm aimed at assuring the security during rail or road freight. In particular, the European Standard EUMOS 40509 [3] is devoted to quantify the rigidity of the pallet when it is subject to a force (due to an acceleration) along a direction. Its goal is to investigate the motion dynamic of a truck loaded with one or more EUR pallets. The stability of the load unit, in fact, depends on the rigidity of the load, i.e., how much the load is sensitive to permanent deformations or excessive shifting [4], and on the holding strength of the external envelope, which could be deformed or ripped during motion. The Acceleration Bench Test of EUMOS 40509 defines a physical test setup and some test acceptance criteria. Such a test consists in subjecting the load unit to an acceleration impulse that immediately stops and gives rise to a constant deceleration, until the unit stops. Typical tests are performed

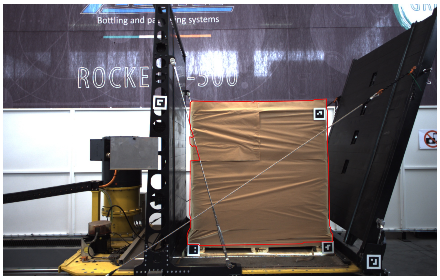

Figure 1: The ESTL Machine in the R&D Department of ACMI S.p.A. The acceleration bench holds on a sleight where a wooden pallet is loaded with some layers of products, and wrapped with Kraft paper.

with constant accelerations from $0.2g$ up to $0.5g$, which is the acceleration to be supported. The acceleration may cause *elastic* deformations and *permanent* deformations, which are, respectively, the deformation of the load unit during the test, and the residual deformations of the load unit after the test ended.

EUMOS 40509 quantifies thresholds and limits of these deformations and shifting, beyond which the unit could impact severely on the stability of the overall truck, thus making unsafe the transport. The specifications of EUMOS 40509 state that *(i)* the permanent displacement of all parts of the test load unit (after the test) must not exceed 5% of the total height of load unit; *(ii)* permanent displacement in the lowest $20cm$ of the test load unit is to be less than $4cm$ on the wooden pallet; *(iii)* the elastic displacement of all parts of the test load unit (during the test) must not exceed 10% of the total height of load unit; *(iv)* there must be no visible structural damage and/or leakage of products at the end of the test.

Such a physical test could be made by actually loading a truck with the load units to be tested, and driving that truck in a safe environment. The alternative, adopted by us and ACMI S.p.A., is to use a special testing machinery, produced by ESTL Company [5] and shown in Figure 1, which consists in a movable platform, the sleight, on which is carried the pallet, and an engine that can generate a constant and controlled horizontal acceleration impulse over such a platform. The acceleration can be set between $0m/s^2$ and $10m/s^2$ in steps of $0.5m/s^2$. The duration of the acceleration is at least $500ms$. Usually the pallets are tested at different acceleration levels: tests start at a low acceleration level of $0.2g$ or $0.3g$ (about $1.962m/s^s$ and $2.943m/s^2$, respectively). Then, if the result is successful (w.r.t. EUMOS 40509), the constant acceleration impulse is increased by a value of $0.1g$, heading for the legal requirement of $0.5g$ for load safety. These parameters permit to simulate road transport events such as diverting maneuvers and/or emergency stops. Such a machinery also records high-quality video of the motion, providing summarized information about elastic and permanent deformations. Three markers are attached to the load unit and two markers on the sleight, as in Figure 1, so that the ESTL Machine vision system can detect fluctuations of the pallet. Acceleration and displacement of the sleight is known at each time instant thanks to an X-Y accelerometer, and the acceleration profile data are recorded and plotted as well.

The ESTL machine method is surely advantageous w.r.t. the road test in terms of both fuel saving and safety. Also, recordings from high-speed cameras and data from the accelerometer allow to precisely identify when EUMOS 40509 conditions are matched. However, the very high energy consumption of each ESTL machine test makes it not convenient to perform extensive experimentation. Therefore the necessity of providing a virtual—simulated—testing bench at the core of our work.

3. Our Proposal

Given the problem of pallet stability, the standard EUMOS 40509, and the ESTL machine as testing bench, our work aims at developing an intelligent and automatic recommendation system that is able to suggest the safer, robust, and most reliable wrapping format to the ACMI paper wrapping machine. To do so, however, it is crucial to gain a thorough knowledge about the dynamics of the acceleration test bench. This paper illustrates the first steps taken in such a direction, i.e., the collection of raw data from the ESTL machine and the usage of such data to virtually reproduce the dynamics of any physical setup. The proposed methodology is AI-based and combines computer vision and machine learning techniques to model a realistic multi-body simulation.

The developing of such a simulation starts from the acquisition by the ESTL machine of raw video recordings of the load unit subjected to horizontal accelerations. Then, a low-level vision system is devoted to extract the centers of gravity of each visible bundle (the wooden pallet and the secondary packages on it) in those videos, and their possible rotations. With the aid of a multi-body physics engine, the base building blocks of the simulation could be modeled. At the beginning of this phase, however, the simulation cannot be realistic, due to the lack of knowledge about crucial physical parameters such as static and dynamic friction forces at work. Using measurements retrieved by the vision system, and centers of gravity of each body of the simulation, those parameters are learned.

3.1. The Vision System

The problems of object detection and tracking of their position are well-known tasks in the field of computer vision, for which there are plenty of AI approaches. Most of the recent literature focuses on deep learning techniques, given the fact that novel neural architectures and learning algorithms outperform standard vision methods in mainstream (i.e. standard) settings, such as the recognition of common objects [6, 7]. Unfortunately, except for some notable examples [8], deep learning methods usually require a huge amount of homogeneous data, that have to be carefully annotated in case of supervised learning. In our case, the object to be recognized are not standard ones, and even if we could fine-tune some pretrained networks, mainstream datasets on which those networks are trained are too general, and making efforts to switch from their to our domain could result in a global lowering of the network performances. Moreover, our raw data are produced by physical experiment with the acceleration testing machine, and each experiment has a high cost in terms of time and power consumption, failing the requirement of a vast amount of data to be available for network training and testing. Hence, our system was crafted for the specific task, with the aid of standard computer vision algorithms and techniques.

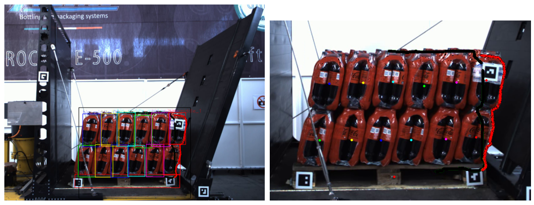

Figure 2: An example result of the tracking phase of the vision system, searching for bundles in a two layer non-wrapped configuration. Then, optical flow is used to extract centers of gravity of pallet and packages.

The experimental settings regard the selection of few influential tests to be physically performed with the acceleration testing machine, their analysis with the computer vision system and the modeling of the relative virtual multi-body simulation. We take two different kind of experiments, with or without the external envelope. We choose bundles of six Coca-Cola Zero™ and maintained the same product for all the experiments. Different products would have different shapes and dynamics, thus making it impossible to compare experiments among each other. The absence of wrapping, in this early phase, allows us to analyse more accurately the behavior of each package above the pallet. Then, tests with the same configurations but wrapped with paper and/or plastic are also taken, to compare them with unwrapped configurations. Recordings of experiments last 8–10 seconds, at a rate of 20 frames per second, with 1920×1200 frames size.

The computer vision system we developed consists of a program which processes raw videos frame by frame.

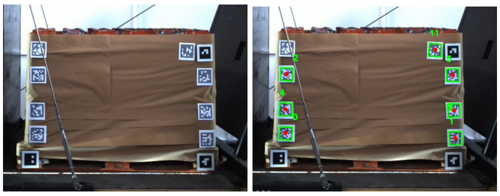

Figure 3: Use of ArUco markers to detect the movement of layers.

The system was developed in Python 3.8.12, using the Python versions of OpenCV [9] open-source library. For the first type of tests, i.e. without wrapping, the goal is to retrieve the centers of gravity of each package and the pallet. The program thus identifies a Region of Interest (ROI), which is automatically detected based on the acceleration profile data of the physical test, from which the position of the load unit in the video is predicted. Then, normalized cross-correlation is used to perform a multiple matching of a template (an example of a package to be recognized) inside the ROI, and to prevent the explosion of computational times, a Non-Maximum Suppression (NMS) algorithm [10] follows the template matching. We used several state-of-the-art methods for multi-object tracking [11, 12, 13, 14, 15, 16, 17] to maintain the recognition of the template along the frames. An example is shown in Figure 2 on the left. Finally, an optical flow detection methods [18] is employed to measure the actual displacements and rotations of packages. For each bundle and the wooden pallet, an approximation of the center of gravity is computed, by taking a weighted mean of all displacements centered in the center of the bounding box of the tracked object. Figure 2 on the right shows an example of results, where colored dots are the computed centers of gravity of bundles and the wooden pallet.

For the second type of tests, i.e. with wrapping, the goal is to retrieve the movement of each layer of packages separately. During the setup of the experiments, we put ArUco markers over the paper wrapping, at the corners of each layer. We also performed tests with plastic wrapping, to compare the two materials. In such a case, ArUco markers were posed inside the wrapping, on each bundle. Using ArUco detection [19], approximated positions of layer were extracted, as in Figure 3.

3.2. The Simulation

Simulations like the one we developed are called multi-(rigid)-body dynamics simulations. There are both free and commercial softwares capable of reproducing rigid bodies motion, such as AutoDesk AutoCAD [20], MathWorks Simscape Multibody [21], NVIDIA PhysX [22], and so on. Nevertheless, our system needs also a non-rigid part to model the external envelope and the engine must be flexible enough to shape the parameters of such an envelope. Kraft paper, and in general, paper dynamics are still an open challenge for those types of engines. For these reasons, we choose a multi-physics engine that is also able to perform Finite Element Analysis (FEA) and model smooth collisions. Such an engine is called Project Chrono [23, 24], developed by University of Wisconsin (Madison) and University of Parma.

Each body can be a simple shape (e.g. a box, a sphere) and/or a user defined 3D model. Each body has a center of gravity, a mass, a moment of inertia and a collision model. Masses of pallets and bundles are easily obtainable from real measurements. Initial centers of gravity of objects depend on their shape and their initial position in the simulation. We choose to approximate bundles with boxes, so the center of gravity could be easily calculated. A flexible material developed using FEA consists of a set of nodes that composes a mesh, and elements that connect the nodes among each others. On each node could be applied a resulting force, and a link to other elements of the simulation. Then, a linear motor engine is initialized to model the sleight. Chrono has a facility to create functions for vertical and/or horizontal motion, and in our case a constant x acceleration could be modeled by imposing the ramp length and height (of the speed function), the ending time of the acceleration and the starting time of the deceleration. All such parameters could be easily obtained from the acceleration profile provided by the ESTL machine.

The ESTL machine, the wooden pallet, and each of the bundles are composed of different materials. For each object/material a value of static friction and a value of kinetic friction must be set. From previously analysed videos, it is easy to see that the motion of the load unit is delayed compared to the motion of the sleight, due to the friction force between those two rigid body, depending on their material. So, we used the sleight displacement as a reference and computer the difference between that and the displacement of the load unit. The same was done for the relative displacements of the pallet w.r.t. the packages of product of the first layer, then the first w.r.t. the second layer and so on. Such differences are strongly related to friction coefficients (static and dynamic) of the pallet over the sleight, of each bundle over the pallet, and of bundles with each other. The displacements are measured using the extracted data (centers of gravity) $p_i(t) = (x_{p_i}^{(t)}, y_{p_i}^{(t)})$ of each relevant object i visible in the video recordings of ESTL machine at time t. We choose to use the centers of gravity retrieved by Boosting tracker [13], that achieves the better accuracy in the previous phase. The *predicted* centers of gravity are the computed positions $c_j(t) = (x_{c_j}^{(t)}, y_{c_j}^{(t)}, z_{c_j}^{(t)})$ of all the objects in the simulation at time t. To made such predicted coordinates to match with video data, we run the simulation with the

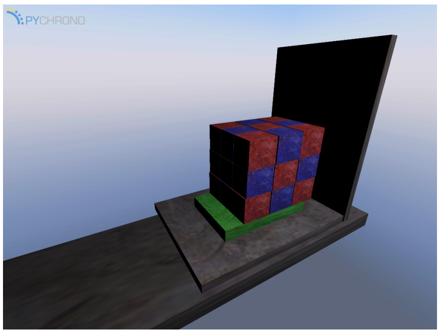

Figure 4: A frame of a simulation with a columnar schema and three layers, without wrapping envelope.

exact same configuration of the previous experiments, and measure with the aid of Chrono all calculated relative displacements. Unfortunately, only the visible bundles, those in front of the camera, could be compared to simulated data. Thus, we consider only $c_i(t) = (x_{c_i}^{(t)}, y_{c_i}^{(t)})$, as if the bundle maintains a constant position on the z axis. The objective is to minimize the distance between real position and simulated positions, for each time instant t, with a L_2 loss. We applied a gradient descent algorithm with L_2 cost function to find static friction μ_s and kinetic friction μ_k for the pallet and each package. However, input data frames are few (~ 50 positions for each bundle) and also very noisy due to previous calculations (matching, tracking, optical flow detection). A statistical method to denoise data is Exponential Moving Average (EMA), that defines a novel sequence from raw data depending on the value of a parameter $\beta \in (0, 1)$. Larger (close to 1) values of β produce smoother sequences. The machine learning method is a variation of the gradient descent algorithm, which uses EMA on gradient sequence. In deep learning field, such a method is known as gradient descent with momentum [25].

The simulated wrapping envelope is a Chrono's FEA object modeled as a closed mesh around the whole load unit. Given the standard dimensions of an EUR pallet, i.e. 800×1200 mm, nodes are generated with a ratio of $4/6$ around the perimeter of the box shape of the pallet. In fact, the lower round of nodes is anchored to the wooden pallet, while the upper ones are shrinked (in negative or positive, depending on the dimensions of the layers of product) around each layer. Project Chrono provides several materials to model elements between FEA nodes, and we choose the ElastoPlastic one to deal with the behavior of a well-known material during the fitting phase. The actual envelope produced by ACMI's wrapping machine has a different number of layers at different heights, i.e. the material is stratified to be more resistant at critical points. Such a stratification is modeled by increasing the pulling tension of nodes. A standard wrapping format divides into three phases: the lower layer phase, where many stratification occur to attach the products to the wooden pallet; the central layers phase, which usually requires a single passage; and the upper layer phase, when some overlaps are made to close the envelope. Therefore, we choose to highly increase the pulling tension of lower nodes and slightly increase the pulling tension of upper nodes, w.r.t. the central ones. At the current stage of development, it is not possible to know the real value of the tensions working on the paper envelope, to compare them with those applied to each point of the mesh and computed by Chrono. In particular, the forces we want to retrieve are those acting at each corner of the wrapping. These forces have to be obtained by observation, as centers of gravity in tests without wrapping. Similarly, we look at the displacements between layers in wrapping experiments, where we put ArUco markers. We plan to use the positions of selected key points of FEA mesh as input of a gradient descent algorithm with momentum, as in previous discussion.

The virtual simulation was developed using the Python versions of Project Chrono, PyChrono [26], with IRRLicht [27] engine to render the simulation. Both programs run on an Anaconda environment on a laptop with a 6-cores 10^{th} gen. i7 CPU, base speed 1.61 GHz up to 3.60 GHz, and 16 GB of RAM. Figure 4 shows a snapshot of a simulation that reproduces the behavior of a three layer unit with a columnar layout, without wrapping.

4. Conclusions and Future Works

This paper proposes the development of a virtual multibody simulation to analyse the stability of pallets wrapped with Kraft paper instead of LLDPE. The design of the simulation strongly relies on the EUMOS 40509 requirements of safety for rail and road transport of packages. The main idea is to simulate the acceleration test bench, to minimize high energy and time consumption experiments that have to be done with the ESTL machine. A complete AI pipeline is proposed to study the dynamics of pallet and packages without wrapping. The analysis starts from the actual physical experiment and combines standard computer vision and machine learning techniques to reproduce the behavior of elements. Preliminary results are encouraging, and thanks to the convergence of a gradient descent method, the simulation is able to adhere to the reality of tests, at least for visible bundles. Even if further investigations are needed, future work will be devoted to the identification of critical points of tension which could impact the paper wrapping and the use of such insight to tell wrapping machine software engineers how many wrapping layers are needed, at which heights, with what tension, and so on.

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