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Capacity analysis of wireless mesh networks with omni or directional antennas

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***Abstract*—In this paper we analyze the capacity of wireless mesh networks that use omni or directional antennas. The capacity in our analysis is the end-to-end per-node throughput. Our analysis is based on the assumption that there is only one gateway in the network and all end-users' traffics go through the gateway. Non-gateway nodes are uniformly distributed in a two-dimensional region centered at the gateway. The main results of our analysis can be summarized as: 1) The capacity is $O(1/N)$ for both omni and directional antennas, where N is number of nodes in the network. 2) the capacity is $O(\frac{\lg m}{\theta})$ for $m = 2$, and $O(\frac{\lg m}{\theta^2 \lg(1/\theta)})$ for $m > 2$, where m is the number of antennas on each node, and θ is the beamwidth of antennas.**[1]

I. Introduction

Wireless mesh networks receive much research interest in recent years [1]. A wireless mesh network [2] is a wireless network consists of a set of nodes, some of them connected to the Internet directly by wired links, called gateway nodes, and the rest are non-gateway nodes. The clients in wireless mesh networks are directly connected with the non-gateway nodes or gateway nodes by wireless links and access the Internet by the mesh backbone, which is formed by gateway nodes and non-gateway nodes via wireless links.

Most of the traffic of a wireless mesh network is to or from the gateway nodes, and the nodes do not have mobility in general. Thus, previous research results on the throughput of the wireless networks, such as [3], [4] cannot be applied to the wireless mesh networks directly. Also, as far as we know, there is no theoretic analysis on throughput of mesh networks. In recent years, the deployment of directional antennas in wireless networks is of increasing research interest, while some works on the performance of directional antennas in ad hoc networks can be found in [5]–[9]. Nevertheless, there is no performance analysis of directional antennas in wireless mesh networks yet. Therefore, the relationship between mesh networks' throughput with the number of nodes, the transmission distance, the number of antennas on each node and the beamwidth of antennas is unclear.

In this paper, we analyze the capacity, that is, the maximal per-node end-to-end throughput, of a single channel wireless mesh network. The antennas on each node can be omni-antennas or directional antennas. We show that the capacity is $O(1/N)$, where N is the number of non-gateway nodes. The capacity is an increasing function of the transmission distance. Directional antennas can achieve higher capacity compared with omni-antennas, and the capacity increases as the increase of the number of directional antennas on each node, and the decrease of the beamwidth of antennas.

The remainder of the paper is organized as follows. The related work on the capacity of the wireless networks is shown in section II. The framework of the analysis of capacity is presented in section III. The analysis of the capacity with omni-antennas is presented in section IV. The analysis of the capacity with directional antennas is presented in section V. We conclude the paper in section VI.

II. Related work

In our analysis, we use per-node capacity, which is the maximal per-node end-to-end throughput. For easy comparison, in the following related work, we have converted all analytical results of total capacity to per-node capacity and we simply use capacity to refer per-node capacity in the rest of the paper.

The capacity of wireless ad hoc networks was analyzed in [3], [4], [10]–[13]. The capacity of a random ad hoc network was shown to be $\Theta(\sqrt{1/(N \lg N)})$ in [3], where N is the number of nodes. Later in [4] and [12], it was found that the capacity of a delay tolerant network is possible to reach $\Theta(1)$ when nodes have mobility. When the network has multi-channels and multi-interfaces, it was shown in [14] that the capacity remains as $\Theta(\sqrt{1/(N \lg N})$, as long as the number of channels is $O(\lg N)$. The work in [15]–[19] analyzed how capacity was improved by adding K base stations into a network. The base stations have both wireless and wired connections. It was shown that capacity can be improved to $\Theta(1)$ by adding sufficient base stations [19], [15], or employing power control [17]. The work in [20] analyze the capacity of wireless mesh networks, assuming that all traffic goes through gateway nodes, and showed that the capacity is $O(1/N)$ through examples. In recent years, there are some works about the capacity analysis [5]–[9] and capacity improvement [21], [22] by using directional antennas. It was shown in [6] that directional antennas can gain $4\pi^2/(\alpha\beta)$ more capacity in random ad hoc networks, compared with omni-antennas, where α and β are the beamwidths of the transmitter and receiver.

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III. The framework of capacity analysis

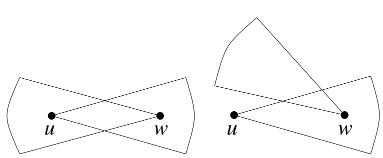
In this section, we present the network configuration of the mesh network we studied and the framework of the analysis of the per-node end-to-end capacity.

A. System model

The wireless mesh network under our analysis consists of one gateway node and N non-gateway nodes. The non-gateway node's traffic is destined to or originated from the gateway. Nodes are evenly distributed in a region of circle, where the gateway node is located in the center. The diameter of the circular region is D. Our analysis includes the cases of omni-antennas and directional antennas. For the case of directional antennas, we assume each node is equipped with m radios and each radio is associated with a directional antenna. Radios on the same node can be active at the same time. All antennas have the same beamwidth θ ($m\theta \leq 2\pi$) and are fixed after configuration. For the case of omni-antennas, we assume each node is equipped with a single radio. All nodes have fixed transmission distance r_t. We consider one channel in the system and communication links are bi-directional. Since each antenna corresponds to a radio on a node, we use antenna and radio interchangeably in the rest of the paper.

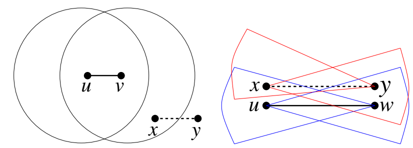
We start with some definitions of interference. The transmission area of an omni-antenna with transmission distance r_t is a disk with radius r_t, and its interference area is a disk with radius qr_t, where q is the ratio of interference distance to the transmission distance, and $q \geq 1$. q is assumed to be an integer in this paper. The transmission area of a directional antenna with beamwidth θ and transmission distance r_t is a sector with angle θ and radius r_t, and its interference area is a sector with angle θ and radius qr_t. For directional antennas, we assume both transmission and reception are directional, as discussed in paper [6]. Thus two directional antennas interfere with each other iff they are face to each other and inside each other's interference area, as shown by the examples in Fig. 1.

Now, we look at the interference between two communication links. Generally, the interference area of a link is the joint area covered by the two interference areas of the end-nodes of the link. For omni-antennas, link l_i interferes with link l_j if at least one end-node of l_j is inside l_i's interference area, as shown in Fig. 2(a). For directional antennas, link l_i is said to interfere with link l_j iff one end-antenna of l_i interferes with one end-antenna of l_j. As shown in the example in Fig. 2(b), link uv interferes with xy, because node v interferes with node x.



(a) u interferes with v. (b) u does not interfere with w.

Fig. 1. Directional reception/interference with directional antennas.



(a) Omni-antennas (b) Directional antennas

Fig. 2. Interference area of a link between node u and v.

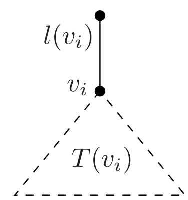


Fig. 3. Node v_i, its corresponding link l_{v_i}, and its subtree $T(v_i)$.

B. Performance metric

The capacity in the analysis of this paper is the per-node end-to-end capacity. We assume each node has the same end-to-end traffic demand, denoted by α. The capacity is defined as the maximal value of α that the system can support.

Since we consider only one gateway in the analysis and all end-users' traffic is destined to or originated from the gateway node, the topology of the network can be represented as a tree where the root of the tree is the gateway node. We assume the tree is the shortest path tree in the analysis. The end-users' traffic is merged at the tree nodes who further pass the traffic to their parent nodes towards the gateway. Let us denote $T(v_i)$ as the subtree rooted at node v_i, $l(v_i)$ as the link connecting subtree $T(v_i)$ to its parent node, and $L(v_i)$ as the load of link $l(v_i)$, as show in Fig. 3. Thus

$$L(v_i) = \sum_{v_j \in T(v_i)} \alpha = \alpha |T(v_i)|, \tag{1}$$

where $|T(v_i)|$ is the size of $T(v_i)$, which is the number of nodes in $T(v_i)$, including v_i itself.

We define the collision set of link $l(v_i)$ as a set of links interfering with link $l(v_i)$, including link $l(v_i)$ itself, and denote it as $I(l(v_i))$. We consider the most conservative case that no two links in the same collision set can be active at the same time in order to guarantee successful transmissions.

Therefore

$$\sum_{l(v_j) \in I(l(v_i))} L(v_j) = \alpha \sum_{l(v_j) \in I(l(v_i))} |T(v_j)| \leq C, \tag{2}$$

where C is the channel bandwidth.

From ineq. (2), we have

$$\alpha \leq \frac{C}{\displaystyle\sum_{l(v_j) \in I(l(v_i))} |T(v_j)|}. \tag{3}$$

The capacity is the maximal possible value of α that meets

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ineq. (3) for all node v_i. That is,

$$Cap = \min_{v_i} \frac{C}{\sum_{l(v_j) \in I(l(v_i))} |T(v_j)|} = \frac{C}{\max_{v_i} \sum_{l(v_j) \in I(l(v_i))} |T(v_j)|}. \quad (4)$$

Let

$$L_{I(l(v_i))} = \sum_{l(v_j) \in I(l(v_i))} |T(v_j)|. \quad (5)$$

We call $L_{I(l(v_i))}$ the (normalized) collision load of link $l(v_i)$.

Then eq. (4) becomes

$$Cap = \min_{v_i} \frac{C}{L_{I(l(v_i))}} = \frac{C}{\max_{v_i} L_{I(l(v_i))}}. \quad (6)$$

In the next two sections, we first analyze the network capacity with omni-antennas. Then, we extend our work to the case of directional antennas.

IV. Analysis of wireless mesh networks of omni-directional antennas

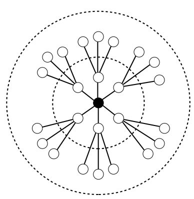

Fig. 4. Examples of the topology of 2-dimensional region deployment

All N non-gateway nodes are assumed to be evenly distributed in a region of circle of diameter D and the gateway node is located in the center of the circle. Let $k = \lceil D/(2r_t) \rceil|$, which represents the minimal hops from the furthest node in the network to the gateway node. We divide the region into rings by circles centered at the gateway and with radius from r_t, $2r_t$, $3r_t$,..., to kr_t. We denote ring i as the area between circles with radius $(i-1)r_t$ and ir_t, $i \in [1, k]$. To be consistent, ring 0 contains only the gateway node. Let R_i denote the set of nodes in ring i. Since nodes are evenly distributed, the number of nodes in ring i, except the ring 0, is

$$\frac{\pi(i^2 - (i-1)^2)}{\pi k^2} N = N(2i-1)/k^2. \quad (7)$$

To minimize the end-to-end hops, we assume that the links always connects nodes in the adjacent rings, as shown in Fig. 4. The subtrees rooted from nodes in R_i contains all the nodes outside ring i (including R_i itself). Thus, we have for $i \geq 1$

$$\sum_{v \in R_i} |T(v)| = \sum_{j=i}^{j=k} N(2j-1)/k^2 = N(1 - \frac{(i-1)^2}{k^2}). \quad (8)$$

The number of links in the collision set of link $l(v)$ remains as a constant when the distance between two end-nodes is fixed, since nodes are evenly distributed. Also load of links increases when links get closer to the gateway, due to more relayed traffic. Thus, the collision load reaches the maximum for a link that is directly connected to the gateway. Let $l(v)$ be such a link. The interference area of $l(v)$ is the joint area of two circles with radius qr_t and centered at the two end-nodes of $l(v)$. The interference area of $l(v)$ is larger than the circle with radius qr_t and smaller than the circle with radius $(q+1)r_t$ all centered at the gateway. Thus, we use the circle of radius qr_t centered at the gateway as a close lower bound to approximate the interference area of $l(v)$. The collision set of $l(v)$ contains all links between rings $i-1$ and i, for $i = 1, \ldots, \min(q+1, k)$. That is

$$I(l(v)) \supseteq \{l(u) : u \in R_i, i \in [1, \min(q+1, k)]\}. \quad (9)$$

Therefore, we have

$$\max_v L_{I(l(v))} \geq \sum_{i=1}^{\min(k,q+1)} \sum_{v \in R_i} |T(v)| \quad (10)$$

$$\geq \sum_{i=1}^{\min(k,q+1)} N(1 - \frac{(i-1)^2}{k^2}.) \quad (11)$$

Therefore according to eq. (6), we have

$$Cap \leq \begin{cases} \frac{C}{kN - N\sum_{i=1}^{i=k-1} i^2/k^2} & \text{if } k \leq q+1 \\ \frac{C}{(q+1)N - N\sum_{i=1}^{i=q} i^2/k^2} & \text{otherwise.} \end{cases} \quad (12)$$

From (12) we have the following observations:

a) When $k \leq q+1$, the capacity is independent from q, because the link of the maximal collision load interferes all links in the network.

b) The capacity is $O(1/N)$, and decreases as the increase of k.

c) When k = 1, the network has the highest capacity, which is C/N; when $\lim_{k \to \infty} \frac{C}{(q+1)N - N\sum_{i=1}^{i=q} i^2/k^2} = C/((q+1)N)$. Therefore, the capacity is in the range of $[C/((q+1)N), C/N]$.

V. Analysis of wireless mesh networks of directional antennas

In the analysis in this section, each node is equipped with m directional antennas of beamwidth θ, and $m \geq 2$. All the other network parameters are the same as the section for omni antennas.

Similar to the analysis for omni-antennas, the region is divided by circles centered at the gateway and with radius from r_t, $2r_t$, $3r_t$,..., to kr_t. Now we look at the maximum collision load of links in the system. Different from the case of omni-antennas, the link with the maximum collision load may not appear on the links that are directly connected to the gateway. This is because the collision set of a link depends on the orientation of antennas of this link and the orientation of other nearby links. Nevertheless, since load of links increases as they get closer to the gateway, the collision load of a link that directly connects to the gateway is a close approximation of the maximum collision load. Let $u \in R_1$ be a node that directly connects to the gateway and $l(u)$ is the link connecting u to the gateway. The interference area of link $l(u)$ is fully covered by a circle from the gateway with radius qr_t. To compute the collision load of $l(u)$, we use the average size of subtrees whose parent links have at least one end-node in this circle as an estimation. Let $L_q = \sum_{i=1}^{\min(k,q+1)} \sum_{v \in R_i} |T(v)|$, which is the total size of all the subtrees whose parent links have at least one end-node falling into the circle with radius $\min(k,q)$ and centered at the gateway. L_q is actually the total load of the links that have at least one end-node in the circle. The total load of the links that have at least one end-node in the interference area of $l(u)$ is $L_q\phi$, where ϕ is the ratio between the size of the interference area of $l(u)$ and the size of this circle. According to the link interference definition of antennas, not all the links that have an end-node in the interference area of $l(u)$ interfere with $l(u)$. Let ρ_0 be the probability that a link that has an end-node in the interference area of $l(u)$ interferes with $l(u)$. The average total-load of links in the collision set of $l(u)$ is

$$L_q \phi \rho_0, \tag{13}$$

We use this average collision load as a close lower bound of the maximum collision load in the system. That is

$$\max_v L_{I(l(v))} \geq L_q \phi \rho_0. \tag{14}$$

On the other hand, the gateway has N nodes in its tree and it has m antennas for use. Thus, we have

$$\max_v L_{I(l(v))} \geq N/m. \tag{15}$$

In the following, we will show how to obtain L_q, ϕ and ρ_0.

First, we consider computing L_q. We divide the region into two parts, the area inside ring 1 and the area outside ring 1. Let L_{R_1} denote the total size of all the subtrees whose roots are inside ring 1, and $L_{R_{2*}}$ the total size of all the subtrees whose roots are outside of ring 1. That is:

$$L_{R_1} = \sum_{v \in R_1} |T(v)|, \tag{16}$$

$$L_{R_{2*}} = \sum_{i=2}^{\min(q+1,k)} \sum_{v \in R_i} |T(v)|. \tag{17}$$

We have

$$L_q = L_{R_1} + L_{R_{2*}}. \tag{18}$$

We first calculate L_{R_1} as follows. Let h denote the number of hops necessary to cover all nodes in ring 1. At least $\frac{\theta}{2\pi}\frac{mN}{k^2}$ nodes in ring 1 can be covered by the m antennas from the gateway. For those nodes that are directly connected to the gateway, each of them uses 1 antenna for the uplink to the gateway, and $m-1$ antennas to link children. They can establish links to cover at least $\frac{\theta}{2\pi}\frac{mN}{k^2}(m-1)$ child nodes, which can further link $\frac{\theta}{2\pi}\frac{mN}{k^2}(m-1)^2$ child nodes. We assume that all the link child nodes are still ring 1 nodes before they are all linked to the tree. Repeat the above operation, let H_i denote the set of nodes that are i hops to the gateway, then we have

$$|H_i| = \frac{Nm\theta}{2\pi k^2}(m-1)^{i-1}. \tag{19}$$

Since

$$\sum_{i=1}^{i=h-1} |H_i| \leq |R_1| \leq \sum_{i=1}^{i=h} |H_i|, \tag{20}$$

we have

$$\frac{Nm\theta}{2\pi k^2}\sum_{i=0}^{h-2}(m-1)^i \leq N/k^2 \leq \frac{Nm\theta}{2\pi k^2}\sum_{i=0}^{h-1}(m-1)^i. \tag{21}$$

Thus, we have

$$\begin{cases} \frac{\lg(\frac{2\pi(m-2)}{\theta m}+1)}{\lg(m-1)} \leq h \leq \frac{\lg(\frac{2\pi(m-2)}{\theta m}+1)}{\lg(m-1)} + 1 & \text{if } m > 2 \\ \pi/\theta \leq h \leq \pi/\theta + 1 & \text{if } m = 2. \end{cases} \tag{22}$$

.

Since the total number of nodes in the subtrees whose roots are in H_i plus the number of the nodes that are less than i hops to the gateway is N, we have, for each $i \leq h$,

$$\sum_{v \in H_i} |T(v)| = N - \sum_{j=1}^{j=i-1} |H_j| = N - \frac{N\theta m}{2\pi k^2}\sum_{j=0}^{i-2}(m-1)^j. \tag{23}$$

Summing up the size of the subtrees whose roots are in H_i for all $1 \leq i \leq h$, we have

$$\begin{aligned} L_{R_1} &= \sum_{i=1}^{i=h}\sum_{v \in H_i} |T(v)| \\ &= \begin{cases} Nh - N\frac{1}{(m-2)k^2}(1 - \frac{m\theta h}{2\pi}) & \text{if } m > 2 \\ Nh - Nh(h-1)\frac{\theta}{2\pi k^2} & \text{if } m = 2. \end{cases} \end{aligned} \tag{24}$$

Next we calculate $L_{R_{2*}}$. $L_{R_{2*}}$ reaches its minimum when all nodes in ring $i+1$ are directly connected to the nodes in ring i, for all $i > 1$, due to the least relayed load. With the similar technique as in section IV, we have

$$L_{R_{2*}} \geq \begin{cases} N\sum_{i=1}^{i=k-1}(1 - i^2/k^2) & \text{if } k \leq q+1 \\ N\sum_{i=1}^{i=q}(1 - i^2/k^2) & \text{otherwise.} \end{cases} \tag{25}$$

We calculate ϕ as follows. Let a and b are the two end-nodes of a link, and A and B are the two sectors centered at a and b, respectively, with angle θ and radius qr_t, as shown in Fig. 5. Since the distance between a and b is less than r_t, we can always find a sector of angle θ and radius $(q-1)r_t$, inside sector A and outside sector B. Let us denote this sector as sector as A'. The size of the joint area of sector A and B is at least

$$\frac{\theta q}{2} r_t^2 + \frac{\theta(q-1)}{2} r_t^2. \tag{26}$$

Therefore, we have

$$\phi \geq \frac{2q-1}{2q}\frac{\theta}{\pi}. \tag{27}$$

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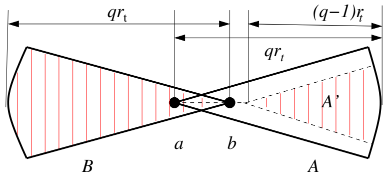

Fig. 5. The joint area of two sectors A and B of angle θ and radius q. $||ab|| \leq qr_t$.

We calculate ρ_0 as follows. Let $P_I(u,v)$ be the probability that a node u is in the interference sector of an antenna at a node v with beamwidth θ, $||uv|| \leq qr_t$. We have

$$P_I(u,v) = \theta/(2\pi). \tag{28}$$

Suppose that link $l = (s,t)$ has at least one end-node, say node s, inside the interference area of link $l(u) = (u,v)$, $||su|| \leq qr_t$ and $||sv|| \leq qr_t$. The probability ρ_0 that link l interferes with link $l(u)$ is at least the probability that u or v falling into the interference sector of s. Thus, we have

$$\rho_0 \geq 1-(1-P_I(u,s))(1-P_I(v,s)) = \theta/\pi-(\theta/(2\pi))^2. \tag{29}$$

Substitute L_q, ϕ, ρ_0 into eq. (14) from eq. (18),(27),(29) and consider eq. (15), the capacity according to (6) is

$$Cap \leq \frac{C}{\max(N/m, \frac{2q-1}{2q}((\frac{\theta}{\pi})^2 - (\frac{1}{4}(\frac{\theta}{\pi}))^3)(L_{R_1} + L_{R_{2*}}))}. \tag{30}$$

From (30) we have the following observations:

a) The capacity with directional antennas depends on q.

b) The directional antennas achieve higher capacity than omni-antennas, and this capacity increases as the increase of number of antennas m, and the decrease of beamwidth θ. The capacity is $O(\frac{\lg m}{\theta})$ for $m = 2$, and $O(\frac{\lg m}{\theta^2 \lg(1/\theta)})$ for $m > 2$.

However, the capacity does not increase to infinity with the decrease of θ. According to eq. (15), it is upper bounded by

$$Cm/N. \tag{31}$$

c) When $k \rightarrow \infty$, the ratio between the capacity of directional antennas and omni-antennas is upper bounded by

$$\frac{q+1}{\max(1/m, \frac{2q-1}{2q}(q+h)\frac{\theta}{\pi}(\frac{\theta}{\pi} - (\frac{\theta}{2\pi})^2))}. \tag{32}$$

d) When θ is sufficiently small, the capacity is upper bounded by Cm/N. Recall that the nodes in the ring 1 is N/k^2, and an antenna will not be used when it does not cover any node in its transmission sector. Thus the number of used antennas on the gateway is at most N/k^2, and the capacity is upper bounded by C/k^2. This means that, when $m = \Theta(N/k^2)$ and θ is sufficiently small, the mesh network is possible to achieve a capacity of $O(C/k^2)$.

VI. Conclusion

We presented a model to analyze the per-node end-end capacity of the single channel wireless mesh networks. We analyzed the case that nodes are uniformly distributed around one gateway. The capacity is shown to be $O(1/N)$, where N is the number of non-gateway nodes. The capacity can be improved by using directional antennas. The capacity is $O(\frac{\lg m}{\theta})$ for $m = 2$, and $O(\frac{\lg m}{\theta^2 \lg(1/\theta)})$ for $m > 2$, where m is the number of antennas on each node, and θ is the beamwidth of antennas. However, the capacity is bounded by Cm/N, no matter how small the beamwidth can be.

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