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# Age of Information Minimization with Power and Distortion Constraints in Multiple Access Channels

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***Abstract*—Emerging fifth generation and beyond networks are expected to deliver accurate information as fresh as possible. In this work, we consider a wireless fading multiple access channel, where $M$ users communicate to a base station (BS) in a time-slotted system. Each user can sample an information packet in any slot of interest, compress it to a finite number of bits and then transmit the compressed packet to the BS. The compression and transmission result in distortion and power consumption, respectively. Using the age of information (AoI) metric for quantifying freshness of information, we consider minimization of a long-term weighted average AoI across the users, subject to average power and distortion constraints at each user, for obtaining the number of bits to be transmitted by a user in a given slot. We cast the problem as a constrained Markov decision process (CMDP) and solve it via Lagrange relaxation. We show that a *threshold-type* policy is optimal for the relaxed problem. We also propose a convex optimization problem to obtain a suboptimal but simpler stationary randomized policy, whose minimum achievable average AoI is within twice that of the optimal policy. Via numerical simulations, we illustrate the threshold structure of the CMDP based solution and study variation of the average AoIs achieved by the proposed policies when the bounds on the average power and distortion constraints are varied.**

## I. Introduction

Emerging technologies such as the fifth generation (5G) and beyond networks demand diverse capabilities including low communication latency, high throughput, timely and accurate delivery of information [1]. In this work, we adopt a metric called age of information (AoI), defined as the time elapsed since the generation of the last successful update generated, for quantifying the timeliness of information [2]–[5], and a distortion metric to measure the accuracy of information. We consider an AoI minimization problem subject to average power and distortion constraints in a wireless multiple access channel. Our aim is to derive a policy to decide the number of bits a user must transmit in a slot.

Optimization of AoI and related metrics has received tremendous interest in the recent years. Specifically, trade-offs between AoI and distortion have been studied in [6]–[9]. The authors in [6] consider an on-off fading channel and optimize data freshness at the receiver subject to a constraint on the product of an AoI term and a distortion term. The authors in [7] consider a scenario where the destination sends requests for updates at random times and the source generates an update packet over a period of time and sends it to the destination over a noiseless channel with zero delay. The distortion is modeled as a decreasing function of the time taken to generate an update after a request. Under this setting, the work minimizes an average AoI subject to a per update distortion constraint. The authors in [8] study trade-offs among AoI, distortion and energy in a communication network and obtain a greedy policy which is 2-competitive, independent of all parameters of the problem, for minimizing a linear combination of AoI, distortion and energy terms. Different from the above works, we consider that communication occurs over a wireless fading channel. A related work, [9] investigates timeliness-distortion trade-off in an energy harvesting, single-user fading channel by minimizing the average weighted sum AoI and distortion over all possible transmit powers and transmission intervals. Different from [9], we consider a multi-user fading channel, where each user is subjected to an average power constraint.

In this work, we consider a single-hop multi-user wireless network in which status update packets can be generated in any slot of interest, as in [10]. The users are required to communicate the packets to a BS over fading channels subject to average power and distortion constraints at each user. The goal is to minimize a weighted average AoI across the users. At each slot, our goal is to find the user and the number of bits to be transmitted based on the channel power gain realizations of the users and their instantaneous AoIs.

The trade-off involved is the following: For a given number of bits to be communicated to meet average distortion constraints, transmitting over *poor* channels requires more power. However, waiting for *good* channels may increase the AoIs. Similarly, transmitting a lower number of bits in a poor channel requires a lower transmit power, but results in a higher distortion. Hence, there exists a trade-off among AoI, distortion and power consumption terms, as in [8]. However, unlike in [8], we consider that communication occurs over a fading multiple access channel, where the users can first observe instantaneous channel power gain realizations and AoIs and then decide to transmit a certain number of bits by adapting transmit powers. Concretely, we consider an AoI minimization problem subject to average power and distortion constraints, in a fading multiple access channel. The main contributions of the paper are as follows:

- We cast the above problem as a constrained Markov decision process, which we solve via Lagrange relaxation. By proving that the state-action value function of the relaxed

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problem is sub-modular, we show that a *threshold-type* policy is optimal for the relaxed problem. This solution may be adopted only when the number of users in the network is low, due to the curse of dimensionality of the CMDP.
- To cater to the situation when the number of users in a network is high, we propose a convex optimization problem, solving which, we can obtain a stationary randomized policy. Based on the technique developed in [11], we prove that the average AoI achieved by the proposed stationary randomized policy is at most twice that of the optimal policy. Moreover, using a dual decomposition technique, we suggest a method to solve the proposed convex optimization problem efficiently by decoupling the problem into $M$ different problems which can be solved in parallel.
- Using numerical simulations, we illustrate the threshold structure of the CMDP based solution. We also compare performance of the proposed policies when the bounds on the average distortion and power constraints are varied.

## II. System Model

We consider $M$ sources updating statuses to a base station (BS). Time is slotted with unit slot duration. The slots are indexed by $n \in \{1, 2..., N\}$. In the below, we present the channel, power consumption, distortion and AoI evolution models, and formulate a long-term average AoI minimization problem subject to a long-term average power and distortion constraints.

### *A. Channel Model*

We consider that the channel power gain of user $i \in \{1, \dots, M\}$ is independent and identically distributed according to the random variable $H_i$ across the slots. $H_i$ takes values from a finite set $\mathcal{H}$. We denote the realization of $H_i$ in slot $n$ as $h_i(n)$, which is estimated at the start of the transmission slot $n$ perfectly, using pilot signals. Hence, at the start of any slot, $\mathbf{h}(n) \triangleq (h_1(n), h_2(n), \dots, h_M(n))$ is known.

### *B. Power and Distortion Models*

For reliably delivering $\rho$ bits when the channel realization is $h$ in user $i$, let $f_i(\rho; h)$ be the transmit power required, where $f_i(x; h)$ is a convex increasing function of $x$, parameterized by $h$ with $f_i(0; h) = 0$ for any $h \in \mathcal{H}$. Moreover, $f_i(x; h)$ is submodular in $(x, h)$, i.e., $f_i(x+1; h+\delta) - f_i(x; h+\delta) \leq f_i(r+1; h) - f_i(r; h)$ holds for any $x$ and $h$, for any $\delta > 0$. When the receiver receives $\rho$ bits from user $i$, let the distortion incurred be $d_i(\rho)$, where $d_i(x) : \mathbb{R}^+ \to \mathbb{R}^+$ is a convex decreasing function of $x$, where $\mathbb{R}^+$ is the set of all non-negative real numbers. For instance, $f_i(x; h) = (\exp(x)-1)/h$ and $d_i(x) = \exp(-x)$ are valid functions that satisfy the above properties. For simplicity, we consider that the number of bits that can be delivered in a slot is a finite positive integer, i.e., $\rho \in \{1, 2, \dots, r_{\max}\}$, where $r_{\max}$ is a finite positive integer.

### *C. Decision Variables and Constraints*

As mentioned, our goal in this work is to decide the number of bits a user must deliver to the BS in a slot, in order to minimize an average AoI subject to average power and distortion constraints at each user. That is, our goal is to obtain $u_{i,\rho}(n) \in \{0, 1\}$, where

$$u_{i,\rho}(n) = \begin{cases} 1 & \text{indicates that user } i \text{ transmits } \rho \text{ bits in slot } n, \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

for $n \in \{1, 2, \dots\}$, $i \in \{1, \dots, M\}$ and $\rho \in \{1, 2, \dots, r_{\max}\}$. We refer to $u_{i,\rho}(n)$ as the decision variable. In the above definition, $u_{i,\rho}(n) = 0$ for all $\rho \in \{1, 2, \dots, r_{\max}\}$ means that user $i$ does not transmit in slot $n$.

In this case, the instantaneous power consumed and the distortion incurred in slot $n$ is $\sum_{\rho=1}^{r_{\max}} f_i(\rho; h_i(n)) u_{i,\rho}(n)$ and $\sum_{\rho=1}^{r_{\max}} d_i(\rho) u_{i,\rho}(n)$, respectively. For $i \in \{1, \dots, M\}$, we consider the following average power constraint:

$$\lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} \sum_{\rho=1}^{r_{\max}} \mathbb{E}\left[f_i(\rho; h_i(n)) u_{i,\rho}(n)\right] \leq \bar{P}_i, \quad (2)$$

and the following average distortion constraint:

$$\lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} \sum_{\rho=1}^{r_{\max}} \mathbb{E}[d_i(\rho) u_{i,\rho}(n)] \leq \alpha_i, \quad (3)$$

where the expectations are with respect to randomness in the channel power gain process and $u_{i,\rho}$ values.

As in many prior works [10], [11], we also consider a time-division multiple access (TDMA) constraint that at most one user can transmit in a slot, i.e., $u_{i,\rho}$ values must satisfy:

$$\sum_{i=1}^{M} \sum_{\rho=1}^{r_{\max}} u_{i,\rho}(n) \leq 1, \ \ \forall n = 1, 2, \dots. \quad (4)$$

### *D. AoI Model*

Let $a_i(n)$ be the instantaneous age of information at the destination in slot $n$ for user $i$. It evolves as follows:

$$a_i(n+1) = \begin{cases} 1 & \text{if } u_{i,\rho}(n) = 1 \text{ for some } \rho > 0 \\ a_i(n) + 1 & \text{otherwise.} \end{cases} \quad (5)$$

The weighted average AoI experienced by the BS is given by

$$\lim_{N \to \infty} \frac{1}{MN} \sum_{i=1}^{M} \sum_{n=1}^{N} w_i \mathbb{E}[a_i(n)] \ , \quad (6)$$

where the $w_i$ represents the weights offered to different sources and $\mathbb{E}[\cdot]$ is the expectation with respect to the randomness in $a_i(n)$.

### *E. Problem Formulation*

In this work, we aim to minimize the long-term average age of information subject to average power and distortion

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constraints. Concretely, our aim is to solve the following optimization problem:

$$A^* = \min_{\pi} \lim_{N\to\infty} \frac{1}{MN} \sum_{i=1}^{M} \sum_{n=1}^{N} w_i \mathbb{E}\left[a_i(n)\right], \tag{7}$$
$$\text{subject to } (2), (3), (4),$$

where $\pi$ is a policy, a specification of the decision rule to be used at each slot $n \in \{1, 2 \ldots\}$ for choosing $u_{i,\rho}(n)$. All the expectations are with respect to the chosen policy $\pi$.

## III. Solution

The optimization problem in (7) has structure of a constrained Markov decision process (CMDP). In this section, we first formally cast (7) as a CMDP and provide a Lagrangian relaxation based solution. Next, we propose a convex optimization problem to obtain a stationary randomized policy, whose maximum achievable average AoI is at most twice that of $A^*$ in (7).

### A. CMDP Based Solution

We first cast (7) as a CMDP by associating its parameters and decision variables to the standard components of a CMDP. We then relax it via Lagrangian method and obtain the optimal solution for the relaxed problem. The components of a CMDP associated with (7) are:

*a) States:* We define the state of user $i$ at time slot $n$ to be $(a_i(n), h_i(n))$, where $a_i(n) \in \{1, 2, \ldots\}$ and $h_i(n) \in \mathcal{H}$ are respectively the instantaneous age of information and the channel power gain realization of user $i$ in slot $n$. The state of the system in slot $n$ is $s(n) = (\mathbf{a}(n), \mathbf{h}(n))$, where $\mathbf{a} = (a_1, \ldots, a_M)$ and $\mathbf{h} = (h_1, \ldots, h_M)$. The state space of the system, $\mathcal{S} = \{1, 2, \ldots\}^M \times \mathcal{H}^M$.

*b) Actions:* Recall that the instantaneous decision variables in slot $n \in \{1, 2, \ldots\}$ in (7) are $u_{i,\rho}(n)$ values for $i \in \{1, \ldots, M\}$ and $\rho \in \{1, 2, \ldots, r_{\max}\}$, defined in (1). In our CMDP formulation, we equivalently let $\mathbf{r}(n) \triangleq (r_1(n), \ldots, r_M(n))$ as the instantaneous action (decision) variable, where $r_i(n) \in \{0, 1, \ldots, r_{\max}\}$ is the number of bits transmitted by user $i$ in slot $n$. The equivalence follows because $u_{i,\rho}(n) = 1$ implies $r_i(n) = \rho$ and vice versa. In the above, we consider that not transmitting is equivalent to transmitting zero bits. That is, $u_{i,\rho}(n) = 0$ for all $\rho \in \{1, 2, \ldots, r_{\max}\}$ is equivalent to $r_i(n) = 0$ for any user $i$ and slot $n$. Moreover, due to the TDMA constraint in (4), in any slot, at most one user can transmit greater than zero bits, i.e., we can have $r_j(n) \geq 0$ for some $j \in \{1, \ldots, M\}$ and $r_k(n) = 0$ for all $k \neq j$. Hence, the action space, $\mathcal{A} = \{(x_1, 0, \ldots, 0), (0, x_2, \ldots, 0), \ldots, (0, 0, \ldots, x_M)\}$, where $x_i \in \{0, 1, 2, \ldots, r_{\max}\}$.

*c) Transition Probabilities:* Noting that the channel power gains evolve in independent and identically distributed manner and using (5), the state transition probabilities for all the states can be easily written down.

*d) Instantaneous Costs:* The instantaneous costs incurred in state $s(n)$ when action $\mathbf{r}(n)$ is taken are: (i) individual AoI cost, $a_i(n+1)$ and (ii) individual power cost, $f_i(r_i(n); h_i(n))$ and (iii) individual distortion cost, $d_i(r_i(n))$ for $i \in \{1, \ldots, M\}$.

A stationary policy, $\pi$, is a mapping from $\mathcal{S}$ to $\mathcal{A}$. For a stationary policy $\pi$ and initial state $s_0 \in \mathcal{S}$, we define the following long-term average expected cost functions:

$$A^{\pi}(s_0) = \lim_{N\to\infty} \frac{1}{NM} \mathbb{E}_{\pi}\left[\sum_{n=1}^{N} \sum_{i=1}^{M} w_i a_i(n) | s_0\right],$$

$$P_i^{\pi}(s_0) = \lim_{N\to\infty} \frac{1}{N} \mathbb{E}_{\pi}\left[\sum_{n=1}^{N} f_i(r_i(n); h_i(n)) | s_0\right],$$

and

$$D_i^{\pi}(s_0) = \lim_{N\to\infty} \frac{1}{N} \mathbb{E}_{\pi}\left[\sum_{n=1}^{N} d_i(r_i(n)) | s_0\right],$$

for all $i \in \{1, \ldots, M\}$, which are the long-term average expected AoI, power and distortion costs, respectively. Now, (7) can be equivalently restated as:

$$\underset{\pi}{\text{minimize}} \quad A^{\pi}(s_0), \tag{8a}$$
$$\text{subject to } \quad P_i^{\pi}(s_0) \leq P_i, \tag{8b}$$
$$D_i^{\pi}(s_0) \leq \alpha_i, \tag{8c}$$

for all $i \in \{1, \ldots, M\}$. In the following, we provide a Lagrangian relaxation based solution to (8).

*1) Lagrange Relaxation of the CMDP:* We now relax (8) via Lagrange relaxation. For this, define,

$$c_{\boldsymbol{\lambda},\boldsymbol{\beta}}(s(n), \mathbf{r}(n)) = \frac{1}{M} \sum_{i=1}^{M} a_i(n+1) + \sum_{i=1}^{M} \lambda_i f_i(r_i(n); h_i(n)) + \sum_{i=1}^{M} \beta_i d_i(r_i(n)),$$

where $\lambda_i, \beta_i \geq 0$ are the parameters that trade-off the cost due to the AoI, the power consumed and the distortion incurred in user $i$, and $\boldsymbol{\lambda} \triangleq (\lambda_1, \ldots, \lambda_M)$ and $\boldsymbol{\beta} \triangleq (\beta_1, \ldots, \beta_M)$. Define

$$V^{\pi}_{\boldsymbol{\lambda},\boldsymbol{\beta}}(s_0) = \lim_{N\to\infty} \sum_{n=1}^{N} \mathbb{E}_{\pi}[c_{\boldsymbol{\lambda},\boldsymbol{\beta}}(\cdot,\cdot)|s_0]. \tag{9}$$

Let $V^*_{\boldsymbol{\lambda},\boldsymbol{\beta}}(s_0) = \min_{\pi} V^{\pi}_{\boldsymbol{\lambda},\boldsymbol{\beta}}(s_0)$. It can be shown that $V^*_{\boldsymbol{\lambda},\boldsymbol{\beta}}(s_0)$ is identical for any initial state, $s_0$, along the lines in [12, Lemma 6]. For any fixed $\boldsymbol{\lambda}$ and $\boldsymbol{\beta}$, we first obtain $V^*_{\boldsymbol{\lambda},\boldsymbol{\beta}}(s_0)$ by solving $\min_{\pi} V^{\pi}_{\boldsymbol{\lambda}}(s_0)$ via the following value iteration of the Bellman equation:

$$V_{n+1,\boldsymbol{\lambda},\boldsymbol{\beta}}(s) = \min_{\mathbf{r}} \ \mathbb{E}[c_{\boldsymbol{\lambda},\boldsymbol{\beta}}(\cdot) + V_{n,\boldsymbol{\lambda},\boldsymbol{\beta}}(s')|s, \mathbf{r}], \tag{10}$$

starting at $n = 0$, where we define $V_{0,\boldsymbol{\lambda},\boldsymbol{\beta}} \triangleq 0$. Then, we have, $V^*_{\boldsymbol{\lambda},\boldsymbol{\beta}}(s_0) = \lim_{n\to\infty} V_{n,\boldsymbol{\lambda},\boldsymbol{\beta}}(s_0)/n$.

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*2) A Threshold Structure of the Optimal Policy for* (9)*:* For any fixed $\boldsymbol{\lambda}$ and $\boldsymbol{\beta}$, we prove that the optimal actions of (9) have a threshold structure in the states. Note that the value function in (9) can be written as

$$V(s) = \min_{\mathbf{r}} \ Q(s, \mathbf{r}), \tag{11}$$

where

$$Q(s, \mathbf{r}) = \mathbb{E}[c|s, \mathbf{r}] + \gamma \mathbb{E}[V(s')|s, \mathbf{r}], \tag{12}$$

is the state-action value function. In the above, we dropped the subscripts $\boldsymbol{\lambda}$ and $\boldsymbol{\beta}$ and the superscript, $\pi$, for notational simplicity. We now have the following result:

**Theorem 1.** *Let* $\mathbf{x}_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_M)$*. When* $\mathbf{a}_{-i}$*,* $\mathbf{h}$ *and* $\mathbf{r}_{-i}$ *are fixed, the optimal number of bits to be transmitted in user i,* $r_i$ *is non-decreasing in AoI,* $a_i$*. Similarly, when* $\mathbf{a}$*,* $\mathbf{h}_{-i}$ *and* $\mathbf{r}_{-i}$ *are fixed,* $r_i$ *is non-decreasing in the channel power gain,* $h_i$*.*

*Proof.* We prove the first and the second part of the result by showing that $Q(s, \mathbf{r})$ is submodular in $(a_i, r_i)$ and $(h_i, r_i)$, respectively, when all other state and action variables are fixed. See Appendix A for details. □

The above theorem says that if it is optimal to transmit $r_i$ bits when the AoI of user $i$ is $a_i$ (the channel power gain is $h_i$), we must transmit at least $r_i$ bits when its AoI is greater than $a_i$ (the channel power gain is greater than $h_i$), given all other state and action variables are fixed. In other words, the optimal policy is monotonic. This knowledge of the optimal policy being monotonic can be exploited to reduce computational complexity in solving the value iteration algorithm in (10) [13]. Moreover, since $r_i \in \{0, 1, \ldots, r_{\max}\}$, the monotonicity of the optimal policy implies existence of $r_{\max}$ thresholds on the state variables, such that when $a_i$ $(h_i)$[1] is below $k^{\text{th}}$ threshold and above $(k-1)^{\text{th}}$ threshold, it is optimal to transmit $k-1$ bits. Such a threshold structure simplifies the real-time implementation of the policy.

### *B. A Stationary Randomized Policy for Solving* (7)

The CMDP based solution can be easily used when the number of users in the network is low. However, due to its curse of dimensionality, when the number of users is large, it is prohibitive to adopt the CMDP solution. In order to address such scenarios, we propose a convex optimization problem to derive a stationary randomized policy that sub-optimally solves (7). Specifically, we re-formulate (7) under the following class of policies:

**Policy** $R$**:** *In a slot, when the channel power gain realization vector,* $\mathbf{H} = \mathbf{h}$*, user* $i$ *transmits* $\rho$ *bits with probability,* $\mu_{i,\rho}(\mathbf{h})$ *for* $i \in \{1, \ldots, M\}$ *and* $\rho \in \{1, \ldots, r_{\max}\}$*.*

In the above policy, $\mu_{i,\rho}(\mathbf{h}) = 0$ for all $\rho \in \{1, \ldots, r_{\max}\}$ implies that user $i$ is not transmitting. When user $i$ transmits, since the number of bits delivered, $\rho \geq 1$, the update is considered successful and the instantaneous AoI drops to 1 (see (5)). The conditional probability of success, conditioned on the event that the channel power gain realization, $\mathbf{H} = \mathbf{h}$, is $\sum_{\rho=1}^{r_{\max}} \mu_{i,\rho}(\mathbf{h})$. Hence, the probability of success in a slot is $\mathbb{E}_{\mathbf{H}}\left[\sum_{\rho=1}^{r_{\max}} \mu_{i,\rho}(\mathbf{h})\right]$. In this case, the inter-success intervals are geometrically distributed random variables with mean, $1/\mathbb{E}_{\mathbf{H}}\left[\sum_{\rho=1}^{r_{\max}} \mu_{i,\rho}(\mathbf{h})\right]$. Hence, the long-term average AoI can be written as $1/\mathbb{E}_{\mathbf{H}}\left[\sum_{\rho=1}^{r_{\max}} \mu_{i,\rho}(\mathbf{h})\right]$ [11]. Moreover, under the class of policies $R$, we can re-formulate (7) as the following optimization problem:

$$A_R^* = \min_{\mu_{i,\rho}(\mathbf{h})} \ \frac{1}{M} \sum_{i=1}^{M} \frac{w_i}{\mathbb{E}_{\mathbf{H}}\left[\sum_{\rho=1}^{r_{\max}} \mu_{i,\rho}(\mathbf{h})\right]}, \tag{13a}$$

$$\text{subject to} \ \ \mathbb{E}_{\mathbf{H}}\left[\sum_{\rho=1}^{r_{\max}} f_i(\rho; h_i)\mu_{i,\rho}(\mathbf{h})\right] \leq \bar{P}_i, \tag{13b}$$

$$\mathbb{E}_{\mathbf{H}}\left[\sum_{\rho=1}^{r_{\max}} d_i(\rho)\mu_{i,\rho}(\mathbf{h})\right] \leq \alpha_i, \tag{13c}$$

$$\mathbb{E}_{\mathbf{H}}\left[\sum_{i=1}^{M}\sum_{\rho=1}^{r_{\max}} \mu_{i,\rho}(\mathbf{h})\right] \leq 1, \tag{13d}$$

$$0 \leq \mu_{i,\rho}(\mathbf{h}) \leq 1, \tag{13e}$$

for all $i \in \{1, \ldots, M\}$ and $\rho \in \{1, \ldots, r_{\max}\}$, where (13b), (13c) and (13d) are respectively obtained by specializing (2), (3) and (4) to the class of stationary randomized policies, $R$. The above problem in (13) is a convex optimization problem. Hence, it can be solved optimally via standard numerical techniques, with a computational complexity proportional to $Mr_{\max}|\mathcal{H}|^M$, where $|\mathcal{H}|$ is the number of possible channel states in a user.

In the below, we propose a method to obtain the optimal solution to (13) with a lower complexity via dual decomposition [14]. For this, note that the only constraint that is common to all the users is (13d). Define $\delta \geq 0$ as the Lagrange variable corresponding to constraint (13d) and we can write down the following problem for obtaining the dual function:

$$\begin{aligned} g(\delta) = \min_{\mu_{i,\rho}(\mathbf{h})} \ & \frac{1}{M} \sum_{i=1}^{M} \frac{w_i}{\mathbb{E}\left[\sum_{\rho=1}^{r_{\max}} \mu_{i,\rho}(\mathbf{h})\right]} + \\ & \delta\left(\mathbb{E}_{\mathbf{H}}\left[\sum_{i=1}^{M}\sum_{\rho=1}^{r_{\max}} \mu_{i,\rho}(\mathbf{h})\right] - 1\right), \end{aligned} \tag{14}$$
$$\text{subject to} \ \ (13\text{b}), (13\text{c}),$$

for all $i \in \{1, \ldots, M\}$ and $\rho \in \{1, \ldots, r_{\max}\}$. For a given $\delta$, as (13b), (13c) and (13d) are applicable to individual users separately, (14) can be decoupled into $M$ optimization problems which can be solved in parallel. Specifically, the decoupled problem for user $i$ is the following:

$g_i(\delta) =$

[1]The thresholds on AoI and channel power gain can be different.

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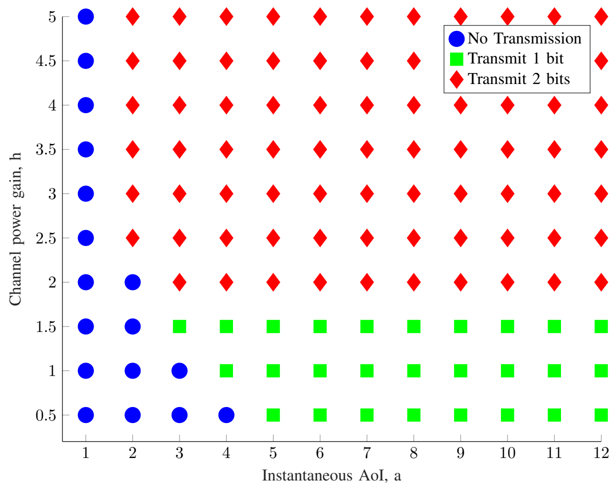


Fig. 1: Optimal actions for a single user case for different states, $(a, h)$ when $\lambda = 1, \beta = 10$ and $r_{\max} = 9$. We observe monotonicity of the optimal solution proved in Theorem 1.

$$\min_{\mu_{i,\rho}(\mathbf{h})} \frac{w_i}{M\mathbb{E}_{\mathbf{H}}\left[\sum_{\rho=1}^{r_{\max}} \mu_{i,\rho}(\mathbf{h})\right]} + \delta\mathbb{E}_{\mathbf{H}}\left[\sum_{\rho=1}^{r_{\max}} \mu_{i,\rho}(\mathbf{h})\right] - \frac{\delta}{M}, \tag{15}$$

subject to (13b), (13c),

for all $\rho \in \{1, \ldots, r_{\max}\}$. Clearly, we have, $g(\delta) = \sum_{i=1}^{M} g_i(\delta)$. The optimal $\delta$ value is the one that maximizes the dual function $g(\delta) = \sum_{i=1}^{M} g_i(\delta)$, i.e.,

$$\delta^* = \arg\max_{\delta \geq 0} \sum_{i=1}^{M} g_i(\delta). \tag{16}$$

The optimal $\mu_{i,\rho}(\mathbf{h})$ can be obtained by solving (15) for $\delta = \delta^*$. For any $\delta$, the optimization problem in (15) can be solved in parallel, simultaneously for all the users, with computational complexity proportional to $r_{\max}|\mathcal{H}|$.

In the following theorem, we show that the average AoI achieved in (13) is at most twice the optimal AoI.

**Theorem 2.** *The optimal objective value of* (13)*, $A_R^*$ is at most* 2 *times that of* (7)*, $A^*$, i.e., $A_R^* < 2A^*$.*

*Proof.* We adopt the technique developed in [11] for proving the result. Specifically, we first obtain an optimization problem, called the lower-bound problem, whose optimal objective value, $L_B$, is a lower bound to $A^*$ in (7). We then show that the optimal solution to the lower-bound problem is a feasible solution to (13). Then, we relate the objective functions of the lower bound problem, $L_B$, and (13), under the optimal solution to the lower-bound problem. Through this, we establish that $A_R^* < 2A^*$. See Appendix B for details. □

The proof of Theorem 2 is based on [11]. The main contribution in the proof is to show that the optimal policy of the lower-bound problem is a feasible policy for (13).

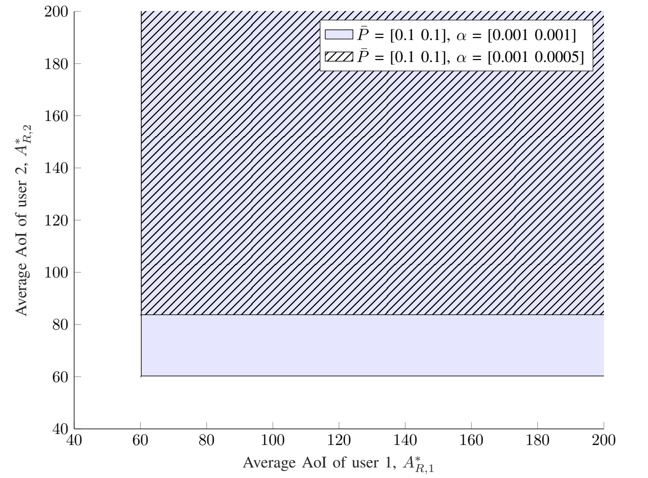


Fig. 2: The long-term achievable AoI regions in a two-user case for different bounds on average power and distortion constraints under the proposed stationary randomized policy, $R$, obtained by solving (13). We consider that the channel power gain $h$ takes values, $1, 2, 3, 4$ and $10$ with probabilities $0.63581$, $0.23024$, $0.0847$, $0.03116$ and $0.01809$, respectively, and $r_{\max} = 10$.

## IV. Numerical Results

In this section, we obtain the numerical results assuming that the power required to deliver $x$ bits with the channel power gain of $h$ is $f(x; h) = (e^x - 1)/h$ and that the distortion incurred when $x$ bits are transmitted is, $d(x) = 2^{-x}$ for all the users.

In Fig. 1, we show the optimal actions taken for different states, $(a, h)$, in a single user case, where $a$ is the instantaneous AoI and $h$ is the channel power gain. From the figure, we observe that when the instantaneous AoI is 1, it is optimal to not transmit irrespective of the channel gain realization, as it is the minimum AoI that can be achieved. For the states with small channel power gains, as the AoI increases, it is optimal to transmit only one bit, at the expense of high distortion, as transmitting a single bit consumes a lower power compared to transmitting higher bits. For the states with high channel power gains, as the AoI increases, it is optimal to transmit two bits, which leads to a lower distortion. Note that the power consumed when the channel power gains are high is lower than that when the channel power gains are low. Moreover, from the figure, we can also observe the result of Theorem 1 that for fixed $h$ (and respectively, $a$), the number of bits transmitted is non-decreasing in $a$ ($h$).

In Fig. 2, we present long-term average achievable AoI regions in a two-user case. For obtaining the plot, we let $w_1 = w$ and $w_2 = 1 - w$ in (13) and vary $w$ over $[0, 1]$. For a specific $w$, suppose $\mu_{i,\rho}(\mathbf{h})$ is the optimal policy, then $A_{R,i}^* \triangleq 1/\mathbb{E}_{\mathbf{H}}\left[\sum_{\rho=1}^{r_{\max}} \mu_{i,\rho}(\mathbf{h})\right]$ for $i \in \{1, 2\}$. From the figure, it can be seen that if the bound on the distortion constraints of user 1 and user 2, $\alpha_1$ and $\alpha_2$, respectively, are lower, the achievable AoI region is smaller. This is because, when the distortion constraint is stringent, one needs to transmit more bits to meet the constraint. This leads to less frequent transmission in order to meet the average power constraint,

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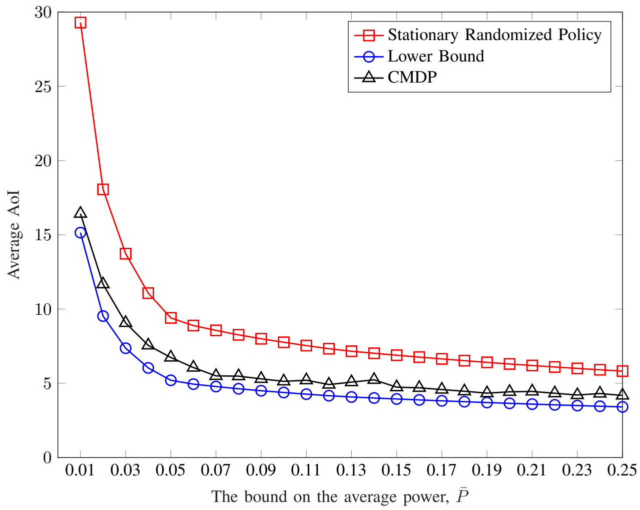
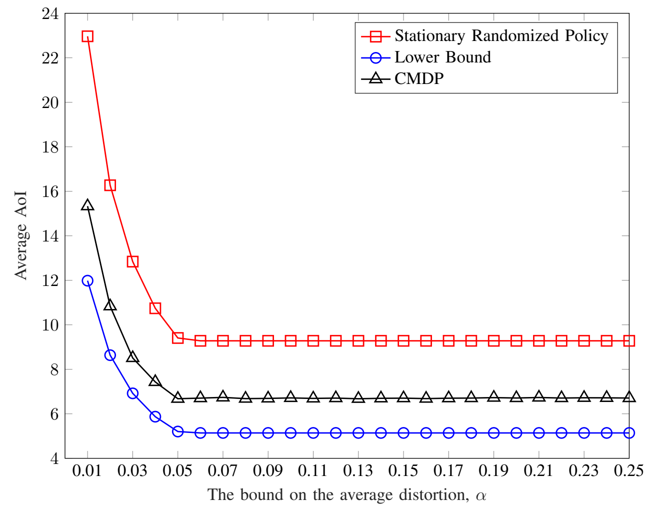



(a) Variation of average AoI with the bound on the average transmission power, $\bar{P}$, when the bound on the average distortion, $\alpha = 0.05$.




(b) Variation of the average AoI with the bound on the average distortion, $\alpha$, when the bound on the average transmission power, $\bar{P} = 0.05$.

Fig. 3: Variation of the average AoI achieved by different policies with bounds on the average power and distortion, for $r_{\max} = 9$ and the channel power gain $h$ takes values, $1, 2, 3, 4$ and $10$ with probabilities $0.63581$, $0.23024$, $0.0847$, $0.03116$ and $0.01809$, respectively,

leading to a higher average AoI.

In Fig. 3a and Fig. 3b, we plot the variation of the average AoI achieved by the CMDP based solution and the stationary randomized policy with the bound on the average power and distortion, respectively. We also plot the average AoI achieved by the lower-bound problem in (18). As expected, in all the curves, the average AoI decreases with increasing bound on the average power and distortion. This is because with a higher transmission power, we can transmit frequently, even when the channel power gains are low while maintaining the required average distortion by transmitting certain minimum number of bits. Similarly, when the maximum average distortion is increased, we can afford to transmit frequently, a lower number of bits so that the average power constraint is satisfied.

## V. Conclusion

In this work, we considered a base station receiving status update packets from multiple users over a fading multiple access channel. In a slot, a user can sample a packet, compress it to a finite number of bits and transmit them. The process of compression and transmission result in distortion and power consumption, respectively. Under this setting, our goal was to obtain the number of bits a user must transmit in a slot for minimizing an average AoI subject to average power and distortion constraints at each user. We provided a CMDP based solution to the above problem, which can be used in a low-density network, and showed that the optimal actions of the Lagrange relaxation of the CMDP exhibits a threshold structure in the state variables. In order to cater to high-density networks, we proposed a convex optimization problem, and presented a method to efficiently solve it for obtaining a simpler, 2-competitive stationary randomized policy. Through numerical simulations, we illustrated the threshold structure of the CMDP based solution and studied variation of the average AoIs achieved by policies when the bounds on the average power and distortion constraints are varied.

## VI. Appendix

### A. *Proof of Theorem 1*

To prove the first part of the theorem, it is sufficient to prove $Q(a_i, \mathbf{a}_{-i}, \mathbf{h}, r_i, \mathbf{r}_{-i})$ is submodular in $(a_i, r_i)$, when $\mathbf{a}_{-i}$, $\mathbf{h}$ and $\mathbf{r}_{-i}$ are fixed [13], which we prove below. We replace all the fixed quantities with a dot $(\cdot)$ for brevity.

Consider

$$\begin{aligned}
&Q(a_i+1, r_i+1, \cdot) - Q(a_i+1, r_i, \cdot) \\
&= 1 + \lambda_i f_i(r_i+1; h_i) + \beta_i d_i(r_i+1) \\
&\quad + \gamma \mathbb{E}[V(1, \mathbf{a}'_{-i}, \mathbf{h}')|a_i+1, r_i+1, \cdot] \\
&\quad - ((a_i+1)(1-\mathbb{I}_{r_i>0}) + 1 + \lambda_i f_i(r_i; h_i) + \beta_i d_i(r_i) \\
&\quad + \gamma \mathbb{E}[V((a_i+1)(1-\mathbb{I}_{r_i>0}) + 1, \mathbf{a}'_{-i}, \mathbf{h}')|a_i+1, r_i, \cdot]) \\
&\overset{a}{\leq} 1 + \lambda_i f_i(r_i+1; h_i) + \beta_i d_i(r_i+1) \\
&\quad + \gamma \mathbb{E}[V(1, \mathbf{a}'_{-i}, \mathbf{h}')|a_i, r_i+1, \cdot] \\
&\quad - (a_i(1-\mathbb{I}_{r_i>0}) + 1 + \lambda_i f_i(r_i; h_i) + \beta_i d_i(r_i) \\
&\quad + \gamma \mathbb{E}[V(a_i(1-\mathbb{I}_{r_i>0}) + 1, \mathbf{a}'_{-i}, \mathbf{h}')|a_i, r_i, \cdot]) , \qquad (17)
\end{aligned}$$

where (a) holds true with equality when $r_i > 0$. However, when $r_i = 0$, (a) is true because $V(a, \cdot)$ is non-decreasing in $a$, which we prove in the below via induction. The instantaneous cost, $c_{\boldsymbol{\lambda},\boldsymbol{\beta}}(n) = \sum_{i=1}^{M} a_i(n+1) + \sum_{i=1}^{M} \lambda_i f_i(r_i(n); h_i(n)) + \sum_{i=1}^{M} \beta_i \mathbb{E}[d_i(r_i(n))]$ is non-decreasing function of $a_i$. Assume that $V_n(a_i(1-\mathbb{I}_{r_i>0})+1, \cdot)$ is non-decreasing in $a_i$. Note that

$$\begin{aligned}
&\mathbb{P}(a_i(1-\mathbb{I}_{r_i>0}) + 1, \mathbf{a}'_{-i}, \mathbf{h}'|a_i, \cdot, r_i, \mathbf{r}_{-i}) = \mathbb{P}(\mathbf{h}'|\mathbf{h}) \\
&= \mathbb{P}((a_i+1)(1-\mathbb{I}_{r_i>0}) + 1, \mathbf{a}'_{-i}, \mathbf{h}'|a_i+1, \cdot, r_i, \mathbf{r}_{-i}),
\end{aligned}$$

as the realization of $a_{i+1}$ is deterministic for a given action $r_i$. Since the positive weighted sum of non-decreasing functions is

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non-decreasing, $\mathbb{E}[V(a_i(1-\mathbb{I}_{r_i>0})+1, \mathbf{a}'_{-i}, \mathbf{h}')|a_i, r_i, \cdot]$ is non-decreasing in $a_i$. Now, since the minimum operator preserve the monotonicity, $V(a, \cdot)$ is non-decreasing in $a$. This proves the first part of the result.

Similarly, for proving the second part of the theorem, it is sufficient to prove that $Q(a_i, h_i, r_i, \cdot)$ is submodular in $(h_i, r_i)$. For some $\delta > 0$, consider

$$\begin{aligned}
&Q(a_i, h_i+\delta, r_i+1, \cdot) - Q(a_i, h_i, r_i, \cdot)\\
&= 1 + \lambda_i f_i(r_i+1; h_i+\delta) + \beta_i d_i(r_i+1)\\
&\quad + \gamma\mathbb{E}[V(1, \mathbf{a}'_{-i}, \mathbf{h}')|a_i, \mathbf{a}_{-i}, h_i+\delta, r_i+1, \cdot]\\
&\quad - (a_i(1-\mathbb{I}_{r_i>0}) + 1 + \lambda_i f_i(r_i; h_i+\delta) + \beta_i d_i(r_i)\\
&\quad + \gamma\mathbb{E}[V(a_i(1-\mathbb{I}_{r_i>0})+1, \mathbf{a}'_{-i}, \mathbf{h}')|a_i, \mathbf{a}_{-i}, h_i, r_i, \cdot])\\
&\overset{a}{\leq} 1 + \lambda_i f_i(r_i+1; h_i) + \beta_i d_i(r_i+1)\\
&\quad + \gamma\mathbb{E}[V(1, \mathbf{a}'_{-i}, \mathbf{h}')|a_i, \mathbf{a}_{-i}, h_i, r_i+1, \cdot]\\
&\quad - (a_i(1-\mathbb{I}_{r_i>0}) + 1 + \lambda_i f_i(r_i; h_i) + \beta_i d_i(r_i)\\
&\quad + \gamma\mathbb{E}[V(a_i(1-\mathbb{I}_{r_i>0})+1, \mathbf{a}'_{-i}, \mathbf{h}')|a_i, \mathbf{a}_{-i}, h_i, r_i, \cdot]),
\end{aligned}$$

where (a) is because $f_i(r_i+1; h_i+\delta) - f_i(r_i; h_i+\delta) \leq f_i(r_i+1; h_i) - f_i(r_i; h_i)$ due to the assumption of sub-modularity of the power function in $r$ and $h$. Moreover, since the channel realizations transition independently, $\mathbb{E}[V(a_i(1-\mathbb{I}_{r_i>0}) + 1, \mathbf{a}'_{-i}, \mathbf{h}')|a_i, \mathbf{a}_{-i}, x, r_i+1, \cdot]$ is identical for $x = h_i + \delta$ and $x = h_i$, for any $r_i \geq 0$. Hence the proof.

### *B. Proof of Theorem 2*

Consider the following optimization problem:

$$L_B = \min_{\pi} \lim_{N\to\infty} \frac{1}{2M} \sum_{i=1}^{M} w_i \left( \frac{1}{\frac{1}{N}\mathbb{E}\left[\sum_{n=1}^{N}\sum_{\rho=1}^{r_{\max}} u_{i,\rho}(n)\right]} + 1 \right), \tag{18}$$

subject to $(2), (3), (4)$,

where $\frac{1}{N}\mathbb{E}\left[\sum_{n=1}^{N}\sum_{\rho=1}^{r_{\max}} u_{i,\rho}(n)\right]$ gives the time-average of the expected number of successful updates over $N$ slots. Along the lines in [15], we can obtain that $L_B \leq A^*$. For proving the result, we construct a feasible policy to (13) from the optimal policy to (18). Let $\tilde{u}_{i,\rho}(n)$ be the optimal solution to (18). Define $\mathcal{N}_N(\mathbf{h}) = \{n : \mathbf{h}(n) = \mathbf{h}\}_{n=1}^{N}$ and

$$\tilde{\mu}_{i,\rho}(\mathbf{h}) = \lim_{N\to\infty} \frac{1}{|\mathcal{N}_N(\mathbf{h})|} \sum_{n\in\mathcal{N}_N(\mathbf{h})} \tilde{u}_{i,\rho}(n).$$

Consider

$$\begin{aligned}
&\mathbb{E}\left[\lim_{N\to\infty} \frac{1}{N}\sum_{n=1}^{N} f_i(\rho; h_i(n))\tilde{u}_{i,\rho}(n)\right]\\
&= \mathbb{E}\left[\lim_{N\to\infty} \frac{1}{N}\sum_{\mathbf{h}}\sum_{n\in\mathcal{N}_N(\mathbf{h})} f_i(\rho; h_i(n))\tilde{u}_{i,\rho}(n)\right]\\
&= \mathbb{E}\left[\lim_{N\to\infty} \sum_{\mathbf{h}} \left(\frac{|\mathcal{N}_N(\mathbf{h})|}{N}\right) \left(\frac{f_i(\rho; h_i)}{|\mathcal{N}_N(\mathbf{h})|}\sum_{n\in\mathcal{N}_N(\mathbf{h})} \tilde{u}_{i,\rho}(n)\right)\right]\\
&= \mathbb{E}\left[\sum_{\{\mathbf{h}\}} \mathbb{P}(\mathbf{h}) f_i(\rho; h_i)\tilde{\mu}_{i,\rho}(\mathbf{h})\right] = \mathbb{E}_{\mathbf{H}}\left[f_i(\rho; h_i)\tilde{\mu}_{i,\rho}(\mathbf{h})\right].
\end{aligned}$$

Hence,

$$\begin{aligned}
&\lim_{N\to\infty} \mathbb{E}\left[\frac{1}{N}\sum_{n=1}^{N}\sum_{\rho=1}^{r_{\max}} f_i(\rho; h)\tilde{u}_{i,\rho}(n)\right] \leq \bar{P}_i\\
&\implies \mathbb{E}_{\mathbf{H}}\left[\sum_{\rho=1}^{r_{\max}} f_i(\rho; h_i)\tilde{\mu}_{i,\rho}(\mathbf{h})\right] \leq \bar{P}_i.
\end{aligned}$$

Similarly, we can prove that (13c) and (13d) are satisfied. Now, comparing the objective functions of (18) and (13), we have, $A_R < 2L_B$. Now, noting that $L_B \leq A^* \leq A_R \leq A_R^*$, we get, $A_R^* < 2A^*$.

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