Formation of Indicators for Evaluating the Model Based on a Set of Interconnected Data Sets in the Tasks of Communication Technologies in Healthcare

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Abstract
One of the tasks of building and evaluating an analytical model, which requires the implementation of the advantages of the idea of remote recovery and the introduction of communication technologies in health care, is considered. It is proposed to use an analytical model of data on state indicators and the course of procedures as a tool for effective lossless compression of transmitted data. The task of building an analytical model is set as a task of evidentiary model selection. The apparatus of functional analysis was used to substantiate the quantitative type selection tools and to determine the best model based only on sets of experimental patient observation data.

Keywords [1]
Medical data, monitoring, approximation, analytical model, evidence selection, quantitative tools

1. Introduction

Realization of the benefits of the idea of remote recovery, especially in the conditions of the formation of the system of family doctors and achievements in the field of miniature wireless surveillance and monitoring devices, requires a revision of the recovery paradigm [1]. The development of the idea of creating modules as an elementary base consists of the automation of autonomous wireless devices for the recovery of post-infarction, post-stroke patients in individual conditions of remote rehabilitation [2]. The success of the implementation of remotely controlled procedures [2] is based on the hypothesis of ensuring wireless communication of all elements under the conditions of two-level authentication of the device and the patient and determination of their state parameters [3]. Open and time-limited access for a specialist and family doctor is achieved due to the security of continuous monitoring data [4] and logging of parameters that determine the patient's condition [5] and the course of procedures [6]. Another working hypothesis is that duplicating types of communication with patients, measurement and diagnostic channels, implementation of a support system and decision-making will increase the efficiency of the recovery process, but will also significantly increase the amount of information [2]. According to the work [2], one of the components is the intelligent compression of information without loss. In addition, it is common knowledge that any analytical model is inherently a more efficient way of presenting information in a compressed manner. In this regard, the quality of the formed mathematical model of the object becomes crucial in the management of automated systems (AS) of communication technologies (CT) of health care [7, 8], and reducing time costs becomes an urgent need. The formation of a mathematical model due to the establishment and use of physical regularities and practical data is a tool for its construction, including with the help of approximation. Thus, the stage of building a model and proving its adequacy becomes an integral part of all research aimed at providing analytical compression, especially for the functioning of CT in the healthcare AS. Traditionally, approximation included such stages of its creation as organizing input information, searching for the most suitable type of model, finding integration constants, and analyzing the achieved accuracy and adequacy. The number of types of models considered simultaneously determined the complexity of the process. For remote rehabilitation, the problem of finding a new way to significantly reduce labor intensity is acute. In this regard, the

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problem of choosing a model based on only the analysis of data of measurement sets with artificial intelligence tools and determining parameters only for the best model becomes an urgent task.

2. Analysis of the latest literary data on the formulation of approximation problems for building a model and assessing its adequacy

In most of the works devoted to the search for general approaches to choosing a type of model, the authors focus on the justification and regularities of approximation of one of the types, using for this purpose various notions of measure and methods of quantitative measurement: - metrics. Types of metrics and norms and distances in the hierarchy of categorical semantics and functions, which are key in mathematics, including the theory of approximations [9]. The work reinterprets and reorients these concepts to practical approximations and applications to the theory of graphs in general and trees, in particular [9]. The second example of studying the possibilities of approximation is demonstrated by work [10]. Its authors study approximation by functions. The results of their work will determine the efficiency of the approximation of complex functions, for which simple and analytical expressions are chosen, and the accuracy of which will not be inferior to spline approximation [10]. This work also considers and proves the uniqueness of the best approximation by generalized polynomials. Proposals using approximation to simplify operator and nonlinear differential equations [11] are no less important. Its author demonstrates the applicability of approximation to the solution of differential and integral equations due to the use of approximation for the iterative approximate solution of equations with analytical conditions. It is important to apply approximation to the solution of linear differential equations with multi-term coefficients or to the formation of new integral representations of the function due to approximation [11]. The third example of works that investigate the possibilities of approximation by a combination of two types of analytical models (the sum of a polynomial and an exponent) is the work [12], which studies the nature of the approximation. Based on the analysis of the approximation process, the author established that a sufficient condition for the existence of a uniform approximation of a function by the sum of a polynomial and an exponent is the continuity of the function and the limitation of its derivative at the beginning and end of the definition domain [12]. The order of the polynomial and the unknown constants in it and two additional constants (a multiplier before the exponent function and a constant in the exponent) provide the best uniform approximation of the function. Such a model with an accurate reproduction of its value at the extreme points of the segment is suitable for constructing continuous minimax spline approximations [12].

However, as the analysis of works [10] [11] and [12] shows, the question of proving the better suitability of the model for approximation when considering several types of models is not even raised in them. The application of approximation for the structure of models in dynamic programming problems or for problems in which the models are represented by nonlinear differential equations is made in [13]. Regardless of the time of its emergence, the fundamentality of the idea of piecewise linear approximation, which was highlighted in it, has positive consequences for the formation of models in the form of a convergent sequence.

However, the application of these results to the development of numerical methods slowed down the process of their introduction to the structure of analytical models. One of the examples of their further development and application is the idea of piecewise-quadratic and piecewise-cubic approximations, which expanded the applicability of the approximation to the structure of nonlinear models in the form of analytical solutions, including recurrent networks [14]. The resulting recurrent models have analytical forms and allow fast calculation, which makes them particularly attractive for applied problems in nonlinear systems [14]. However, despite such results, the question of evidential choice of the type of model was not raised, and therefore the choice among the types of approximation was not made at the first stage. The determination of constants based on the results of the application of several types of approximation significantly increases the complexity of solving model selection problems [14].

A review of the works of recent years allows us to state that uncertainty, as a feature of modern types of models, is increasingly dominating [15]. One of the examples of the implementation of systems whose models are described by fuzzy sets is a system of whose models are described by fuzzy sets is a system of support and decision-making in the automated management of processes in marine technologies. The experience of such a system will serve as an alternative prototype for further developments in which tools and means of simplified presentation of complex models are used [15].

It is obvious that further development will be accompanied by the implementation of the concept of a modular cyber-physical system. As the authors of the work [16] prove, early diagnostics will become a priority for industrial equipment based on Industry 4.0 standards and should be considered as an integral structural element. It is expected, in particular, that the Internet of Things methodologies will also play their role in the formation of new requirements for compression and will contribute their positive role in the process of information protection [16].

As a development trend, the use of a neural-fuzzy observer can be traced, but it will also require the expansion of computer libraries and new forms of fuzzy models for operations with asymmetric membership functions [17]. However, regardless of the forms, the problem of approximation and evaluation of its results with quantitative indicators of adequacy will remain relevant. The effectiveness of an attempt to consider approximation as a complex process of building a neural network using a vector indicator is presented in [18]. The coefficients of synaptic weights are analytically determined as a convergent sequence - the solution of a system of nonlinear algebraic equations, which is presented through the initial data for training [18].

An example of the development of a practical assessment of the quality of a model is the work [19], which considers the process of building a model as a process of creation. In the work, it is proposed to evaluate the efficiency by evaluating the generally known factors, which are inherently defined as the component properties of the adequacy of the model in relation to the object that needs to be described on the basis of data about the function and its derivatives [19]. The level and degree of adequacy is evaluated according to separate criteria from the group or the entire population as a whole: reliability; accuracy and completeness; depth and essence, simplicity; applicability to a convenient solution to the problem of investigating the phenomenon [19].

The work [20] is also devoted to the study of the approximation of the function and its derivatives. Practical successes of simultaneous approximation are presented, which confirm the possibility of approximating derivatives of several orders. The obtained results of work [19] taking into account the results of work [20] in the implementation of the depth adequacy criterion [19] acquire a more realizable form. The simultaneous multifunctional application of approximation as a description of sets of points, functions, and their derivatives in the formation of the network and its calibration and training gives positive results and expands the possibilities of creating and simplifying the model [21]. It was also demonstrated that due to the use of the vector indicator, its functionality is expanded [21]. The analysis of recent works on approximation shows that its application has been extended to the solutions of nonlinear differential equations by the mixed finite-element approximation of the solutions of the Hilbert space, as well as to the fully nonlinear Hamiltonian Jacobi Bellman equation [22]. In it, there is proposed to use indirect means of informing about the error values exceeding the limits during the solution, as a result of which the grid is adjusted [22].

However, the applied indicator has a practical value for choosing a mesh but does not evaluate the adequacy of the model creation results [23].

Thus, recent works investigate and justify the properties and capabilities of certain types of models. The existing approximation methodology does not contain tools for evidentiary selection of the best model based on the data of the sets formed during the experiment. The rationale for the creation of quantitative model selection tools is the main unsolved problem.

3. The purpose and objectives of the research

The purpose of the work is to build scientifically based quantitative tools that will provide an evidence-based selection of the type of approximation model for available experimental data.

To achieve this goal, the following tasks were formulated:

- To create a system that allows taking into account the intervals of the existence of permissible values in the presence of additional conditions determined by the accuracy class of the device and the measurement methodology;
- To form tools for quantitative proof of the better belonging of the model type of a given set of data values to the description on the corresponding set of definitions.

4. Build a sequence of actions that will ensure a proven choice of the type of approximation model and an assessment of its adequacy

Quantitative proof of the best fit of the model type of a given set of data values to the description of the corresponding set of definitions on the basis few hypotheses. As a basic hypothesis, we assumed that D is the domain of operators Fs, where s=1,…,S, that define the types of models, and the function $\Phi(\bar{x})$ also belongs to the Banach space of functions in R. It was also assumed that the values of these operators applied to the function $\Phi(\bar{x})$ are reflected in the general Banach space of values W. We will continue to assume that which is an m component vector, and the function is continuous and integrated with the square or for each of its components there is a square norm:

$$\|\Phi\| = \sqrt{\int_0^1 \Phi^2\left(x_j\right) dx_j}$$

Note that if f and h are two such functions, then the Buniakovsky and Cauchy inequalities are valid for them:

$$\left|\int_0^1 fhdx_j\right| \leq \|f\| \cdot \|h\| \quad and \quad \|f+h\| \leq \|f\| + \|h\| \ .$$

In addition, the following properties are satisfied:

$$\|f\| \leq |f|_{\max} ; \|f\| |h|_{\min} \leq \|fh\| \leq \|f\| |h|_{\max}$$

and

$$\|af\| = |a| \|f\|, \qquad \text{if } \alpha - \text{ is a constant.}$$

The main hypothesis was formulated as follows: There is at least one form from the set S of known functions that is suitable to describe the set of values as a continuous or discrete operation on N elements of the definition set D to ensure convergence with a given set of elements and satisfy m conditions from the set of constraints. Let us also assume that each of the S known operators forms N values for each of the points from the definition set.

Under the specified hypotheses and conditions, a given array of real numbers is formed from the set of definitions and their measurement functions as the output signal of the device $\Phi[i]$ and the magnitude of the output signal, which represents the result $y[i]$ for all N measurements:

$$x[i], \Phi[i], y[i], i = \overline{1,N} \tag{1}$$

Thus, for each of its elements i it defines sets the corresponding array of values that determine for each element $x[i]$ the only element $\Phi[i]$ and the only element $y[i]$. Denote by the index s belonging to the s type of the model, we present a system of expressions that forms. For each from the S types of known operators, for each i-th element of the set of definitions, respectively, the element of the set of values $f_s(i)$, its largest value of the module $F_{s\max}$, absolute $\Delta_s(i)$ and relative $\varepsilon_s(i)$ errors:

$$\begin{cases} F_S\left[\Phi(i)\right] = f_S(i), s = \overline{1,S}; F_{S\max} = \left|F_S\left[\Phi(i)\right]\right|_{\max}; \\ \Delta_S(i) = y(i) - F_S\left[\Phi(i)\right]; \varepsilon_S(i) = \Delta_S(i) / F_{S\max}. \end{cases} \tag{2}$$

Under these conditions and assumptions, a linear form was formed for the relative error on the entire set of the original representation (1) of the set of experimental data for the model structure:

$$\varepsilon_s(i) = \frac{y(i) - F_s[\Phi(i)]}{F_{s\max}} \tag{3}$$

The expression of the relative error (3), written in the notations (1)-(2), is represented by a set of experimental data. The error that occurs as a result of measurement by a device of accuracy class K determines the interval of values of the measured value and the limit $|y(i)|_{\max}$:

$$y(i) = y(i) \pm \frac{K|y(i)|_{\max}}{100} \tag{4}$$

For each model of type s, for each i - th element of the domain of definition, there corresponds a number from the array $f_s(i)$. An important sign of the quality of the model for each i-th element of the domain of definition D is its possible relation to the experimental interval of values (4), the value of which is determined by the error.

Let's formulate a lemma that summarizes experimental experience and does not require proof.

Lemma. If a model given by an operator F_s on a set S set of known functions, as an operation on N elements on a set of definitions D of a Banach space, forms N values from a set of values W of the same Banach space and whose values lie within the measurement interval of each element, it is suitable for description .

This statement will be represented on the basis of notations (3) and (4) by the expression for the interval of existence of admissible values:

$$y(i) - \frac{K|y(i)|_{\max}}{100} < F_s[\Phi(i)] + \varepsilon_s(i) F_{s\max} < y(i) + \frac{K|y(i)|_{\max}}{100} \tag{5}$$

Applying the quadratic norm to the left, central and right parts of inequality (5) taking into account the Buniakovsky and Cauchy inequalities, we obtained:

$$\|y(i)\| - \frac{K|y(i)|_{\max}}{100} < \|F_s[\Phi(i)] + \varepsilon_s(i) F_{s\max}\| < \|y(i)\| + \frac{K|y(i)|_{\max}}{100} \tag{6}$$

Based on the spread of values, as a characteristic of the properties of this inequality, the choice of model type is proved. The accuracy class of the device K, for which the interval of values of the maximum possible error is determined by (6), limits the choice of model types suitable for analysis and allows choosing the best of those suitable for approximation. Such properties are objective, but not so obvious from expression (6). To present them in a more visual form, let's transform inequality (6) by taking into account the fair norms of the sum and the Buniakovsky and Cauchy inequalities:

$$0 < \|\varepsilon_s(i) F_{s\max}\| + \|F_s[\Phi(i)]\| < \frac{2K|y(i)|_{\max}}{100}, \quad \|\varepsilon_s(i)\| < \frac{2K|y(i)|_{\max}}{100|F_{s\max}|} - \frac{\|F_s[\Phi(i)]\|}{|F_{s\max}|}., \tag{7}$$

Now the expression of the norm of relative error in (7) quantitatively expresses a feature whose value represents a quantitative selection criterion. This expression, as a further formal basis for the application of artificial intelligence, learns from the set S of complex images of models and the formation of an unambiguous conclusion. Note that the size of the interval of the error field is determined by the double error.

To simplify the further implementation of these results, we will form a definition.

Definition 1. A model satisfying inequalities (7) for quadratic norms and methodical measurement error does not go beyond the margin of error.

Thus, if the quantitative parameters of the model are known in the form of constants, which have already been found during the approximation, as well as the data found according to the system (2), then it is possible to compare the models. However, this approach is time-consuming. In this regard, the search and comparison at the first stage of model selection of their possible properties, without previously solving the approximation problem for each of the model types, is a task of artificial intelligence according to three main features.

Let's pose the problem of comparing models at the stage when the approximation constants are unknown. Suppose that it was possible to bring the models from the set S to one formal space W in the formal coordinates of the space of functions [23], for example, by the method of straightening the coordinates to a linear form in new coordinates *X* and *Y* of the type:

$$Y = AX + B$$

Under these conditions, inequalities (6) will be rewritten as follows:

$$\left\|y(i)\right\| - \frac{K\left|y(i)\right|_{\max}}{100} < \left\|A_{sj}\Phi_{sj}(i) + B_{sj} + \varepsilon_s(i)F_{s\max}\right\| < \left\|y(i)\right\| + \frac{K\left|y(i)\right|_{\max}}{100} \tag{8}$$

where $\Phi_{sj}(i)$ - is a law defining a new argument *X* with the old *x* for s – th model in each i – th point j – that method of calculation, its approximation constants . According to definition 1, all elements of the value space of *W* must not fall outside the margin of error. In addition, algebraic transformations will determine this interval: An Example of equation

$$0 < \left\|A_{sj}\Phi_{sj}(i) + B_{sj} + \varepsilon_s(i)F_{s\max}\right\| < \frac{2K\left|y(i)\right|_{\max}}{100}. \tag{9}$$

Next, we will use the properties of the norm of the sum and take into account the Buniakovsky and Cauchy inequalities, which will lead (9) to the expression:

$$\left|Asj\right|_{\min}\left\|\Phi_{sj}(i)\right\| < \frac{2K\left|y(i)\right|_{\max}}{100} - \left\|B_{sj}\right\| - \left\|\varepsilon_s(i)\right\|F_{s\max} < \left|Asj\right|_{\max}\left\|\Phi_{sj}(i)\right\|. \tag{10}$$

Thus, if it was possible to bring the models from the set *S* to one formal space *W*, in the formal coordinates of the function space $\Phi(i)$, and their norms exist, then the spread of the difference of the modules of the constants $\left|A_{sj}\right|_{\max}$ and $\left|A_{sj}\right|_{\min}$ could possibly be suitable as an indicator of the quality of the models already at the previous stage of its selection.

However, for their application, it would be necessary to calculate, even for a linear form, two constants of approximation. The latter leads to the conclusion that the solution to the selection problem does not meet the original goal: comparing models without solving the approximation problem. For further search, we will indicate that the calculation of the constants is carried out in some *j*-th way, then the linear form is reduced to:

$$Y_{sj}[i] = A_{sj}X_{sj}[i] + B_{sj} \tag{11}$$

For these conditions, two pairwise versions of the points for which the two conditions for the coincidence of the model with the data of the set of values in the new coordinates are recorded give the solution of the formed system. The latter precisely defines a constant on the interval $\left[X_{sj}[i], X_{sj}[i+1]\right]$, which will additionally be characterized by the value of the point of its definition, and can be denoted A_{sj} [*i,i+1*]. However, we will continue to write the expression of the constant calculated for two adjacent points in a simplified manner, and its value is equal to:

$$A_{sj} = \frac{Y_{sj}[i+1] - Y_{sj}[i]}{X_{sj}[i+1] - X_{sj}[i]} = const \tag{12}$$

and due to the linearity of the form of its derivative:

$$Y'_{sj} = \frac{Y_{sj}[i+1] - Y_{sj}[i]}{X_{sj}[i+1] - X_{sj}[i]} \tag{13}$$

In addition, the function increment on this interval

$$\Delta Y_{sj} = Y'_{sj}\left\{X_{sj}[i+1] - X_{sj}[i]\right\} = A_{sj}\left\{X_{sj}[i+1] - X_{sj}[i]\right\}; \tag{14}$$

For comparison, some element I was taken from the interval $i \in [1, N]$, then the constant was calculated A_{sj} by comparison. It is easy to prove that it will present itself as follows:

$$A_{sj} = A_{sj}\left[(i+1), I\right] - A_{sj}\left[(i), I\right] \tag{15}$$

and the error estimate now becomes determined by only one type of constant, which is determined only by the original data:

$$\Delta Y_{sj} = \left\{A_{sj}\left[(i+1), I\right] - A_{sj}\left[(i), I\right]\right\}\left\{X_k[i+1] - X_k[i]\right\}. \tag{16}$$

This conclusion provides grounds for determining the condition for comparing models. We apply the quadratic norm to conclusions (14) and (16) and obtain two error estimates for any element and set of data:

$$\begin{gathered} \left\|X_{sj}[i+1] - X_{sj}[i]\right\| \left|A_{sj}\right|_{\min} < \left\|\Delta Y_{sj}\right\| < \\ < \left\|X_{sj}[i+1] - X_{sj}[i]\right\| \left|A_{sj}\right|_{\max} \end{gathered} \tag{17}$$

or

$$\begin{gathered} \left\|\left\{X_{sj}[i+1] - X_{sj}[i]\right\}\right\| \left|A_{sj}\left[(i+1), I\right] - A_{sj}\left[(i), I\right]\right|_{\min} < \\ < \left\|\Delta Y_{sj}\right\| < \\ \left\|\left\{X_{sj}[i+1] - X_{sj}[i]\right\}\right\| \left|A_{sj}\left[(i+1), I\right] - A_{sj}\left[(i), I\right]\right|_{\max} \end{gathered} \tag{18}$$

Thus, on the basis of expressions (17) and (18) for the given experimental data, under the conditions of entry into the interval (8), the model will be better for the smallest error.

5. Modeling and discussion of research results

In the course of considering the applicability of several models that allow coordinate correction, it was established that there is a value that is preserved (13). This result is chosen as a criterion that will determine the appropriateness of the approximation form and it`s belonging to the best type of the considered types of approximations if the estimate of the maximum possible error for it is minimal. To demonstrate such an idea and statement, as well as for their transparent quantitative confirmation, let's use the research data of stepper motor models as drivers of rehabilitative devices which was recently published in [24].

Eight values to describe the process of rotating the rotor by the angle Y are taken as a basis as a result of statistically processed samples with five elements volume of each, as a function of the discrete-time factor X.

Table 1 presents experimental data for the rotation angle Y of the shaft [24] and quantitative characteristics of the quality of the approximation by the exponential function that is calculated for the found constant. Column 1 presents the discrete-time factor X. Column 2 presents the value of the mathematical expectation, which was statistically processed and borrowed from the work [24]. The indicator function, as a result of the approximation, which is denoted by Y1, is presented in column 3. The absolute and relative error, as a characteristic of the quality of the approximation, is presented in columns 4 and 5, respectively. Column 6 presents the value of the constant *a* in the space of X,Y directed coordinates, which is determined by (12). Column 7 shows its relative deviation from the value determined by approximation in aligned coordinates: a=0.272099. Column 8 presents the local values of the adequacy of the approximation by the indicator quality of the model, which was calculated according to three criteria from the seven [19].

Table 1

Approximation parameters by the exponential function $y=2.012029*1.312717^{X}$

X	*Y, degree*	Y_1	Y_1 - *Y*	*Ɛ,%*	*a*	*Ɛa%*	Adequacy, Y_1
1	1,8795	2,641225	0,777573	41,72311	0,669153	145,9226	12,49884
2	3,6484	3,467181	-0,17175	-4,71985	0,405589	49,05925	976,7113
3	5,4737	4,551426	-0,90765	-16,6264	0,282378	3,777749	78,70947
4	7,2505	5,974734	-1,26553	-17,479	0,225243	-17,2203	71,21787
5	9,0879	7,843134	-1,22621	-13,5204	0,178187	-34,5139	119,027
6	10,8478	10,29581	-0,54249	-5,00533	0,155447	-42,8712	868,4757
7	12,6765	13,51549	0,854397	6,748205	0,132494	-51,3068	477,8005
8	14,4687	17,74201	3,287198	22,7412	0,333878	22,70479	42,07232

Table 2 presents the data and quantitative characteristics of the approximation by the power function. In column 1, the discrete-time factor X is given. In columns 2 and 3, the value of the mathematical expectation and approximation by a power function, what Y2 denotes, are correspondingly obtained. The absolute and relative error of the power-law approximation is presented in columns 4 and 5. In column 6, the value of the constant *a* in the space of X,Y directed coordinates between two adjacent points is given, which is determined by (12). Columns 6 and 7 show its absolute and relative deviation from the value defined as the solution of the approximation problem in the aligned coordinates: *a*=0.986919. For approximation by a power function, the approximation constants are calculated: A2 = 0.986919; B2=1.850311. Column 8 presents the local values of the adequacy indicator of the power function approximation which was calculated according to three criteria from the seven [19].

Table 2

Approximation parameters by the power function $y=1.850311*X^{0.986919}$

X	Y, degree	Y_2	Y_2- Y	Ɛ%	*a*	Ɛ*a*%	Adequacy, Y_2
1	1,8795	1,850311	-0,01334	-0,71582	0,965383	-2,1821	42463,31
2	3,6484	3,66722	0,028287	0,777341	1,000305	1,356325	26004,73
3	5,4737	5,471731	0,012657	0,231855	0,981563	-0,54265	153890,8
4	7,2505	7,268237	0,027976	0,386401	1,009407	2,278645	101915,5
5	9,0879	9,058815	-0,01053	-0,11609	0,977322	-0,97236	1614494
6	10,8478	10,84468	0,006375	0,058823	1,008409	2,177467	2184052
7	12,6765	12,62664	0,03445	-0,27209	0,992229	0,53806	293896,8
8	14,4687	14,40526	-0,04955	-0,34278	#ЧИСЛО!	#ЧИСЛО!	185178,1

The analysis of the data in Tables 1 and 2 proves that the relative error of the deviation of the constant of approximation in the corrected coordinates (column 7) is similarly changing as the relative error of the model (column 5) and local adequacy (column 8). The high resolution 10^{-3} and the range of its value more than 50 times the max/min ratio will be useful for the application as an indicator. The changes of the module of the maximum relative error of the deviation of the constant of approximation in the corrected coordinate *a*, the best choice for approximation from two models for this data is a power function. However, despite the axiomatic obviousness of such a statement, there are no examples described in the literature when the task of quantitative comparison and selection of the type of approximation form was set, based only on the analysis of numerical data and the selection of the best of them before the start of the approximation process as such. The formulation and solution of such a problem significantly reduces the overall complexity of the approximation process as a whole, since it is performed only for one form. Also, now comparing the results of data processing, it is clear that the smallest error will be given by the model with the smallest error by the constant (12), which is determined according to the single algorithm (14) or (16).

However, before, after finding the approximation constants, it was necessary once again to ensure the correctness of the conclusions for each model, but now there is no need to do this. Thus, the methodology of building and evaluating the model based on the data of connected sets is supplemented by the stage of selection, the basis of which is a quantitative comparison. Each of the traditional stages is now performed only for one before the chosen form of approximation.

This addition and unification into a single methodology allows you to prove which of the forms is better based on the estimation of the given relative error, and after finding the approximation constants and calculating the adequacy of the model, compare the quality of the model by assessing the adequacy according to several of the seven criteria [19]. Undoubtedly, the choice of content and calculation algorithms is subject to standardization, which is the direction of further research into the structure of empirical models.

6. Conclusions

1. A quantitative assessment of the interval of existence of permissible magnitude of the measured value, which is determined by the accuracy class of the device and the measurement methodology, has been formed. The magnitude of the measured value for the correct model is within the range of the existence of permissible values.
2. Formed tools of quantitative proof of better belonging of the type of model to describe the experimental set of data on the corresponding set of definitions, as an absolute or relative value demonstrates the corresponding error of deviation of the constant, which should be preserved according to its original definition. It is similarly changing as the relative error of the model and local adequacy. The high resolution 10^{-3} and the range of its value more than 50 times the max/min ratio will be useful for the application as an indicator.

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