# Space-Time Coding-Assisted Double-Spread Rake Receiver-Based CDMA for Dispersive Rayleigh Fading Environments

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## Abstract

**In this contribution we propose a novel Space-Time Coding-Assisted Double-Spread Rake Receiver (STC-DS-RR) based CDMA scheme for downlink (base to mobile) transmissions over dispersive Rayleigh fading channels. The STC-DS-RR scheme invokes Turbo Convolutional (TC) channel coding and Space-Time Block Coding (STBC) for achieving both time and space diversity gains in addition to the path diversity gain provided by the dispersive Rayleigh fading environments. The double-spreading mechanism of the STC-DS-RR scheme is capable of detecting the wanted user's channel-impaired wideband signals with the aid of a low-complexity Rake receiver, without resorting to complex multi-user detection. The performance of the proposed scheme was investigated using Quadrature-Phase-Shift-Keying (QPSK), when communicating over dispersive channels. It was shown that the wideband scheme advocated is capable of achieving $E_b/N_0$ gains up to 35 dBs in comparison to an uncoded narrowband single-transmitter scheme.**

## 1. INTRODUCTION

Space-Time Trellis Coding (STTC) [1,2] pioneered by Tarokh *et al.* combines signal processing at the receiver with intelligent coding techniques, appropriately mapping the information to be transmitted to multiple transmit antennas. In an attempt to reduce the associated decoding complexity, Alamouti proposed Space-Time Block Coding [3] (STBC) employing two transmit antennas. Alamouti's scheme was later generalized to an arbitrary number of transmit antennas [2,4].

Space-Time Coding (STC) schemes have originally been proposed for transmissions over narrowband fading channels. When encountering wideband channels, Multi-Carrier Code Division Multiple Access (MC-CDMA) and Orthogonal Frequency Division Modulation (OFDM) [5] can be utilised for converting the wideband channel to numerous narrowband channels. However, a detrimental effect of employing STTCs or STBCs is that not only the desired signal, but also the Multiple Access Interference (MAI) and the Co-Channel Interference (CCI) are enhanced due to the multiple antenna based transmission diversity scheme. These are the most important limiting factors of employing space-time coding in CDMA systems [6]. By contrast, Multi-User Detection (MUD) constitutes an efficient way of combating the MAI and CCI, although its complexity may become excessive. On the other hand, a simple Rake receiver can be used for obtaining path diversity in CDMA systems, provided that the maximum path delay is shorter than the original symbol period, and hence the ISI can be neglected. Furthermore, the multi-path intensity profile typically has an exponentially decaying shape in realistic channel conditions, where the longer the path delay the lower the magnitude of the path. Hence the ISI induced by a long delay path is typically insignificant and can be neglected. Based on these arguments the employment of a simple Rake receiver is often more practical, than that of a complex MUD scheme, especially in a downlink scenario.

In our proposed system, a double-spreading assisted Rake receiver is used. Specifically, in the first spreading operation unique user signature Walsh codes are used for distinguishing the users and hence mitigating the effects of MAI, while in the second spreading step a random code is employed for spreading and hence attaining path diversity. Additionally, STBC is invoked for achieving transmit diversity, while a power efficient Turbo Convolutional (TC) channel code [7] is used for obtaining time diversity.

The rest of this treatise is organised as follows. In Section 2 the STC-DS-RR CDMA scheme is described and the required parameter values are investigated. Our simulation results are discussed in Section 3 and finally our conclusions are offered in Section 4.

## 2. SYSTEM DESCRIPTION

The block diagram of the Space-Time Coding-Assisted Double-Spread Rake Receiver (STC-DS-RR) based CDMA scheme proposed for downlink (base to mobile) transmissions is shown in Figure 1. The information destined for user-$i$, where $i \in \{1 \dots K\}$, is first encoded by the TC channel encoder to yield $S_i$. Then, $S_i$ is spread by the Walsh code $c_i^W$, where each user is assigned a unique Walsh code. The sum $S$ of the signals of all users is then passed to the STBC encoder on a chip by chip basis. Here Alamouti's $G_2$ code associated with two transmit antennas is used for STBC. The STBC encoder yields two chips, namely $d_1$ and $d_2$, one for each transmit antenna. Both chips are then spread by the same random code, namely $c^R$ before transmission through the $L$-path dispersive Rayleigh fading channels associated with the two transmission antennas. A single Rake receiver is utilised at the mobile station. The Rake receiver then produces $L$ outputs, $y_1 \dots y_L$, each to be

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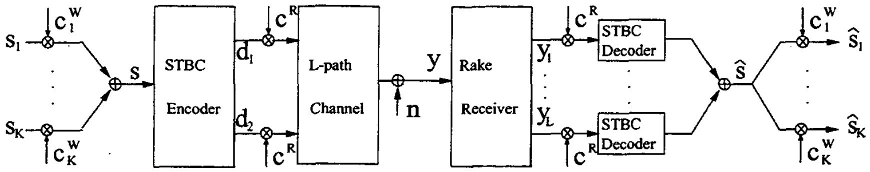

Figure 1: Block diagram of the space-time coding-assisted double-spread Rake receiver based CDMA scheme.

despread by the random code $c^R$, where each of the $L$ despread signals benefits from transmit diversity of order two due to the STBC. Consequently, the STBC decoder is invoked $L$ times, namely once for each of the $L$ despread outputs for obtaining path diversity of order $L$. The $L$ STBC decoded signals are summed, yielding $\hat{S}$, which has a total diversity order of $2L$. Then, the signal $\hat{S}_i$ of user-$i$ can be obtained by despreading $\hat{S}$ with the aid of the unique Walsh signature sequence $c_i^W$. Finally, $\hat{S}_i$ is channel decoded by the TC channel decoder in order to yield the estimate of user-$i$'s information.

## 2.1. Double-Spreading Mechanism

Walsh codes are well-known for their attractive zero cross-correlation property expressed as:

$$\sum_{l=1}^{Q} c_i^W[l] \cdot c_j^W[l] \quad = \quad 0; \text{ for } i \neq j, \tag{1}$$

where $i, j \in \{1 \ldots K\}$ and $c_i[l] \in \{+1, -1\}$. There are $Q$ chips in a code and the maximum number of Walsh codes of length $Q$ is $Q$, which can support a maximum number of $K = Q$ users. Therefore, it is possible to support $Q$ users employing Walsh codes of length $Q$, without encountering any MAI, provided that the codes' orthogonality is not destroyed by the channel. However, the auto-correlation of Walsh codes is relatively high, which is quantified as:

$$\sum_{l=1}^{Q} c_i^W[m] \cdot c_i^W[l] \quad \neq \quad 0; \text{ for } m, i \in \{1 \ldots Q\}. \tag{2}$$

Hence, the good cross-correlation properties of Walsh codes can be easily destroyed by the multi-path interference encountered in dispersive channels. In contrast to Walsh codes, random codes exhibit high cross-correlation but low auto-correlation values. Therefore random codes can be beneficially processed by a Rake receiver for obtaining path diversity due to their low auto-correlation value. However, without additional Walsh-spreading they are only applicable for single user scenarios, since the associated high cross-correlation will result in un-tolerable MAI.

To elaborate a little further, the double-spreading mechanism introduced here is different from the conventional method of invoking Walsh codes plus random or Pseudo-Noise (PN) scrambling codes. More specifically, in the context of the conventional method the chip duration of the Walsh code is identical to that of the random code. For the transmission of a $D$-symbol burst using Walsh codes having a spreading factor of $Q$, there are $Q$ chips per information symbol, resulting in $D \times Q$ chips per transmission block. After each of the information symbols has been spread by the $Q$-chip Walsh code, a random code of length $D \times Q$ chips is multiplied by the $D \times Q$-chip transmission block in order to decrease the auto-correlation of the Walsh codes. Although the resultant spread signal exhibits a reduced auto-correlation, the cross-correlation of the various users' signals is no longer zero.

More quantitatively, the double-spreading mechanism invokes $\bar{Q}$-chip Walsh codes for supporting $K = \bar{Q}$ users and additinally a $\tilde{Q}$-chip random code for attaining multi-path diversity, where there are $Q$ chips per information symbol and we have $\bar{Q} \times \tilde{Q} = Q$. The first spreading operation spreads each information symbol of user-$i$ to $\bar{Q}$ chips, using the Walsh code $c_i^W$. Then the $\tilde{Q}$-chip random code further spreads each of the resultant chips to $\tilde{Q}$ chips. Hence there are a total of $\bar{Q} \times \tilde{Q} = Q$ chips per information symbol according to the conventional method highlighted above, although the spreading mechanism used is different. Here, the same $\tilde{Q}$-chip random code repeats itself every $\tilde{Q}$ chips, namely for each of the Walsh-code chips. Note that the $l^{th}$ chip of the $\bar{Q}$-chip Walsh code of all users is spread by the same random code, which are conveyed via the same channel and experience an identical multi-path interference of:

$$I[l] = I_k[l] = I_i[l], \tag{3}$$

where $k, i \in \{1 \ldots \bar{Q}\}$ and $I_k[l]$ is the interference imposed on the $l^{th}$ chip of user-$k$'s Walsh code, $c_k^W[l]$. If a slow fading channel is encountered, the Channel Impulse Response (CIR) of the multi-path components of the channel can be assumed to be time-invariant for the duration of $Q = \bar{Q} \times \tilde{Q}$ chips. Since each chip of the $\bar{Q}$-chip Walsh code is also spread by the same random code, each of the $\bar{Q}$ chips, again, experiences identical multi-path interference in the slow fading channels considered, yielding:

$$I = I[l] = I[m]; \quad l, m \in \{1 \ldots \bar{Q}\}. \tag{4}$$

With the aid of Equations 1, 3 and 4 the resultant cross-correlation of the codes can be shown to be zero, which is expressed as:

$$\begin{aligned} \sum_{l=1}^{\bar{Q}} (c_i^W[l] \cdot I_i[l]) \cdot (c_k^W[l] \cdot I_k[l]) \quad &= \quad I^2 \cdot \sum_{l=1}^{\bar{Q}} c_i^W[l] \cdot c_k^W[l], \\ &= \quad 0; \text{ for } i \neq k. \end{aligned} \tag{5}$$

This implies encountering no MAI, hence requiring no MUD. Therefore attaining near-single-user performance is feasible with the advent of this simple double-spreading method without the employment of complex MUD schemes in slow-fading dispersive channels.

### 2.2. Space-Time Block Coded Rake Receiver

The main idea behind Rake receivers [8] is that of capturing the energy of several multi-path components of the channels in a spread spectrum system, provided that the different multipath delays are multiples of the chip duration. The Rake receiver consists of a number of matched filters, often referred to as Rake fingers. The maximum number of Rake fingers is equal to the number of the resolvable paths. These Rake receiver fingers are synchronised to each path and the corresponding delayed replicas of the transmitted signals are combined coherently or noncoherently. Assuming a perfect knowledge of the CIRs and multi-path delays, the Rake receiver is equivalent to a Maximal Ratio Combining (MRC) scheme having a diversity order $L$ [9]. In a conventional single transmitter scenario, the outputs of Rake receiver are first despread and then multiplied with the conjugate of the CIRs of the multi-path components. However, when using transmit diversity, each of the despread Rake outputs is conveyed to one of the STBC decoders shown in Figure 1. In other words, there are $L$ STBC decoders, each corresponding to one resolvable path of the dispersive channel.

In the STBC scheme using Alamouti's $G_2$ code, the encoding requires two time slots and two transmit antennas for transmitting two symbols, namely $x_1$ and $x_2$. Specifically, $G_2$ is defined as [4]:

$$G_2 = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix}, \qquad (6)$$

where, $x^*$ denotes the complex conjugate of the signal $x$. In time slot 1, signals $x_1$ and $x_2$ are transmitted, while in time slot 2, signals $-x_2^*$ and $x_1^*$ are emitted, from the transmit antennas 1 and 2, respectively. We assume that the fading amplitude is constant across the two STBC time slots, where each time slot has a duration of $\bar{Q}$ chips, or one chip of $c^W$. Hence we can write:

$$h_p^i(t) = h_p^i(t+T) = h_p^i = \alpha_p^i e^{j\theta_1^i}; \quad p \in \{1,2\}, \qquad (7)$$

where $T$ is the time slot duration, while $h_p^i$ is the CIR associated with path $i$ and transmit antenna $p$. The received signals after despreading are:

$$\begin{aligned} r_1^i = r^i(t) &= h_1^i \cdot x_1 + h_2^i \cdot x_2 + n_1^i, \\ r_2^i = r^i(t+T) &= -h_1^i \cdot x_2^* + h_2^i \cdot x_1^* + n_2^i, \qquad (8) \end{aligned}$$

where $r_t^i$ is the received signal at instant $t$ for path $i$, while $n_t^i$ is the random variable representing the sum of the receiver noise at instant $t$ and the multi-path interference imposed by path $i$ after despreading. Equivalently, the two signals received over path $i$ can be represented in matrix form as:

$$\begin{aligned} \begin{pmatrix} r_1^i \\ (r_2^i)^* \end{pmatrix} &= \begin{pmatrix} h_1^i & h_2^i \\ (h_2^i)^* & -(h_1^i)^* \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} n_1^i \\ (n_2^i)^* \end{pmatrix} \\ \mathbf{r^i} &= \mathbf{A^i} \cdot \mathbf{x} + \mathbf{n^i}, \qquad (9) \end{aligned}$$

where $\mathbf{A^i}$ is termed the system matrix associated with path $i$. Since the transmitted signal vector $\mathbf{x}$ can be factorized due to the orthogonality of the $G_2$ code, transmit diversity can be obtained by multiplying $\mathbf{r^i}$ with the conjugate transpose of $\mathbf{A^i}$, namely with $\mathbf{A_i^H}$ during the STBC decoding process. With the aid of the Rake receiver we can sum all the $L$ STBC decoded outputs for generating the estimate of:

$$\begin{aligned} \hat{\mathbf{x}} &= \sum_{i=1}^{L} (\mathbf{A_i^H} \cdot \mathbf{r^i}), \\ \begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \end{pmatrix} &= \left( \sum_{i=1}^{L} \left( (\alpha_1^i)^2 + (\alpha_2^i)^2 \right) \right) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \mathbf{N}, \qquad (10) \end{aligned}$$

where $\mathbf{N}$ is the resultant noise plus interference for all the $L$ paths, while the transmit diversity of order $2L$ can be observed in the context of the term $\sum_{i=1}^{L} \left( (\alpha_1^i)^2 + (\alpha_2^i)^2 \right)$.

Having exploited both the space and path diversity, now the TC decoder is invoked for attaining further time diversity. Based on Equation 10 the soft inputs for the TC decoder are computed. Finally, the estimates of the original information symbols are obtained from the decoder. Here, two identical 1/2-rate Recursive Systematic Convolutional (RSC) codes having a constraint length of three were utilised by the turbo encoder, while soft decision trellis decoding utilising the binary Log-Maximum A Posteriori (Log-MAP) algorithm [10] was invoked for turbo decoding. For detailed discussions on TC channel coding the interested reader is referred to [7].

### 2.3. Channel Model and System Parameter Design

Table 1: Parameters of the STC-DS-RR CDMA scheme for downlink transmissions.

| Parameter | Value |
|---|---:|
| Doppler frequency (Hz) | 80 |
| Vehicular velocity (mph) | 30 |
| First Spreading ratio, $Q$ | 4 |
| Second Spreading ratio, $Q$ | 8 |
| Total Spreading ratio, $Q$ | 32 |
| Chip rate (MBaud) | 2.167 |
| Modulation mode | QPSK |
| STBC code | $G_2$ [4] |
| Number of transmitters | 2 |
| Number of receivers | 1 |
| TC coding rate | 1/2 |
| TC constraint length | 3 |
| TC decoding iterations | 8 |
| TC interleaver length (bit) | 1000 |
| Channel A power profile | Equal/Constant |
| Channel B power profile | Exponential decay |

In this section the performance of the proposed scheme will be evaluated using the simulation parameters shown in Table 1. We denote the conventional single-transmitter scenario as the $G_1$-coded scheme, for comparison with the two-transmitter based $G_2$ code. We assumed that the receiver determined the CIRs perfectly, while the fading is constant for the duration of two STBC timeslots or for $2Q$=64 chips. Channel A exhibits a CIR having equal-power taps, while the CIR of Channel B exhibits an exponentially decaying power for each of the multi-path components. Each path was faded according to independent Rayleigh fading statistics, as described by the parameters of Table 1. Figure 2 shows the normalised magnitude versus chip delay profile for the 5-path CIR of Channel

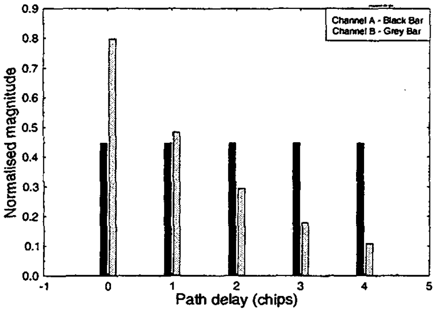

Figure 2: The normalised magnitude versus chip delay profile for the 5-path Channel A and Channel B.

A and Channel B, where the total power of the paths was normalised to unity. The delay of each path is expressed in terms of chip durations, where the chip duration is equal to the reciprocal of the system's chip rate. We will consider one to five resolvable paths for each of the CIRs.

## 3. SIMULATION RESULTS AND DISCUSSIONS

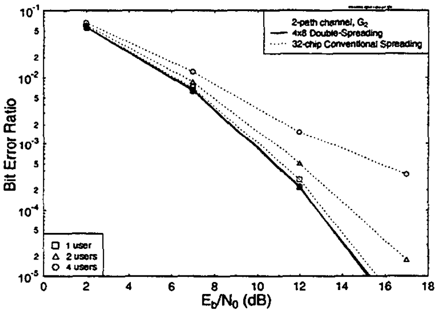

Figure 3: Bit Error Ratio (BER) versus signal to noise ratio per information bit ($E_b/N_0$) performance of the double-spreading and conventional spreading schemes assisted by a Rake receiver in a multi-user scenario for transmissions over Channel A exhibiting two resolvable paths, utilising the simulation parameters of Table 1. The separation of the two CIR taps was one chip interval.

Let us now compare the performance of the proposed double-spreading scheme and the conventional spreading scheme. Figure 3 shows the Bit Error Ratio (BER) versus signal to noise ratio per information bit ($E_b/N_0$) performance of both the double-spreading and that of the conventional spreading schemes in a multi-user scenario for transmissions over Channel A although using only two resolvable paths, utilising the simulation parameters of Table 1. The separation of the two CIR taps was one chip duration.

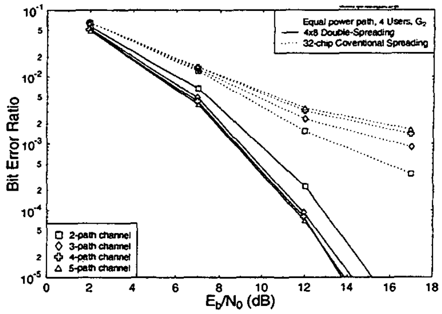

Figure 4: BER versus $E_b/N_0$ performance of the double-spreading and conventional spreading schemes assisted by a Rake receiver in a multi-path scenario for transmissions over Channel A, utilising the simulation parameters of Table 1.

We observe in Figure 3 that conventional spreading experienced performance degradations, when the number of users increased, since the orthogonality of the 32-chip Walsh codes used by the conventional spreading scheme was destroyed by the multi-path interference imposed by the two-path channel. By contrast, the double-spreading scheme experienced no performance degradations when supporting up to $\bar{Q}$=4 users, since the multi-path interference can be rejected, as suggested by Equation 5. Hence the orthogonality of the 4-chip Walsh codes was preserved by the double-spreading scheme.

Although the conventional spreading method is also capable of attaining path diversity with the aid of the signal spreading imposed by the random scrambling code, the multipath components will inflict severe interference rather than yielding beneficial path diversity, when the number of users is high. This is illustrated in Figure 4, where the performance of the conventional spreading scheme degrades, as the channel exhibits an increasing number of multi-path components, when supporting four users. By contrast, the performance of the double-spreading scheme improved, as the channel exhibited a higher number of multi-path components, although the path diversity gains eventually saturated, when a high number of paths was encountered.

Having investigated the performance gains achieved using the double-spreading scheme, let us now apply the double-spreading scheme in conjunction with the $G_2$ STBC utilising two transmitters as well as the single transmitter scenario, which we have denoted as the '$G_1$-coded scheme' in Section 2.3. The coding plus diversity gain versus the number of resolvable paths at a BER of $10^{-5}$ for the STC-DS-RR scheme against the benchmarker of the uncoded STC-DS-RR scheme utilising $G_1$ in a narrowband channel is summarised in Figure 5. The benchmarker required an $E_b/N_0$ of 39.3dB for a BER of $10^{-5}$.

Explicitly, the $G_2$ code exhibits twice the diversity gain in comparison to its $G_1$-coded counterpart. For example, the performance of the $G_2$ code in single-path Rayleigh channel is identical to that of the $G_1$ code in the two-path Channel A, as seen from Figure 5. However, when targeting a total diversity order of four, the performance of the $G_2$ code used for transmission over the chip-spaced two-path Channel A is slightly better, than that of the $G_1$ code in the chip-spaced four-path Channel A. This is because the equal power paths in Channel A result in a higher multi-path interference, when the

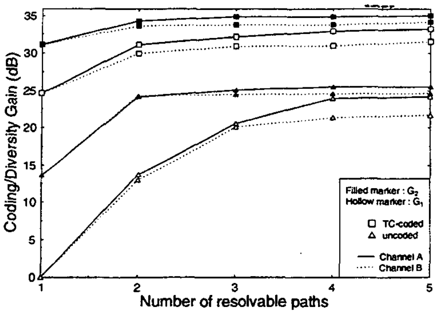

Figure 5: Coding plus diversity gain versus the number of resolvable paths at a BER of $10^{-5}$ of the STC-DS-RR scheme against the uncoded STC-DS-RR scheme utilising the $G_1$ code in a narrowband channel. The simulation parameters of Table 1 were utilised and the number of users was four.

channel exhibits a longer delay spread, where the delay spreads of the four-path and two-path channels constitute 50% and 25% of the duration of the 8-chip random code, respectively.

When transmitting over Channel B, where the power of the paths decays exponentially with time, the multi-path interference will be less severe in a long delay spread environment, although the path diversity gain will be decreased. These effects can be studied in Figure 5, where the performance of the $G_2$ code in a single-path scenario is seen to be better than that of the $G_1$ code employed for transmission over the two-path Channel B. Similarly, the performance of the $G_2$ code used in the two-path Channel B is significantly better, than that of the $G_1$ code in the four-path Channel B.

The employment of power efficient TC channel coding further enhances the coding gain of the STC-DS-RR scheme. As we can see from Figure 5, a high coding gain was obtained by the TC-coded systems compared to the uncoded systems. We can observe that at a BER of $10^{-5}$, the TC-coded STC-DS-RR scheme utilising the $G_2$ code yielded a total diversity plus coding gain of 35.05 dB in the 5-path Channel A and 33.2 dB in the 5-path Channel B. However, we observed from Figure 5 that most of this total diversity plus coding gain was already achieved, when three resolvable paths were encountered.

## 4. CONCLUSION

In this contribution, a novel space-time coding-assisted double-spread Rake receiver-based CDMA scheme was proposed and characterised in performance terms for downlink transmissions over dispersive Rayleigh fading channels. The double-spreading mechanism is capable of attaining a near-single-user performance. Additionally, it is capable of yielding a performance equivalent to that of maximal ratio combining of order $L$ in $L$-path channels, even when utilising only a single Rake receiver. Explicitly, the transmit diversity and path diversity constitute two independent sources of diversity gain.

It was shown in Figure 5 that transmit diversity is more advantageous than path diversity in dispersive Rayleigh fading channels exhibiting an exponentially decaying CIR. When targeting a certain diversity gain, the multi-path interference sensitivity of STBC is lower, than that of a single-transmitter based scheme, when communicating over wideband channels having a long delay spread, as investigated in Figure 5. With the advent of TC channel coding the proposed scheme required a low bit energy for attaining a low BER. **In conclusion, near-single-user performance can be achieved even with the aid of a single Rake receiver, i.e. without MUD, at the cost of supporting a factor $\tilde{Q}$ reduced number of users.**

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