Eviction and Reception for Description Logic Ontologies (Extended Abstract)*

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Abstract
In Belief Change, one studies how to adapt the epistemic state of an agent according to some incoming information. Here, we investigate the case in which the epistemic state of the agent is represented as a knowledge base in description logic. Moreover, we consider that the incoming information is in the format of a set of models and investigate *eviction* (removal of models) and *reception* (addition of models) in this setting. We briefly present the case of $\mathcal{ALC}$ extended with Boolean operators over the axioms.

1. Introduction

In traditional paradigms of Belief Change, such as the AGM paradigm [1] for belief revision, and the KM paradigm [2] for belief update, the agent's epistemic state is represented as a set of formulae logically closed, called a theory, while the incoming information is represented as a single formula. The literature within these paradigms often does not address the question of finite representation of the epistemic state. One can see the representation of the incoming information as a formula as a restriction to the case when the incoming information is a set of models (since there can be sets of models that cannot be finitely represented as a formula). Moreover, the setting where the incoming information is in the format of models is useful in various scenarios, as already established in the learning from interpretations literature [3].

To address these shortcomings, Guimarães et al. [4] recently proposed a Belief Change framework where the incoming information is a set of models. The authors focus on the question of finite representation, which is particularly relevant for formulas representing knowledge bases (KBs), since ontology reasoners are designed to deal with finite input. The framework is proposed with two basic operations: eviction (removal of models) and reception (inclusion of models) [4]. It turns out that for many logics, in particular those in the field of Description Logic (DL), eviction and reception are not always possible (meaning that there are sets of models that cannot be finitely represented in a given DL language).

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In this work, we generalise the framework by Guimarães et al. [4] by introducing the notion of “compartments”. A compartment is a pair where the first element is a (potentially infinite) subset of (finite) KBs that can be expressed in a given DL and the second element is a (potentially infinite) set of sets of models taken from the whole set of DL models. This can be used to restrict the more general case of eviction and reception in a given DL, for example, by just considering finite models or by just considering a particular set of formulae where eviction and reception can be performed while preserving finiteness of the agent’s epistemic state. There have been works on repairing ontologies, where the goal is both to preserve the syntax of the ontology and perform the change operation [5, 6, 7, 8]. Our work differs from these studies as the goal in our case is to perform the change operation in an optimal way, giving up on the syntax. Regarding earlier approaches within the field of Belief Change focusing on finite representation of the agent’s epistemic state and the format of the incoming information, we refer to the related work section by Guimarães et al. [4].

In Section 2 we recall the framework by Guimarães et al. [4] and present our generalised framework, with the already mentioned notion of compartments. In Section 3, we consider the case of $\mathcal{ALC}_{bool}$.

2. Eviction and Reception Modulo Compartments

We represent a logic using a satisfaction system $\Lambda(\mathcal{L})$ [7, 9, 4], where $\mathcal{L}$ is the language of the logic. More specifically, a satisfaction system is a triple $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$, where $\mathcal{L}$ is a language, $\mathfrak{M}$ is a set of models, also called interpretations, and $\models$ is a satisfaction relation which contains all pairs $(M, \mathcal{B})$, where M is an interpretation and $\mathcal{B}$ is a *base* (that is, a subset of $\mathcal{L}$), such that M satisfies $\mathcal{B}$ (i.e., $M \models \mathcal{B}$).

The belief change framework originally proposed by Guimarães et al. [4] assumes that an agent’s corpus of beliefs is represented as a finite set of formulae $\mathcal{B}$ (a finite base), while incoming information is represented as a set of models $\mathbb{M}$. There are two main kinds of operations addressed in such a framework: eviction, which consists in removing from $\mathcal{B}$ all models in $\mathbb{M}$; and reception, which consists in incorporating to $\mathcal{B}$ all models from $\mathbb{M}$. In both operations, the output is a finite base.

Both eviction and reception operations are characterized by a set of rationality postulates that capture the notion of minimal change: eviction is characterized by the postulates success, inclusion, finite retainment, and uniformity; while reception is characterized by the postulates success, persistence, finite temperance, and uniformity. These postulates are inspired by the traditional postulates in Belief Change and express desirable (and often expected) properties of operations in this field. For the complete definition and discussion on the rationale of these postulates see [4].

Here, we give an intuitive explanation of the postulates that characterise eviction and reception There are four postulates for eviction: *success*, indicating that no input models satisfy the resulting base; *inclusion*, the new finite base is produced by removing models from the original finite base; *finite retainment*, models are lost either if they are in the input or if they are necessary to reach a finite base; and *uniformity*, ensures that the operation is not sensitive to syntax. Analogously, we have four postulates for reception: *success*, all the new models satisfy the input base; *persistence*, all the old models satisfy the new base; *finite temperance*, a model will satisfy the resulting base if it was in the input, if it satisfied the original base, or if it is required to ensure finite representability; and *uniformity*, the result of the reception depends only on the sets of models that satisfy the input base and the set of input models.

Satisfaction systems in which eviction operations can be defined are called *eviction-compatible*, while satisfaction systems in which reception can be defined are called *reception-compatible* [4]. Unfortunately, there are satisfaction systems that are neither eviction- nor reception-compatible, as it is the case of the DL $\mathcal{ALC}$ [4]. These impossibility results, however, are proved specifically for each of the investigated logics. We identify sufficient conditions for reception-compatibility.

Theorem 1. Let $\mathcal{L}$ be a monotonic DL with $\top$, that can represent inconsistencies and it is interpreted over an infinite signature. The satisfaction system $\Lambda(\mathcal{L})$ is not reception-compatible.

The main obstacle to eviction- and reception-compatibility is that eviction and reception functions might be undefined for some inputs in the domain $\mathcal{P}_{\mathrm{f}}(\mathcal{L}) \times \mathcal{P}(\mathfrak{M})$, for a satisfaction system $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$.

Thus, we will attempt to circumvent the incompatibilities of a satisfaction system by placing restrictions on which bases and sets of models are allowed as input, restricting the domain of the eviction and reception functions. We formalise these additional constraints with the notion of *compartment*. In the following, the power set of a set A is denoted by $\mathcal{P}(A)$, while the set of all finite subsets of A is denoted by $\mathcal{P}_{\mathrm{f}}(A)$.

Definition 2. A *compartment* of a satisfaction system $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$ is a pair $(\mathfrak{B}, \mathfrak{I})$ such that $\mathfrak{B} \subseteq \mathcal{P}_{\mathrm{f}}(\mathcal{L})$ and $\mathfrak{I} \subseteq \mathcal{P}(\mathfrak{M})$.

We say that a compartment $(\mathfrak{B}, \mathfrak{I})$ of a satisfaction system $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$ is eviction-compatible (resp. reception-compatible) iff it defines a subset of $\mathcal{P}_{\mathrm{f}}(\mathcal{L}) \times \mathcal{P}(\mathfrak{M})$ in which eviction (resp. reception) functions can be defined over $\mathfrak{B} \times \mathfrak{I}$. As a result, we can perform eviction and reception functions to potentially interesting portions of satisfaction systems that are, otherwise, incompatible with either eviction or reception. Furthermore, since we preserve the underlying constructions of the operations proposed by Guimarães et al. [10], we also retain their formal characterisation via postulates.

Theorem 3. Let $(\mathfrak{B}, \mathfrak{I})$ be an eviction-compatible compartment of a satisfaction system Λ and evc be a model change operator modulo $(\mathfrak{B}, \mathfrak{I})$. The operator evc is a maxichoice eviction function modulo $(\mathfrak{B}, \mathfrak{I})$ iff it satisfies the postulates success, inclusion, finite retainment, and uniformity from Theorem 5 of Guimarães et al. [4], for all $\mathcal{B} \in \mathfrak{B}$ and $\mathbb{M} \in \mathfrak{I}$.

Theorem 4. Let $(\mathfrak{B}, \mathfrak{I})$ be a reception-compatible compartment of a satisfaction system Λ and rcp be a model change operator modulo $(\mathfrak{B}, \mathfrak{I})$. The operator rcp is a maxichoice reception function modulo $(\mathfrak{B}, \mathfrak{I})$ iff it satisfies success, persistence, finite temperance, and uniformity from Theorem 10 from Guimarães et al. [4], for all $\mathcal{B} \in \mathfrak{B}$ and $\mathbb{M} \in \mathfrak{I}$.

Since the negative result in Theorem 1 involves infinite signatures and several DLs are interpreted over infinite signatures, we consider, unless otherwise stated, only compartments such that (1) bases and models are defined over a finite signature, (2) the class of models is composed of singleton sets, and (3) every singleton corresponds to a finite model.

3. The Case of $\mathcal{ALC}_{bool}$

The framework we presented is general enough to cover several satisfaction systems without imposing much constraints upon the logics being used to represent an agent's beliefs. However, there are interesting logics used for knowledge representation that are not reception-compatible, as it is the case of some DLs (Theorem 5). In this section, we investigate how to extend model change operations to one such logic as a study case. We look precisely at the logic $\mathcal{ALC}_{bool}$[1], which corresponds to the DL $\mathcal{ALC}$ enriched with boolean operators over $\mathcal{ALC}$ axioms. As $\mathcal{ALC}$ is a prototypical DL, it shares many similarities other logics in the of DL family. Our results are built on proofs for the $\mathcal{ALC}$ case without boolean operators over the axioms [4].

We establish negative results for eviction compatibility. We denote by $\Lambda(\mathcal{ALC}_{bool})$ the satisfaction system with the entailment relation given by the standard semantics of $\mathcal{ALC}_{bool}$ [11].

Theorem 5. $\Lambda(\mathcal{ALC}_{bool})$ is neither eviction-compatible nor reception-compatible.

Theorem 6 establishes the connection between quasimodels [11] and formulae in $\mathcal{ALC}_{bool}$.

Theorem 6 (Theorem 2.27 [11])**.** An $\mathcal{ALC}_{bool}$-formula φ is satisfiable iff φ has a quasimodel.

One can associate a model $\mathcal{I}_\mathcal{Q}$ to each quasimodel $\mathcal{Q}$ for φ such that $\mathcal{I}_\mathcal{Q} \models \varphi$. The class of $\mathcal{ALC}_{bool}$-formulae *induced by* φ is the class of $\mathcal{ALC}_{bool}$-formulae that contains φ and any $\mathcal{ALC}_{bool}$-formulae that is a boolean combination of atoms in φ.

Theorem 7. Consider the compartment $(\mathfrak{B}_\varphi, \mathfrak{I}_\varphi)$ where $\mathfrak{I}_\varphi$ is the class of models $\mathcal{I}_\mathcal{Q}$ with $\mathcal{Q}$ a quasimodel for an $\mathcal{ALC}_{bool}$-formula φ and $\mathfrak{B}_\varphi$ is the class of $\mathcal{ALC}_{bool}$-formulae induced by φ. Then, the compartment $(\mathfrak{B}_\varphi, \mathfrak{I}_\varphi)$ is both eviction and reception-compatible.

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[1] $\mathcal{ALC}_{bool}$ is also called $\mathcal{ALC}$-formula in [11], the former nomenclature facilitates the distinction between the logic and its formulae.

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