Leveraging Coherent Distributed Space-Time Codes for Noncoherent Communication in Relay Networks via Training

G. Susinder Rajan and B. Sundar Rajan

Abstract

For point to point multiple input multiple output systems, Dayal-Brehler-Varanasi have proved that training codes achieve the same diversity order as that of the underlying coherent space time block code (STBC) if a simple minimum mean squared error estimate of the channel formed using the training part is employed for coherent detection of the underlying STBC. In this letter, a similar strategy involving a combination of training, channel estimation and detection in conjunction with existing coherent distributed STBCs is proposed for noncoherent communication in AF relay networks. Simulation results show that the proposed simple strategy outperforms distributed differential space-time coding for AF relay networks. Finally, the proposed strategy is extended to asynchronous relay networks using orthogonal frequency division multiplexing.

Index Terms

Cooperative diversity, distributed STBC, noncoherent communication, training.

I. Introduction

Recently the idea of space time coding has been applied in wireless relay networks in the name of distributed space time coding to extract similar benefit as in point to point multiple input multiple output (MIMO) systems. Mainly there are two types of distributed space time coding techniques discussed in the literature: (i) decode and forward (DF) based distributed space time coding [1], wherein a subset (chosen based on some criteria) of the relay nodes decode the symbols from the source and transmit a column of a distributed

G. Susinder Rajan and B. Sundar Rajan are with the Department of Electrical Communication Engineering, Indian Institute of Science, Bangalore-560012, India. Email:{susinder,bsrajan}@ece.iisc.ernet.in.

space time block code (STBC) and (ii) amplify and forward (AF) based distributed space time coding [2], where all the relay nodes perform linear processing on the received symbols according to a distributed space time block code (DSTBC) and transmit the resulting symbols to the destination. AF based distributed space time coding is of special interest because the operations at the relay nodes are greatly simplified and moreover there is no need for every relay node to inform the destination once every quasi-static duration whether it will be participating in the distributed space time coding process as is the case in DF based distributed space time coding [1]. However, in [2], the destination was assumed to have perfect knowledge of all the channel fading gains from the source to the relays and those from the relays to the destination. To overcome the need for channel knowledge, distributed differential space time coding was studied in [3], [4], [5], [6], which is essentially an extension of differential unitary space time coding for point to point MIMO systems to the relay network case. But distributed differential space time block code (DDSTBC) design is difficult compared to coherent DSTBC design because of the extra stringent conditions (we refer readers to [4], [6] for exact conditions) that need to be met by the codes. Moreover, all the codes in [3], [4], [5] for more than two relays have exponential encoding complexity. On the other hand, coherent DSTBCs with reduced maximum likelihood (ML) decoding complexity are available in [8], [10], [13].

Interestingly in [9], it was proved that for point to point MIMO systems, training codes[1] achieve the same diversity order as that of the underlying coherent STBC if a minimum mean squared error (MMSE) estimate of the channel formed using the training part of the code is employed as if it were error free for coherent detection of the underlying STBC. Also, it was shown that training codes have an error rate comparable to the best performing differential unitary STBCs. The contributions of this letter are summarized as follows.

- Motivated by the results of [9], a similar training and channel estimation scheme is proposed to be used in conjunction with coherent distributed space time coding in AF relay networks as described in [2]. An interesting feature of the proposed training scheme is that the relay nodes do not perform any channel estimation using the training symbols transmitted by the source but instead simply amplify and forward the received training symbols. The proposed strategy is shown to outperform the best known DDSTBCs [3],

[1]Each codeword of a training code consists of a part known to the receiver (pilot) and a part that contains codeword(s) of a STBC designed for the coherent channel (in which receiver has perfect knowledge of the channel)

[4], [5], [6] using simulations. Also, it is shown that appropriate power allocation among the training and data symbols can further improve the error performance marginally.

- Finally, this training based strategy is extended to asynchronous relay networks with no knowledge of the timing errors using the recently proposed Orthogonal Frequency Division Multiplexing (OFDM) based distributed space time coding [7].

The rest of this letter is organized as follows. The proposed training scheme along with channel estimation is described in Section II. Extension to the asynchronous relay network case is addressed in Section III. Simulation results comprise Section IV and conclusions are presented in Section V.

Notation: A complex Gaussian vector with zero mean and covariance matrix Ω will be denoted by $\mathcal{CN}(0, \Omega)$.

II. Proposed Training Based Strategy

In this section, we briefly review the distributed space time coding protocol for AF relay networks in [2], make some crucial observations and then proceed to describe the proposed training based strategy.

A. *Observations from Coherent Distributed Space Time Coding*

Consider a wireless relay network consisting of a source node, a destination node and R relay nodes $U_1, U_2, \ldots, U_R$ which aid the source in communicating information to the destination. All the nodes are assumed to be equipped with a half duplex constrained, single antenna transceiver. The wireless channels between the terminals are assumed to be quasi-static and flat fading. The channel fading gains from the source to the i-th relay, f_i and those from the j-th relay to the destination g_j are all assumed to be independent and identically distributed complex Gaussian random variables with zero mean and unit variance. Symbol synchronization and carrier frequency synchronization are assumed among all the nodes. Moreover, the destination is assumed to have perfect knowledge of all the channel fading gains $f_i, g_i, \;\; i = 1, \ldots, R$.

Every transmission cycle from the source to the destination is comprised of two phases. In the first phase, the source transmits a vector $\mathbf{z} = \begin{bmatrix} z_1 & z_2 & \ldots & z_{T_1} \end{bmatrix}^T$ composed of T_1 complex symbols $z_i, \;\; i = 1, \ldots, T_1$ to all the R relays using a fraction π_1 of the total power P_d for data transmission. The vector $\mathbf{z}$ satisfies $\mathrm{E}[\mathbf{z}^H \mathbf{z}] = T_1$ and P_d denotes the total average power spent by the source and the relays for communicating data to the destination. The received vector at the i-th relay is then given by $r_i = \sqrt{\pi_1 P_d} f_i \mathbf{z} + \mathbf{v_i}$ where, $\mathbf{v_i} \sim \mathcal{CN}(0, I_{T_1})$ represents the additive noise at the i-th relay.

In the second phase, the i-th relay transmits $\mathbf{t_i} = \sqrt{\frac{\pi_2 P_d}{\pi_1 P_d + 1}} \mathbf{B_i r_i}$ or $\mathbf{t_i} = \sqrt{\frac{\pi_2 P_d}{\pi_1 P_d + 1}} \mathbf{B_i r_i}^*$ to the destination, where $\mathbf{B_i} \in \mathbb{C}^{T_2 \times T_1}$ is called the relay matrix. Without loss of generality we may assume that the first M relays linearly process $\mathbf{r_i}$ and the remaining $R-M$ relays linearly process $\mathbf{r_i}^*$. Under the assumption that the quasi-static duration of the channel is much greater than $2R$ channel uses, the received vector at the destination can be expressed as $\mathbf{y} = \sum_{j=1}^{R} g_j \mathbf{t_j} + \mathbf{w} = \sqrt{\frac{\pi_1 \pi_2 P_d^2}{\pi_1 P_d + 1}} \mathbf{Xh} + \mathbf{n}$ where, $\mathbf{X} = \begin{bmatrix} \mathbf{B_1 z} & \ldots & \mathbf{B_M z} & \mathbf{B_{M+1} z}^* & \ldots & \mathbf{B_R z}^* \end{bmatrix}$,

$$\mathbf{h} = \begin{bmatrix} f_1 g_1 & f_2 g_2 & \ldots f_M g_M & f_{M+1}^* g_{M+1} & \ldots & f_R^* g_R \end{bmatrix}^T, \tag{1}$$

$\mathbf{n} = \sqrt{\frac{\pi_2 P_d}{\pi_1 P_d + 1}} \left(\sum_{j=1}^{M} g_j \mathbf{B_j v_j} + \sum_{j=M+1}^{R} g_j \mathbf{B_j v_j}^* \right) + w$ and $\mathbf{w} \sim \mathcal{CN}(0, I_{T_2})$ represents the additive noise at the destination. The power allocation factors π_1 and π_2 are chosen to satisfy $\pi_1 P_d + \pi_2 P_d R = 2P_d$. The covariance matrix of $\mathbf{n}$ is given by $\mathbf{\Gamma} = \mathrm{E}[\mathbf{nn}^H] = \mathbf{I_{T_2}} + \frac{\pi_2 P_d}{\pi_1 P_d + 1} (\sum_{i=1}^{R} |g_i|^2 \mathbf{B_i B_i}^H)$. Let the DSTBC $\mathscr{C}$ denote the set of all possible codeword matrices $\mathbf{X}$. Then the ML decoder is given by

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X} \in \mathscr{C}} \parallel \mathbf{\Gamma}^{-\frac{1}{2}} (\mathbf{y} - \sqrt{\frac{\pi_1 \pi_2 P_d^2}{\pi_1 P_d + 1}} \mathbf{Xh}) \parallel_F^2 . \tag{2}$$

Note from (2) that the ML decoder in general requires the knowledge[2] of all the channel fading gains $f_i, g_i, \ i = 1, \ldots, R$. Consider the following decoder:

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X} \in \mathscr{C}} \parallel \mathbf{y} - \sqrt{\frac{\pi_1 \pi_2 P_d^2}{\pi_1 P_d + 1}} \mathbf{Xh} \parallel_F^2 . \tag{3}$$

Remark 1: The decoder in (3) is suboptimal in general and coincides with the ML decoder for the case when Γ is a scaled identity matrix. The relay matrices for all the codes in [2], [10], [12], [13] and some of the codes in [8] are unitary. For the case when $\mathbf{B_i B_i}^H$ is a diagonal matrix for all $i = 1, 2, \ldots, R$ (Γ is a diagonal matrix for this case), the performance of the suboptimal decoder in (3) differs from that of the ML decoder (2) only by coding gain and the diversity gain is retained. This can be proved on similar lines as in the proof of Theorem 7 in [8]. The class of DSTBCs from precoded co-ordinate interleaved orthogonal designs in [8] is an example for the case of diagonal Γ matrix.

[2] Γ requires knowledge of the g_i's and $\mathbf{h}$ requires knowledge of $f_i g_i, \ i = 1, \ldots, M$ and $f_i^* g_i, \ i = M+1, \ldots, R$ which together imply knowledge of $f_i, g_i, \ i = 1, \ldots, R$.

The decoder in (3) requires only the knowledge of $\mathbf{h}$ and not necessarily the knowledge of all the individual channel fading gains $f_i, g_i, \ i = 1, 2, \ldots, R$. The training strategy to be described in the sequel essentially **exploits this crucial observation**.

B. Training cycle

Note from the previous subsection that one data transmission cycle comprises of $2R$ channel uses. In the proposed training strategy, we introduce a training cycle comprising of $R+1$ channel uses for channel estimation before the start of data transmission cycle. We assume that the quasi-static duration of the channel is greater than $(R+1)+F(2R)$ channel uses where F denotes the total number of data transmission cycles that can be accommodated within the channel quasi-static duration. Thus, the minimum channel quasi-static duration required for the proposed strategy is $3R+1$ channel uses. Let P_t be the total average power spent by the source and the relays during the training cycle. Thus, the total average power P used by the source and the relays is $P = \frac{P_t(R+1)+P_d(F2R)}{R(2F+1)+1}$.

In the first phase of the training cycle, the source transmits the complex number 1 to all the relays using a fraction π_1 of the total power P_t dedicated for training. The received symbol at the i-th relay denoted by $\mathbf{r_i^t}$ is given by $\mathbf{r_i^t} = \sqrt{\pi_1 P_t} f_i + n_i$ where $n_i \sim \mathcal{CN}(0,1)$ is the additive noise at the i-th relay.

The second phase of the training cycle comprises of R channel uses, out of which one channel use is assigned to every relay node. Without loss of generality, we may assume that the i-th time slot is assigned to the i-th relay. Furthermore, we assume that the value of M to be used during the data transmission cycle is already decided. During its assigned time slot, the i-th relay transmits $t_i^t = \begin{cases} \sqrt{\frac{\pi_2 P_t R}{\pi_1 P_t+1}}\mathbf{r_i^t} = \sqrt{\frac{\pi_1\pi_2 P_t^2 R}{\pi_1 P_t+1}} f_i + \sqrt{\frac{\pi_2 P_t R}{\pi_1 P_t+1}} n_i, & \text{if } i \leq M \\ \sqrt{\frac{\pi_2 P_t R}{\pi_1 P_t+1}}\mathbf{r_i^{t^*}} = \sqrt{\frac{\pi_1\pi_2 P_t^2 R}{\pi_1 P_t+1}} f_i^* + \sqrt{\frac{\pi_2 P_t R}{\pi_1 P_t+1}} n_i^*, & \text{if } i > M \end{cases}$.
At the end of the training cycle, the received vector y_t at the destination is given as follows:

$$\mathbf{y_t} = \sqrt{\frac{\pi_1\pi_2 P_t^2 R}{\pi_1 P_t + 1}}\mathbf{I_R}\mathbf{h} + \mathbf{n_t} \tag{4}$$

where $\mathbf{n_t} = \sqrt{\frac{\pi_2 P_t R}{\pi_1 P_t+1}} \begin{bmatrix} g_1 n_1 & \ldots & g_M n_M & g_{M+1} n_{M+1}^* & \ldots & g_R n_R^* \end{bmatrix}^T + \mathbf{w_t}$, $\mathbf{h}$ is same as that given in (1) and $\mathbf{w_t} \sim \mathcal{CN}(0, \mathbf{I_R})$ is the additive noise at the destination. The entire transmission from source to destination is illustrated pictorially in Fig. 1 and Fig. 2.

Treating the entries of $\mathbf{h}$ as independent, identically distributed (i.i.d) complex Gaussian random variables and also $\mathbf{n_t}$ as complex Gaussian with mean 0 and covariance $(\frac{\pi_2 P_t R}{\pi_1 P_t+1}+1)\mathbf{I_R}$, we propose to estimate the equivalent channel matrix $\mathbf{h}$ (similar to point to point MIMO case [9]) as follows:

$$\hat{\mathbf{h}} = \sqrt{\frac{\pi_1\pi_2 P_t^2 R}{\pi_1 P_t + 1}} \left(\frac{\pi_2 P_t R + \pi_1\pi_2 P_t^2 R}{\pi_1 P_t + 1} + 1 \right)^{-1} \mathbf{y_t} \tag{5}$$

Now using the estimate $\hat{\mathbf{h}}$, coherent DSTBC decoding can be done in every data transmission cycle, as $\hat{\mathbf{X}} = \arg\min_{\mathbf{X}\in\mathscr{C}} \parallel \mathbf{y} - \sqrt{\frac{\pi_1\pi_2 P_d^2}{\pi_1 P_d+1}}\mathbf{X}\hat{\mathbf{h}} \parallel_F^2$. Thus, coherent DSTBCs [8], [10], [11], [12], [13] can be employed in non-coherent relay networks via the proposed training scheme. We would like to mention that there may be better channel estimation techniques than the one described by (5) but this is beyond the scope of this letter. But the simulation results in section V show that a simple channel estimator as in (5) is good enough to outperform the best known DDSTBCs.

III. Training strategy for asynchronous relay networks

The training strategy described in the previous section assumes that the transmissions from all the relays are symbol synchronous with reference to the destination. In this section, we relax this assumption and extend the proposed training strategy to asynchronous relay networks with no knowledge of the timing errors of the relay transmissions. However we shall assume that the maximum of the relative timing errors from the source to the destination is known.

An asynchronous wireless relay network is depicted in Fig. 4. Let τ_i denote the overall relative timing error of the signals arrived at the destination node from the i-th relay node. Without loss of generality, we assume that $\tau_1 = 0$, $\tau_{i+1} \geq \tau_i, i = 1, \ldots, R-1$. Perfect carrier synchronization is assumed among all the nodes. This scheme relies on the recently proposed OFDM based distributed space time coding in [7], [8] which is essentially distributed space time coding over OFDM symbols and the cyclic prefix (CP) of OFDM is used to mitigate the effects of symbol asynchronism. The number of sub-carriers N and the length of the cyclic prefix (CP) l_{cp} are chosen such that $l_{cp} \geq \max_{i=1,2,\ldots,R}\{\tau_i\}$. The channel quasi-static duration assumed for this strategy is $((R+1) + F(2R))\,(N + l_{cp})$ channel uses. For the sake of brevity, we shall only outline the main idea here and refer the readers to [7], Section IV of [8] for a detailed description.

As for the synchronous case, there will be a training cycle before the start of data transmission from the source. In the first phase of the training cycle, the source takes the N point inverse discrete Fourier transform (IDFT) of the N length vector $\mathbf{p} = \begin{bmatrix} 1 & 1 & \ldots & 1 \end{bmatrix}^T$

and adds a CP of length l_{cp} to form a OFDM symbol $\bar{\mathbf{p}}$. This OFDM symbol is transmitted to the relays using a fraction π_1 of the total power P_t. The i-th relay receives $\mathbf{r_i^t} = \sqrt{\pi_1 P_t}\bar{\mathbf{p}} + \bar{\mathbf{n}}_\mathbf{i}^\mathbf{t}$ where $\bar{\mathbf{n}}_\mathbf{i}^\mathbf{t} \sim \mathcal{CN}(0, I_N)$ is the additive noise at the i-th relay. During the second phase of the training cycle, each relay is allotted a unique OFDM time slot during which only that relay transmits. Let us assume that the i-th relay is allotted the i-th OFDM time slot. Thus, the second phase comprises of R OFDM time slots. Similar to the synchronous case, let us assume that the first M relays linearly process the received vector and the remaining $R-M$ relays linearly process the conjugate of the received vector. During its scheduled time slot, the i-th relay transmits $\mathbf{t_i^t} = \begin{cases} \sqrt{\frac{\pi_2 R P_t}{\pi_1 P_t+1}}\mathbf{r_i^t}, & \text{if } i \leq M \\ \sqrt{\frac{\pi_2 R P_t}{\pi_1 P_t+1}}\zeta\left((\mathbf{r_i^t})^*\right), & \text{if } i > M \end{cases}$ where $\zeta(.)$ denotes the time reversal operation, i.e., $\zeta(\mathbf{r}(n)) \triangleq \mathbf{r}(N + l_{cp} - n)$. The destination receives R OFDM symbols which are processed as follows:

1) Remove the CP for the first M OFDM symbols.
2) For the remaining OFDM symbols, remove CP to get a N-length vector. Then shift the last l_{cp} samples of the N-length vector as the first l_{cp} samples.

Discrete Fourier transform (DFT) is then applied on the resulting R vectors to obtain $\mathbf{y_j^t} = \begin{bmatrix} y_{0,j}^t & y_{1,j}^t & \ldots & y_{N-1,j}^t \end{bmatrix}^T, j = 1, 2, \ldots, R$. Let $\mathbf{w_j^t} = \begin{bmatrix} w_{0,j}^t & w_{1,j}^t & \ldots & w_{N-1,j}^t \end{bmatrix}^T$ represent the additive noise at the destination node in the j-th OFDM time slot and let $\mathbf{n_j^t} = \begin{bmatrix} n_{0,j}^t & n_{1,j}^t & \ldots & n_{N-1,j}^t \end{bmatrix}^T$ denote the DFT of $\bar{\mathbf{n}}_\mathbf{j}^\mathbf{t}$ after CP removal. Note that a delay τ in the time domain translates to a corresponding phase change of $e^{-\frac{i2\pi k\tau}{N}}$ in the k-th sub carrier. Now using the identities $(\mathrm{DFT}(\mathbf{x}))^* = \mathrm{IDFT}(\mathbf{x}^*)$, $(\mathrm{IDFT}(\mathbf{x}))^* = \mathrm{DFT}(\mathbf{x}^*)$, $\mathrm{DFT}(\zeta(\mathrm{DFT}(\mathbf{x}))) = \mathbf{x}$, $\mathbf{p}^* = \mathbf{p}$ we have in the j-th OFDM time slot

$$\mathbf{y_j^t} = \begin{cases} f_j g_j \sqrt{\frac{\pi_1\pi_2 R P_t^2}{\pi_1 P_t+1}}\mathbf{p} \circ \mathbf{d}^{\tau_\mathbf{j}} + \sqrt{\frac{\pi_2 R P_t}{\pi_1 P_t+1}} g_j \mathbf{n_j^t} \circ \mathbf{d}^{\tau_\mathbf{j}} + \mathbf{w_j^t} & \text{if } j \leq M \\ f_j^* g_j \sqrt{\frac{\pi_1\pi_2 R P_t^2}{\pi_1 P_t+1}}\mathbf{p} \circ \mathbf{d}^{\tau_\mathbf{j}} + \sqrt{\frac{\pi_2 R P_t}{\pi_1 P_t+1}} g_j \mathbf{n_j^t}^* \circ \mathbf{d}^{\tau_\mathbf{j}} + \mathbf{w_j^t} & \text{if } j > M \end{cases}$$

where $\mathbf{d}^{\tau_\mathbf{j}} = \begin{bmatrix} 1 & e^{-\frac{i2\pi\tau_j}{N}} & \ldots & e^{-\frac{i2\pi\tau_j(N-1)}{N}} \end{bmatrix}^T$ and $\circ$ denotes Hadamard product. Thus, in each sub-carrier $k,\ 0 \leq k \leq N-1$, we get

$$\mathbf{y_k^t} = \begin{bmatrix} y_{k,1}^t & y_{k,2}^t & \ldots & y_{k,R}^t \end{bmatrix}^T = \sqrt{\frac{\pi_1\pi_2 R P_t^2}{\pi_1 P_t + 1}}\mathbf{I_R h_k} + \mathbf{n_k^t} \tag{6}$$

where

$$\mathbf{h_k} = \begin{bmatrix} f_1 g_1 & u_k^{\tau_2} f_2 g_2 & \ldots & u_k^{\tau_M} f_M g_M & u_k^{\tau_{M+1}} f_{M+1}^* g_{M+1} & \ldots & u_k^{\tau_R} f_R^* g_R \end{bmatrix}^T, \tag{7}$$

$u_k^{\tau_i} = e^{-\frac{i2\pi k \tau_i}{N}}$ and

$$\mathbf{n_k^t} = \sqrt{\frac{\pi_2 P_t R}{\pi_1 P_t + 1}} \left[\begin{array}{cccccc} g_1 u_k^{\tau_1} n_{k,1}^t & \ldots & u_k^{\tau_M} g_M n_{k,M}^t & u_k^{\tau_{M+1}} g_{M+1} n_{k,M+1}^{t^*} & \ldots & u_k^{\tau_R} g_R n_{k,R}^{t^*} \end{array} \right] + \left[\begin{array}{ccc} w_{k,1}^t & w_{k,2}^t & \ldots w_{k,R}^t \end{array} \right]^T.$$

Analogous to the synchronous case, we propose to estimate the equivalent channel matrix $\mathbf{h_k}$ from (6) as $\hat{\mathbf{h}}_\mathbf{k} = \sqrt{\frac{\pi_1 \pi_2 R P_t^2 R}{\pi_1 P_t + 1}} \left(\frac{\pi_2 P_t R + \pi_1 \pi_2 P_t^2 R}{\pi_1 P_t + 1} + 1 \right)^{-1} \mathbf{y_k^t}$. After the training cycle, the data transmission cycle starts which involves the transmission of R OFDM symbols containing data from the source which is then time reversed and/or conjugated by the relays before forwarding to the destination. In essence, a DSTBC (similar to synchronous case) is seen by the destination in every sub-carrier and the equivalent channel seen by the destination in the k-th sub-carrier is precisely the matrix $\mathbf{h_k}$ whose estimated value is available at the end of the training cycle. As for the synchronous case (see (3)), we propose to ignore the covariance matrix of the equivalent noise while performing data detection . We refer the readers to [7] and Section IV of [8] for a detailed explanation of the data transmission cycle.

IV. Simulation results

In this section, simulations are used to compare the error performance of the proposed strategy against the best known DDSTBC for 4 relays recently reported in [6]. Note that for 4 relays, the DDSTBCs in [6] were shown (see [6] for simulations) to outperform the codes reported in [3], [4], [5] in both complexity as well as performance.

We consider a 4 relay network and the coherent DSTBC employed in the proposed strategy for simulations is $\left[\begin{array}{cccc} z_1 & z_2 & -z_3^* & -z_4^* \\ z_2 & z_1 & -z_4^* & -z_3^* \\ z_3 & z_4 & z_1^* & z_2^* \\ z_4 & z_3 & z_2^* & z_1^* \end{array} \right]$ where $\{\mathrm{Re}(z_1), \mathrm{Re}(z_2)\}$, $\{\mathrm{Re}(z_3), \mathrm{Re}(z_4)\}$, $\{\mathrm{Im}(z_1), \mathrm{Im}(z_2)\}$ and $\{\mathrm{Im}(z_3), \mathrm{Im}(z_4)\}$ take values from quadrature amplitude modulation (QAM) rotated by $166.7078°$ (QAM constellation size chosen based on transmission rate). The relay matrices corresponding to this coherent DSTBC are unitary and $M = 2$. We set $\pi_1 = 1$, $\pi_2 = \frac{1}{R}$ (as suggested in [2]), $T_1 = T_2 = 4$ and $F = 50$ for all the simulations. We chose $P_t = (1+\alpha)P_d$, where α denotes the power boost factor to allow for power boosting to the pilot symbols. In order to quantify the loss in error performance due to channel estimation errors in the proposed strategy, we take the performance of the corresponding coherent STBC as the reference. The DDSTBC taken for comparison is the one reported recently in [6].

Fig. 3 shows the error performance of the proposed strategy in comparison with [6] and the corresponding coherent DSTBC for $\alpha = 0$, $\alpha = 0.4$ and transmission rates[3] of 1 bits per channel use (bpcu) and 2 bpcu respectively. It can be observed that for a rate of 1 bpcu and codeword error rate (CER) of 10^{-5}, the proposed strategy outperforms the DDSTBC of [6] by approximately 2 dB for $\alpha = 0$. For a transmission rate of 2 bpcu, the performance gap between the proposed strategy and the DDSTBC of [6] increases to 8 dB. Thus, we infer that the performance advantage of the proposed strategy over DDSTBCs increases as the transmission rate increases. Also note that the proposed strategy is better than the DDSTBC of [6] at all signal to noise ratio (SNR). We can attribute three reasons for the proposed strategy to outperform DDSTBCs as follows: (1) lesser equivalent noise power seen by the destination during data transmission cycle as compared to distributed differential space time coding [3], [4], [5], [6], (2) no restriction of coherent DSTBC codewords to unitary/scaled unitary matrices as is the case with DDSTBCs [3], [4], [5], [6] and (3) the relay matrices $\mathbf{B_i},\ i = 1, 2, \ldots, R$ need not satisfy certain algebraic relations involving the codewords (see [4], [6] for exact relations), thus giving more room to optimize the minimum determinant of difference matrices (coding gain). In spite of the simple channel estimation method employed (Eq. (5)), note that the performance loss due to channel estimation errors is only about 3 dB for transmission rates of 1 and 2 bpcu respectively. Finally, observe that a 40% power boost to the pilot symbols gives marginally better performance (gain of 0.7 dB). From our simulations we have observed that the performance begins to degrade for $\alpha > 0.4$. Simulation results are not reported for the asynchronous case because the use of OFDM essentially makes the signal model in every sub-carrier similar to the synchronous case and hence the performace will be same but for a rate loss due to CP.

V. Conclusion

Similar to the results of [9] for point to point MIMO systems, we show that a simple training and channel estimation scheme combined with the protocol in [2] outperforms distributed differential space time coding at all SNR. The proposed strategy leverages existing coherent DSTBCs [8], [10], [11], [12], [13] for noncoherent communication in AF relay networks. We would like to emphasize here that designing coherent DSTBCs with low ML decoding

[3]When calculating transmission rate, the rate loss due to initial few channel uses for training is ignored ($R + 1$ for proposed strategy and $2R$ for DDSTBC [3], [4], [5], [6]).

complexity and/or good coding gain is much simpler compared to designing DDSTBCs wherein there are several stringent constraints [4], [6]. An important feature of the proposed strategy is that the relays do not perform any channel estimation and only amplify and forward the received pilot/data symbols as required. The extra processing required for channel estimation is done only at the destination. Thus, we conclude that the proposed strategy based on training and existing coherent DSTBCs is a good alternative to DDSTBCs for practical AF relay networks in terms of performance as well as complexity.

Finally, the proposed strategy is extended for application in asynchronous relay networks with no knowledge of the timing errors using OFDM. A drawback of this strategy is that it requires a large channel quasi-static duration spanning over multiple OFDM symbols.

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Terminal	Slot 1	Slot 2	...	Slot $M+1$	Slot $M+2$	...	Slot $R+1$
Source	$\sqrt{\pi_1 P_t}$						
Relay 1		$\sqrt{\frac{\pi_1\pi_2 P_t^2 R}{\pi_1 P_t+1}} f_1 + \sqrt{\frac{\pi_2 P_t R}{\pi_1 P_t+1}} n_1$					
$\vdots$			$\ddots$				
Relay M				$\sqrt{\frac{\pi_1\pi_2 P_t^2 R}{\pi_1 P_t+1}} f_M + \sqrt{\frac{\pi_2 P_t R}{\pi_1 P_t+1}} n_M$			
Relay $M+1$					$\sqrt{\frac{\pi_1\pi_2 P_t^2 R}{\pi_1 P_t+1}} f_{M+1}^* + \sqrt{\frac{\pi_2 P_t R}{\pi_1 P_t+1}} n_{M+1}^*$		
$\vdots$						$\ddots$	
Relay R							$\sqrt{\frac{\pi_1\pi_2 P_t^2 R}{\pi_1 P_t+1}} f_R^* + \sqrt{\frac{\pi_2 P_t R}{\pi_1 P_t+1}} n_R^*$

Fig. 1. Training cycle

Terminal	Data transmission cycle 1		...	Data transmission cycle F
	Phase I Slots $R+2$ to $2R+1$	Phase II Slots $2R+2$ to $3R+1$	...	Slots $R(2F-1)+2$ to $R(2F+1)+1$
Source	$\sqrt{\pi_1 P_d}\mathbf{z}$			
Relay 1		$\sqrt{\frac{\pi_1\pi_2 P_d^2}{\pi_1 P_d+1}} f_1 \mathbf{B_1 z} + \sqrt{\frac{\pi_2 P_d}{\pi_1 P_d+1}} \mathbf{B_1 v_1}$		
$\vdots$		$\vdots$		
Relay M		$\sqrt{\frac{\pi_1\pi_2 P_d^2}{\pi_1 P_d+1}} f_M \mathbf{B_M z} + \sqrt{\frac{\pi_2 P_d}{\pi_1 P_d+1}} \mathbf{B_M v_M}$		$\vdots$
Relay $M+1$		$\sqrt{\frac{\pi_1\pi_2 P_d^2}{\pi_1 P_d+1}} f_{M+1}^* \mathbf{B_{M+1} z}^* + \sqrt{\frac{\pi_2 P_d}{\pi_1 P_d+1}} \mathbf{B_{M+1} v_{M+1}}^*$		
$\vdots$		$\vdots$		
Relay R		$\sqrt{\frac{\pi_1\pi_2 P_d^2}{\pi_1 P_d+1}} f_R^* \mathbf{B_R z}^* + \sqrt{\frac{\pi_2 P_d}{\pi_1 P_d+1}} \mathbf{B_R v_R}^*$		

Fig. 2. Data transmission

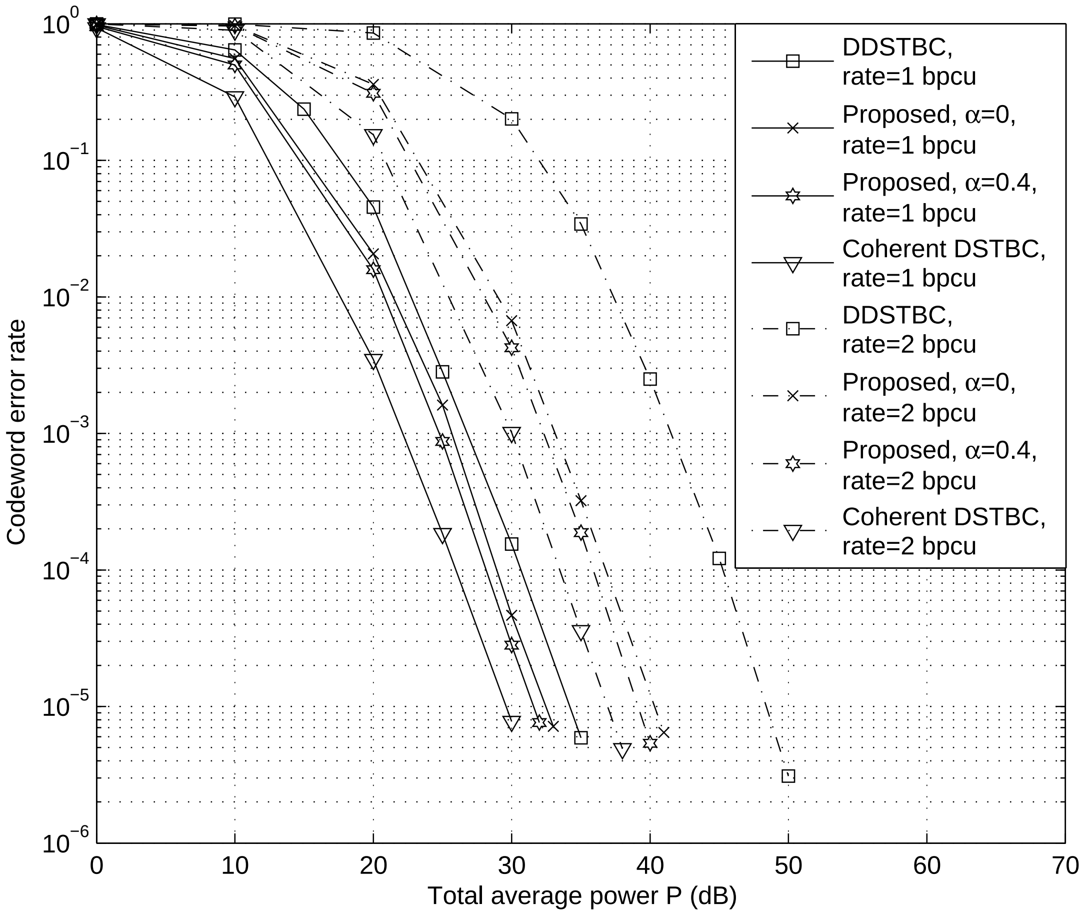

Fig. 3. Error performance comparison for a 4 relay network

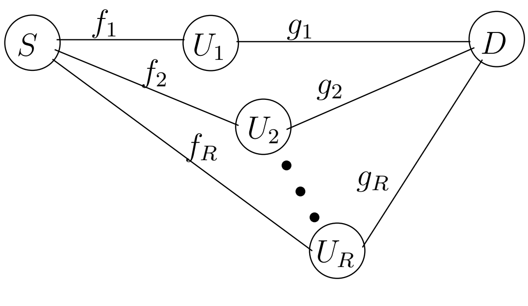

Fig. 4. Asynchronous wireless relay network