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ON THE MAXIMUM AND MINIMUM SIZES OF A GRAPH WITH GIVEN k-CONNECTIVITY

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Abstract

The concept of k-connectivity $\kappa_k(G)$, introduced by Chartrand in 1984, is a generalization of the cut-version of the classical connectivity. For an integer $k \geq 2$, the k-connectivity of a connected graph G with order $n \geq k$ is the smallest number of vertices whose removal from G produces a graph with at least k components or a graph with fewer than k vertices. In this paper, we get a sharp upper bound for the size of G with $\kappa_k(G) = t$, where $1 \leq t \leq n - k$ and $k \geq 3$; moreover, the unique extremal graph is given. Based on this result, we get the exact values for the maximum size, denoted by $g(n, k, t)$, of a connected graph G with order n and $\kappa_k(G) = t$. We also compute the exact values and bounds for another parameter $f(n, k, t)$ which is defined as the minimum size of a connected graph G with order n and $\kappa_k(G) = t$, where $1 \leq t \leq n - k$ and $k \geq 3$.

Keywords: k-connectivity, generalized connectivity, connectivity.

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