Benchmarking Parameter Control Methods in Differential Evolution for Mixed-Integer Black-Box Optimization

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ABSTRACT

Differential evolution (DE) generally requires parameter control methods (PCMs) for the scale factor and crossover rate. Although a better understanding of PCMs provides a useful clue to designing an efficient DE, their effectiveness is poorly understood in mixed-integer black-box optimization. In this context, this paper benchmarks PCMs in DE on the mixed-integer black-box optimization benchmarking function (bbob-mixint) suite in a componentwise manner. First, we demonstrate that the best PCM significantly depends on the combination of the mutation strategy and repair method. Although the PCM of SHADE is state-of-the-art for numerical black-box optimization, our results show its poor performance for mixed-integer black-box optimization. In contrast, our results show that some simple PCMs (e.g., the PCM of CoDE) perform the best in most cases. Then, we demonstrate that a DE with a suitable PCM performs significantly better than CMA-ES with integer handling for larger budgets of function evaluations. Finally, we show how the adaptation in the PCM of SHADE fails.

CCS CONCEPTS

• Mathematics of computing \rightarrow Evolutionary algorithms.

KEYWORDS

Mixed-integer black-box optimization, differential evolution, parameter control, benchmarking

ACM Reference Format:

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1 INTRODUCTION

General context. As in the single-objective mixed-integer blackbox optimization benchmarking function (bbob-mixint) suite [42], this paper considers mixed-integer black-box optimization of an objective function $f : \mathbb{Z}^{n^{\text{int}}} \times \mathbb{R}^{n-n^{\text{int}}} \to \mathbb{R}$, where *n* is the total number of variables, n^{int} is the number of integer variables, and $n-n^{\text{int}}$ is the number of numerical variables. This problem involves finding a solution $\mathbf{x} \in \mathbb{Z}^{n^{\text{int}}} \times \mathbb{R}^{n-n^{\text{int}}}$ with an objective value $f(\mathbf{x})$ as small as possible without any explicit knowledge of f.

Differential evolution (DE) [31, 33] is an efficient evolutionary algorithm for numerical black-box optimization. Here, numerical black-box optimization can be considered as a special case of mixed-integer black-box optimization when $n^{\text{int}} = 0$. Previous studies [35, 40] empirically demonstrated that DE performs as well as or better than covariance matrix adaptation evolution strategy (CMA-ES) [11, 15] under some conditions.

Evolutionary algorithms generally require parameter control methods (PCMs) [6, 17, 26] that automatically adjust one or more parameters during the search. This is true for DE. Previous studies (e.g., [1, 7, 53]) showed that the performance of DE is sensitive to the setting of two parameters: scale factor *s* and crossover rate *c*. To address this issue, a number of DE with PCMs have been proposed [3]. According to [6], PCMs can be classified into three groups: deterministic PCMs, adaptive PCMs, and self-adaptive PCMs. However, most PCMs in DE are deterministic or adaptive [39].

A DE is a complex of many components. For example, as described in [39], "L-SHADE" [40] mainly consists of the following four components: (i) the current-to-*p*best/1 mutation strategy [51], (ii) binomial crossover, (iii) the PCM of SHADE [37] for adaptively adjusting the scale factor *s* and crossover rate *c*, and (iv) linear population size reduction strategy. This complex property makes an analysis of DE algorithms difficult. To address this issue, some previous studies (e.g., [5, 35, 39, 44, 52]) employed component-wise analysis. For example, the previous study [39] analyzed only (iii) the PCM of 24 DE algorithms by fixing the other components.

Some previous studies proposed extensions of DE for mixedinteger black-box optimization. As demonstrated in [20, 22], any DE can handle integer variables by simply using the rounding operator $\mathbb{R} \to \mathbb{Z}$. Some previous studies (e.g., [25, 30]) proposed efficient methods for handling integer variables in DE. A previous study [23] proposed a hybrid method of L-SHADE and ACO_{MV} [21], called L-SHADE_{ACO}.

Motivation. Although some DE algorithms for mixed-integer blackbox optimization have been proposed, their analysis has received little attention in the DE community. In particular, the performance of PCMs in DE is poorly understood in the context of mixed-integer black-box optimization. On the one hand, the importance of PCMs for numerical black-box optimization has been widely accepted in the DE community. On the other hand, most previous studies on DE for mixed-integer black-box optimization (e.g., [20, 22, 24, 25]) did not use any PCM and fixed the two parameters to pre-defined values, e.g., s = 0.5 and c = 0.9.

Of course, some DE algorithms for mixed-integer black-box optimization use PCMs. For example, DE-CaR+S [30] uses a deterministic PCM that randomly generates the scale factor *s* and crossover rate *s*. L-SHADE_{ACO} [23] uses the PCM of SHADE for adaptation

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of *s* and *c*. However, the effectiveness of the PCMs of DE-CaR+S and SHADE is unclear. For example, the previous study [23] investigated the performance of L-SHADE_{ACO} but did not investigate the performance of (iii) the PCM of SHADE. Here, the conclusion "L-SHADE_{ACO} performs well" does not mean "the PCM of SHADE performs well" due to the existence of other components.

Some previous studies (e.g., [9, 10]) proposed extensions of CMA-ES for mixed-integer black-box optimization so that they can handle integer variables. Three previous studies [8, 29, 42] investigated the performance of CMA-ES with integer handling on the bbob-mixint suite [42]. Their results showed that the CMA-ES variants perform significantly better than the SciPy implementation of DE. However, the SciPy implementation of DE is the most classical version of DE [33] and does not use any PCM. Thus, it is unclear whether CMA-ES can outperform a DE with an advanced PCM or not.

Contributions. Motivated by the above discussion, this paper investigates the performance of nine PCMs in DE for mixed-integer black-box optimization. Note that we are interested only in a PCM in DE rather than an adaptive DE algorithm. This paper addresses the following three research questions:

- RQ1: Are PCMs effective in DE for mixed-integer black-box optimization? If so, which PCMs are useful in which situations?
- RQ2: Can a DE algorithm with a suitable PCM outperform CMA-ES with integer handling?
- RQ3: How does a state-of-the-art PCM behave?

Outline. Section 2 gives some preliminaries. Section 3 describes our experimental setup. Section 4 shows the analysis results to answer the three research questions. Section 5 concludes this paper.

Supplementary file. Figure S.*, Table S.*, and Algorithm S.* indicate a figure, table, and algorithm in the supplement, respectively.

Code availability. The Python implementation of DE is available at https://github.com/ryojitanabe/de_bbobmixint.

2 PRELIMINARIES

First, Section 2.1 describes the basic DE. Then, Section 2.2 describes two repair methods: the Baldwinian and Lamarckian repair methods. Finally, Section 2.3 describes nine PCMs in DE investigated.

2.1 Differential evolution

Algorithm 1 shows the procedure of DE. Let $\mathcal{P} = \{\mathbf{x}_i\}_{i=1}^{\mu}$ be the population of size μ at iteration *t*. Each individual $\mathbf{x} = (x_1, \dots, x_n)^{\top}$ in \mathcal{P} consists of the *n*-dimensional numerical vector in \mathbb{R}^n . Since **x** violates the integer constraint, **x** must be repaired so that **x** is a feasible solution for mixed-integer black-box optimization.

At the beginning of the search t = 1, the population \mathcal{P} of size μ is initialized randomly (line 1). The optional external archive \mathcal{A} is also initialized, where \mathcal{A} maintains inferior individuals. \mathcal{A} is used only when using the current-to-*p*best/1 [51] and rand-to-*p*best/1 [50] mutation strategies described later.

After the initialization of \mathcal{P} , the following steps (lines 2–14) are repeatedly performed until the termination conditions are satisfied. For each $i \in \{1, ..., \mu\}$, a parameter pair $\langle s_i, c_i \rangle$ is generated by a PCM (line 4), where $\langle \rangle$ means a tuple. The scale factor s > 0determines the magnitude of differential mutation. The crossover rate $c \in [0, 1]$ determines the number of elements inherited from

Algorithm 1: The basic DE algorithm with a PCM				
1 $t \leftarrow 1$, initialize $\mathcal{P} = \{\mathbf{x}_1, \dots, \mathbf{x}_\mu\}$ randomly, $\mathcal{A} \leftarrow \emptyset$;				
² Initialize inter	² Initialize internal parameters for s and c ;			
3 while The ter	³ while The termination criteria are not met do			
4 Generate	a pair of <i>s</i> and <i>c</i> for each individual in \mathcal{P} ;			
5 for $i \in \{1, \dots, n\}$	$1,, \mu$ do			
$6 v_i \leftarrow$	Apply mutation with s_i to individuals in \mathcal{P} ;			
7 $u_i \leftarrow$	Apply crossover with c_i to \mathbf{x}_i and \mathbf{v}_i ;			
s for $i \in \{1, \dots, n\}$	$1,\ldots,\mu\}$ do			
9 $ $ if $f($	$\mathbf{u}_i) \leq f(\mathbf{x}_i)$ then			
10 3	$\mathcal{A} \leftarrow \mathcal{A} \cup \{\mathbf{x}_i\};$			
11 X	$\mathbf{u}_i \leftarrow \mathbf{u}_i;$			
12 $ \mathbf{if} \mathcal{A} >$	<i>a</i> then Delete randomly selected $ \mathcal{A} - a$			
individu	als in $\mathcal A$;			
13 Update in	ternal parameters for the adaptation of s and c ;			
14 $t \leftarrow t+1$	• ?			

Table 1: Eight representative mutation strategies for DE.

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Strategies	Definitions
rand/1	$\mathbf{v}_i = \mathbf{x}_{r_1} + s_i \ (\mathbf{x}_{r_2} - \mathbf{x}_{r_3})$
rand/2	$\mathbf{v}_i = \mathbf{x}_{r_1} + s_i \ (\mathbf{x}_{r_2} - \mathbf{x}_{r_3}) + s_i \ (\mathbf{x}_{r_4} - \mathbf{x}_{r_5})$
best/1	$\mathbf{v}_i = \mathbf{x}_{\text{best}} + s_i \left(\mathbf{x}_{r_1} - \mathbf{x}_{r_2} \right)$
best/2	$\mathbf{v}_i = \mathbf{x}_{\text{best}} + s_i \left(\mathbf{x}_{r_1} - \mathbf{x}_{r_2} \right) + s_i \left(\mathbf{x}_{r_3} - \mathbf{x}_{r_4} \right)$
current-to-rand/1	$\mathbf{v}_i = \mathbf{x}_i + s_i \left(\mathbf{x}_{r_1} - \mathbf{x}_i \right) + s_i \left(\mathbf{x}_{r_2} - \mathbf{x}_{r_3} \right)$
current-to-best/1	$\mathbf{v}_i = \mathbf{x}_i + s_i \left(\mathbf{x}_{\text{best}} - \mathbf{x}_i \right) + s_i \left(\mathbf{x}_{r_1} - \mathbf{x}_{r_2} \right)$
current-to- <i>p</i> best/1	$\mathbf{v}_i = \mathbf{x}_i + s_i \left(\mathbf{x}_{pbest} - \mathbf{x}_i \right) + s_i \left(\mathbf{x}_{r_1} - \tilde{\mathbf{x}}_{r_2} \right)$
rand-to- <i>p</i> best/1	$\mathbf{v}_i = \mathbf{x}_{r_1} + s_i \left(\mathbf{x}_{p\text{best}} - \mathbf{x}_{r_1} \right) + s_i \left(\mathbf{x}_{r_2} - \tilde{\mathbf{x}}_{r_3} \right)$

each individual **x** to a child **u**. When $\langle s_i, c_i \rangle$ is fixed for each $i \in \{1, ..., \mu\}$ at any *t*, Algorithm 1 becomes the DE with no PCM. Here, Algorithm S.2 shows the DE with no PCM.

For each $i \in \{1, ..., \mu\}$, a mutant vector \mathbf{v}_i is generated by applying differential mutation to randomly selected individuals (line 6). Table 1 shows eight representative DE mutation strategies. If an element of \mathbf{v}_i is outside the bounds, we applied the bound handling method described in [51] to it. In Table 1, the indices r_1 , r_2 , r_3 , r_4 , and r_5 are randomly selected from $\{1, ..., \mu\} \setminus \{i\}$ such that they differ from each other. In Table 1, $\boldsymbol{x}_{\text{best}}$ is the best individual with the lowest objective value in \mathcal{P} . For each $i \in \{1, ..., \mu\}$, \mathbf{x}_{obest} is randomly selected from the top max($\lfloor p \times \mu \rfloor$, 2) individuals in \mathcal{P} , where $p \in [0, 1]$ controls the greediness of the current-to-*p*best/1 and rand-to-pbest/1 strategies. A better individual is likely to be selected as \mathbf{x}_{pbest} when using a smaller p value. For the current-topbest/1 and rand-to-pbest/1 strategies, $\tilde{\mathbf{x}}_{r_2}$ and $\tilde{\mathbf{x}}_{r_3}$ are randomly selected from the union of ${\mathcal P}$ and the external archive ${\mathcal A}.$ The use of inferior individuals in $\mathcal A$ facilitates the diversity of mutant vectors. The rand/1 strategy is the most basic strategy. Since the best/1 and current-to-best/1 strategies are likely to generate mutant vectors near the best individual, they are exploitative. As in the rand/2

strategy, the use of two difference vectors makes the search explorative. The current-to-*p*best/1 strategy is used in state-of-the-art DE algorithms (e.g., [2, 40, 51]).

For each $i \in \{1, ..., \mu\}$, after the mutant vector \mathbf{v}_i has been generated, a child \mathbf{u}_i is generated by applying crossover to \mathbf{x}_i and \mathbf{v}_i (line 7). The binomial crossover [33] is the most representative crossover method in DE, which it can be implemented as follows: for each $j \in \{1, ..., n\}$, if randu $[0, 1] \leq c_i$ or $j = j_{rand}, u_{i,j} = v_{i,j}$. Otherwise, $u_{i,j} = x_{i,j}$. Here, randu[a, b] returns a random value generated from a uniform distribution in the range [a, b]. An index j_{rand} is also randomly selected from $\{1, ..., n\}$ and ensures that at least one element is inherited from \mathbf{v}_i even when $c_i = 0$.

DE performs environmental selection in a pair-wise manner (lines 8–11). For each $i \in \{1, ..., \mu\}$, if $f(\mathbf{u}_i) \leq f(\mathbf{x}_i)$, \mathbf{x}_i is replaced with \mathbf{u}_i (line 11). Thus, the comparison is performed only among the parent \mathbf{x}_i and its child \mathbf{u}_i . Environmental selection in DE allows the replacement of the parent with its child even when they have the same objective value, i.e., $f(\mathbf{u}_i) = f(\mathbf{x}_i)$. As discussed in [31, Section 4.2.3, pp. 192], this property is helpful for DE to escape a plateau, which generally appears in mixed-integer black-box optimization [42, 45].

If the parent \mathbf{x}_i is replaced with its child \mathbf{u}_i , \mathbf{x}_i is added to \mathcal{A} (line 10). When the archive size $|\mathcal{A}|$ exceeds a pre-defined size *a*, randomly selected individuals in \mathcal{A} are deleted to keep the archive size constant (line 12). At the end of each iteration, the internal parameters in the PCM are updated (line 13).

2.2 Lamarckian and Baldwinian repair methods

Let $\mathbf{x} = (x_1, \dots, x_n)^\top \in \mathbb{R}^n$ be an individual in DE. Since \mathbf{x} is an infeasible solution for mixed-integer black-box optimization, \mathbf{x} must be repaired before evaluating \mathbf{x} by the objective function. The rounding operator has been generally used to repair \mathbf{x} in the DE community [20, 22]. Let *i* be an index for an integer variable. In the rounding operator, the *i*-th variable x_i in \mathbf{x} is rounded to the nearest integer. For example, if $x_i = 2.024$, x_i is rounded to 2.

The Lamarckian and Baldwinian repair methods have been considered in the evolutionary computation community [16, 34, 48, 49], where these terms come from the Lamarckian evolution and Baldwin effect in the field of evolutionary biology, respectively. Let \mathbf{x}^{rep} be a repaired feasible version of an individual \mathbf{x} in DE by the rounding operator. In both the Lamarckian and Baldwinian repair methods, $f(\mathbf{x}^{\text{rep}})$ is used as $f(\mathbf{x})$.

On the one hand, the Lamarckian repair method replaces \mathbf{x} with \mathbf{x}^{rep} . Thus, in the Lamarckian repair method, the result of the repair is reflected to the original \mathbf{x} . All individuals in the population are feasible when using the Lamarckian repair method.

On the other hand, the Baldwinian repair method does not make any modifications to **x**. Thus, in the Baldwinian repair method, **x** is infeasible even after the repair. The repaired feasible solution \mathbf{x}^{rep} is used only to compute the objective function f.

Except for [20], most previous studies on DE for mixed-integer black-box optimization did not clearly describe which repair method was used. As pointed out in [32], there is also no clear winner between the Lamarckian and Baldwinian repair methods in evolutionary algorithms. Thus, it is unclear which repair method is suitable for DE for mixed-integer black-box optimization.

2.3 Nine PCMs in DE

This section briefly describes the following nine PCMs in DE: the PCM of CoDE (P-Co) [46], the PCM of SinDE (P-Sin) [4], the PCM of DE-CaR+S (P-CaRS) [30], the PCM of jDE (P-j) [1], the PCM of JADE (P-JA) [51], the PCM of SHADE (P-SHA) [37], the PCM of EPSDE (P-EPS) [28], the PCM of CoBiDE (P-CoBi) [47], and the PCM of cDE (P-c) [43]. Here, our descriptions of PCMs are based on [39]. We re-emphasized that we focus on PCMs in DE (e.g., P-SHA) rather than complex DE algorithms (e.g., SHADE and L-SHADE). While P-Co, P-Sin, and P-CaRS are deterministic PCMs with no feedback, the others are adaptive PCMs. Except for P-CaRS, we selected these PCMs based on the results in [39]. Since DE-CaR+S is one of the latest DE algorithms for mixed-integer black-box optimization, we investigate the performance of P-CaRS. The nine PCMs can be incorporated into Algorithm 1 in a plug-in manner. Although this section briefly describes the nine PCMs due to the paper length limitation, their details can be found in Algorithms S.3-S.12.

Below, for each $i \in \{1, ..., \mu\}$, the pair of s_i and c_i is said to be *successful* if $f(\mathbf{u}_i) \leq f(\mathbf{x}_i)$ in Algorithm 1 (line 9). Otherwise, the pair of s_i and c_i is said to be *failed*. Since the use of successful parameters leads to the improvement of individuals, it is expected that successful parameters are more suitable for a given problem than failed parameters.

P-Co [46]. P-Co is the simplest of the nine PCMs. For each iteration *t*, for each $i \in \{1, ..., \mu\}$, a pair of s_i and c_i is randomly selected from three pre-defined pairs of *s* and *c*: $\langle 1, 0.1 \rangle$, $\langle 1, 0.9 \rangle$, and $\langle 0.8, 0.2 \rangle$.

P-Sin [4]. All individuals use the same *s* and *c* for each iteration *t*. As its name suggests, P-Sin uses the sinusoidal function to generate the *s* and *c* values for each *t* as follows: $s = \frac{1}{2} \left(\frac{t}{t^{\max}} (\sin(2\pi\omega t)) + 1 \right)$ and $c = \frac{1}{2} \left(\frac{t}{t^{\max}} (\sin(2\pi\omega t + \pi)) + 1 \right)$. Here, ω is the angular frequency, and t^{\max} is the maximum number of iterations. In [4], $\omega = 0.25$ was recommended.

P-CaRS [30]. In the nine PCMs, only P-CaRS was designed for mixedinteger black-box optimization. For each iteration *t*, for each individual, s_i is a random value in the range [0.5, 0.55]. In contrast, the same *c* value is assigned to all individuals. For each iteration, *c* is randomly selected from {0.5, 0.6, 0.7, 0.8, 0.9}.

P-j [1]. A pair of s_i and c_i is assigned to each individual, where $s_i = 0.5$ and $c_i = 0.9$ at t = 1. For each iteration t, each individual generates a child by using s_i^{trial} and c_i^{trial} instead of s_i and c_i . With pre-defined probabilities τ_s and τ_c , s_i^{trial} and c_i^{trial} are set to random values as follows: $s_i^{\text{trial}} = \text{randu}[0.1, 1]$ and $c_i^{\text{trial}} = \text{randu}[0, 1]$. Otherwise, $s_i^{\text{trial}} = s_i$ and $c_i^{\text{trial}} = c_i$. In [1], $\tau_s = 0.1$ and $\tau_c = 0.1$ were recommended. If s_i^{trial} and c_i^{trial} are successful, $s_i = s_i^{\text{trial}}$ and $c_i = c_i^{\text{trial}}$ for the next iteration.

P-JA [51]. P-JA adaptively adjusts *s* and *c* by using two metaparameters m_s and m_c , respectively. Both m_s and m_c are initialized to 0.5 for t = 1. For each iteration, for each individual, s_i and c_i are set to values randomly selected from a Cauchy distribution $C(m_s, 0.1)$ and a Normal distribution $N(m_c, 0.1)$, respectively. At the end of each iteration, m_s and m_c are updated based on sets Θ_s and Θ_c of successful *s* and *c* values: $m_s = (1 - \alpha)m_s + \alpha \operatorname{Lmean}(\Theta_s)$ and $m_c = (1 - \alpha)m_c + \alpha \operatorname{mean}(\Theta_c)$. Here, $\alpha \in [0, 1]$ is a learning rate, and $\alpha = 0.1$ was recommended in [51]. Lmean(Θ) and mean(Θ) return the Lehmer mean and mean of the input set Θ , respectively. If $\Theta_s = \emptyset$ and $\Theta_c = \emptyset$ at that *t*, P-JA does not update m_s and m_c .

P-SHA [37]. P-SHA is similar to P-JA. Instead of m_s and m_c , P-SHA adaptively adjusts s and c by using two historical memories $\mathbf{m}_s = (m_{s,1}, ..., m_{s,h})^{\top}$ and $\mathbf{m}_c = (m_{c,1}, ..., m_{c,h})^{\top}$, respectively. Here, h is a memory size, and h = 10 was recommended in [38]. For t = 1, all h elements in \mathbf{m}_s and \mathbf{m}_c are initialized to 0.5. A memory index $k \in \{1, ..., h\}$ is also initialized to 1.

Although some slightly different versions of P-SHA are available, we consider the simplest one described in [39]. For each iteration, for each $i \in \{1, ..., \mu\}$, s_i and c_i are set to values randomly selected from $C(m_{s,r}, 0.1)$ and $N(m_{c,r}, 0.1)$, respectively. Here, r is a random number in $\{1, ..., h\}$. At the end of each iteration, the k-th elements $m_{s,k}$ and $m_{c,k}$ are updated based on sets Θ_s and Θ_c of successful sand c values: $m_{s,k} = \text{Lmean}(\Theta_s)$ and $m_{c,k} = \text{Lmean}(\Theta_c)$. After the update, k is incremented. If k > h, k is re-initialized to 1.

P-EPS [28]. P-EPS uses two parameter sets for the adaptation of *s* and *c*: $Q_s = \{0.4, 0.5, ..., 0.9\}$ and $Q_c = \{0.1, 0.2, ..., 0.9\}$. For t = 1, for each $i \in \{1, ..., \mu\}$, s_i and c_i are initialized with values randomly selected from Q_s and Q_c , respectively. At the end of each iteration, if s_i and c_i are failed, they are re-initialized.

P-*CoBi* [47]. P-CoBi is similar to P-EPS. The only difference between the two is how to generate s_i and c_i . In P-CoBi, s_i and c_i are set to values randomly selected from a bimodal distribution consisting of two Cauchy distributions as follows: $s_i \sim C(0.65, 0.1)$ or C(1, 0.1), and $c_i \sim C(0.1, 0.1)$ or C(0.95, 0.1).

P-c [43]. For each iteration, for each $i \in \{1, ..., \mu\}$, P-c randomly selects a pair of *s* and *c* from nine combinations of values taken from $\{0.5, 0.8, 1\}$ and $\{0, 0.5, 1\}$, i.e., $\mathbf{q}_1 = \langle 0.5, 0 \rangle$, $\mathbf{q}_2 = \langle 0.5, 0.5 \rangle$, ..., $\mathbf{q}_9 = \langle 1, 1 \rangle$. Here, for each $k \in \{1, ..., 9\}$, the probability $\tau_k \in [0, 1]$ of selecting \mathbf{q}_k is given as follows: $\tau_k = (o_k + \epsilon)/(\sum_{l=1}^9 (o_l + \epsilon))$, where ϵ is a parameter to avoid $\tau_k = 0$. In addition, o_k represents the number of successful trials of \mathbf{q}_k from the last initialization. When any τ_k is below the threshold δ , o_1, \ldots, o_9 are reinitialized to 0. The recommended settings of ϵ and δ are 2 and 1/45, respectively.

3 EXPERIMENTAL SETUP

This section describes the experimental setup. We conducted all experiments using the COCO platform [13]. We used a workstation with an Intel(R) 48-Core Xeon Platinum 8260 (24-Core×2) 2.4GHz and 384GB RAM using Ubuntu 22.04. The bbob-mixint suite [42] used in this work consists of the 24 mixed-integer functions f_1, \ldots, f_{24} , which are mixed-integer versions of the 24 noiseless BBOB functions [14]. For each *n*-dimensional problem, 4n/5 and 1n/5 variables are integer and continuous, respectively. The feasible solution space \mathbb{X} consists of $\mathbb{X} = \{0, 1\}^{\frac{n}{5}} \times \{0, 1, 2, 3\}^{\frac{n}{5}} \times \{0, 1, \ldots, 7\}^{\frac{n}{5}} \times \{0, 1, \ldots, 15\}^{\frac{n}{5}} \times [-5, 5]^{\frac{n}{5}}$. Details of the 24 functions can be found in https://numbbo.github.io/gforge/preliminary-bbobmixint-documentation/bbob-mixint-doc.pdf. We set *n* to 5, 10, 20, 40, 80, and 160. According to the COCO platform, we set the number of instances to 15 for each function. In other words, we perform 15 independent runs for each function.

We implemented DE algorithms with the nine PCMs in Python. We used the default settings of the hyper-parameters for the nine PCMs. In addition to them, we evaluate the performance of DE with no PCM as a baseline. We denote this version of DE as "NOPCM". Here, as in most previous studies [1, 51], we set s = 0.5 and c = 0.9 for NOPCM. We set μ to 100. We set p = 0.05 and the archive size $a = \mu$ in the current-to-*p*best/1 and rand-to-*p*best/1 strategies. We set the maximum number of function evaluations to $10^4 \times n$.

4 **RESULTS**

This section describes our analysis results. Through experiments, Sections 4.1–4.3 aim to address the three research questions (**RQ1–RQ3**) described in Section 1, respectively. For the sake of simplicity, we refer to "a DE with a PCM" as "a PCM". For example, we refer to a DE with P-j as P-j.

4.1 Comparisons of PCMs

Figures 1–3 show comparison of the 10 DE algorithms with the nine PCMs (Section 2.3) and NOPCM (Section 3) on the 24 bbob-mixint functions for $n \in \{10, 80, 160\}$. Figures 1 and 2 show the results when using the rand/1 and rand-to-*p*best/1 strategies, respectively. Here, the Baldwinian repair method is used in Figures 1 and 2. In contrast, Figure 3 shows the results when using the rand/1 strategy and Lamarckian repair method. Figures S.1–S.16 show all results of DE algorithms using the eight mutation strategies for $n \in \{5, 10, 20, 40, 80, 160\}$.

Figures 1–3 show the bootstrapped empirical cumulative distribution (ECDF) [13] based on the results on the 24 bbob-mixint functions with each *n*. We used the COCO software [13] to generate all ECDF figures in this paper. Let $f_{\text{target}} = f(\mathbf{x}^*) + f_{\Delta}$ be a target value to reach, where \mathbf{x}^* is the optimal solution, and f_{Δ} is any one of 51 evenly log-spaced f_{Δ} values $\{10^2, 10^{1.8}, \ldots, 10^{-7.8}, 10^{-8}\}$. Thus, 51 f_{target} values are available for each function instance, and 18 360 f_{target} values (= 51 × 15 × 24) are available for all 15 instances of the 24 bbob-mixint functions. In the ECDF figure, the vertical axis represents the proportion of f_{target} values reached by an optimizer within specified function evaluations. The horizontal axis represents the number of function evaluations. For example, Figure 1(b) shows that P-j solved about 35% of the 18 360 f_{target} values within $10^3 \times n$ function evaluations for n = 80. Figure 1(b) shows that P-j is about ten times faster than P-Co to reach the same precision.

Statistical significance is tested with the rank-sum test for $f_{\Delta} \in \{10^1, 10^0, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-5}, 10^{-7}\}$ by using COCO. The statistical test results can be found in https://zenodo.org/doi/10.5281/zenodo.10608500. Due to the paper length limitation, we do not describe the statistical test results, but they are consistent with the ECDF figures in most cases.

Table 2 shows the best PCMs on the 24 bbob-mixint functions with each *n* in terms of the ECDF value at $10^4 n$ evaluations when using each mutation strategy. Table 2(a) and (b) show the results when using the Baldwinian and Lamarckian repair methods, respectively. *4.1.1* Comparison when using rand/1 and the Baldwinian repair method. As shown in Figure 1(a), when using the rand/1 strategy for n = 10, NOPCM performs the best almost until $10^4 n$ function evaluations. P-CoBi performs slightly better than other PCMs exactly at $10^4 n$ function evaluations. These results suggest that DE with the rand/1 mutation strategy does not require any PCM for low dimension. In fact, as shown in Table 2(a), NOPCM is the best performs for $n \ge 5$. However, NOPCM performs poorly for $n \ge 20$.

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Figure 1: Comparison of the nine PCMs and NOPCM using the rand/1 strategy and Baldwinian repair method on the 24 bbob-mixint functions with $n \in \{10, 80, 160\}$.



Figure 2: Comparison of the nine PCMs and NOPCM using the rand-to-*p*best/1 strategy and Baldwinian repair method on the 24 bbob-mixint functions with $n \in \{10, 80, 160\}$.



Figure 3: Comparison of the nine PCMs and NOPCM using the rand/1 strategy and Lamarckian repair method on the 24 bbob-mixint functions with $n \in \{10, 80, 160\}$.

As shown in Figures 1(b) and (c), P-Sin performs the best for n = 80 and 160 at $10^4 n$ function evaluations, followed by P-j. Although P-j is outperformed by P-Sin at the end of the run, P-j shows the better anytime performance than other PCMs, including P-Sin.

4.1.2 Comparison when using rand-to-pbest/1 and the Baldwinian repair method. As shown in Figure 2, the rankings of the PCMs for

the rand/1 and rand-to-*p*best/1 mutation strategies are totally different. NOPCM performs the worst for any *n*. Although P-Sin is the best for n = 160 when using rand/1, P-Sin is the third worst when using rand-to-*p*best/1. P-Co shows the second worst performance for Figures 1(b)–(c), but P-Co performs the best for Figures 2(b)–(c) at 10^4n function evaluations.

Table 2: Best PCMs on the bbob-mixint suite based on the results when using each mutation strategy at 10^4n function evaluations. In the table, ctb/1, ctr/1, ctp/1, and rtp/1 represent the current-to-best/1, current-to-rand/1, current-to-pbest/1, and rand-to-pbest/1 mutation strategies, respectively.

(a) Baldwinian repair method

Strategy	<i>n</i> = 5	<i>n</i> = 10	<i>n</i> = 20	<i>n</i> = 40	<i>n</i> = 80	<i>n</i> = 160
rand/1	NOPCM	P-CoBi	P-c	P-Sin	P-Sin	P-Sin
rand/2	P-Sin	P-Sin	P-Sin	P-Sin	P-j	P-j
best/1	P-Co	P-Co	P-Co	P-Co	P-Čo	P-JĂ
best/2	P-CoBi	P-CoBi	P-EPS	P-CoBi	P-c	P-c
ctb/1	P-CoBi	P-Co	P-Co	P-CoBi	P-Co	P-JA
ctr/1	P-CoBi	P-CoBi	P-CoBi	P-CoBi	P-CoBi	P-ČoBi
ctp/1	P-Co	P-CoBi	P-Co	P-Co	P-Co	P-Co
rt p /1	P-Co	P-Co	P-Co	P-Co	P-Co	P-Co

(b) Lamarckian repair method

Strategy	<i>n</i> = 5	<i>n</i> = 10	<i>n</i> = 20	n = 40	<i>n</i> = 80	n = 160
rand/1	P-CoBi	P-CoBi	P-CoBi	P-CaRS	P-CoBi	P-JA
rand/2	P-Sin	P-Sin	P-CoBi	P-Sin	P-CoBi	P-CoBi
best/1	P-Co	P-Co	P-Co	P-Co	P-Co	P-Co
best/2	P-CoBi	P-CoBi	P-Co	P-CaRS	P-Co	P-Co
ctb/1	P-Co	P-CoBi	P-Co	P-Co	P-Co	P-Co
ctr/1	P-CoBi	P-CoBi	P-Co	P-CoBi	P-CoBi	P-CoBi
ct <i>p</i> /1	P-CoBi	P-CoBi	P-CoBi	P-Co	P-Co	P-Co
rt <i>p</i> /1	P-CoBi	P-SHA	P-Co	P-Co	P-Co	P-Co

Table 3: Top three configurations on the bbob-mixint suite for each *n*. "B" and "L" indicate the Baldwinian and Lamarckian repair methods, respectively.

n	1st	2nd	3rd
n = 5 n = 10 n = 20 n = 40 n = 80 n = 160	$ \begin{array}{l} \langle \text{P-CoBi, rand/1, L} \rangle \\ \langle \text{P-CoBi, ctb/1, L} \rangle \\ \langle \text{P-Co, rt}p/1, \text{B} \rangle \\ \langle \text{P-CoBi, ctr/1, L} \rangle \\ \langle \text{P-Sin, rand/1, B} \rangle \\ \langle \text{P-j, rand/2, B} \rangle \end{array} $	$\begin{array}{l} \left< P\text{-CoBi, ct} p/1, L \right> \\ \left< P\text{-CoBi, rand/1, L} \right> \\ \left< P\text{-CoBi, rand/1, L} \right> \\ \left< P\text{-CoBi, rand/2, B} \right> \\ \left< P\text{-Sin, rand/2, B} \right> \\ \left< P\text{-j, rand/2, B} \right> \\ \left< P\text{-Sin, rand/1, B} \right> \end{array}$	$ \begin{array}{l} \left< P\text{-CoBi, ctr/1, L} \right> \\ \left< P\text{-CoBi, ct}p/1, L \right> \\ \left< P\text{-Co, ctr/1, L} \right> \\ \left< P\text{-CaRS, rand/1, L} \right> \\ \left< P\text{-CoBi, ctr/1, L} \right> \\ \left< P\text{-Co, rt}p/1, L \right> \end{array} $

4.1.3 Comparison when using the Lamarckian repair method. Interestingly, as shown in Figure 3, the use of the Lamarckian repair method significantly deteriorates or improves the performance of some PCMs. For example, as shown in Figures 1(b)–(c) and 3(b)–(c), the use of the Lamarckian repair method significantly deteriorates the performance of P-j with the rand/1 strategy for $n \in \{80, 160\}$. In contrast, as seen from Figures 1 and 3, the performance of P-Co is significantly improved by using the Lamarckian one.

4.1.4 *Summary*. As seen from Table 2, with one exception, any one of the nine PCMs performs the best for each case. Although most previous studies (e.g., [20, 22, 24, 25]) did not use any PCMs, our observation suggests the importance of PCMs in DE for mixed-integer black-box optimization.

As shown in Table 2, the best PCM significantly depends on the combination of the mutation strategy and method. Roughly speaking, P-CoBi and P-Co perform the best in many cases, followed by P-Sin. P-CoBi works especially well for low dimensions, i.e., $n \in \{5, 10\}$. P-CaRS, P-EPS, P-c, P-j, P-JA, and P-SHA show the best performance in a few cases. Table 2 suggests that P-Co is suitable

when using the Lamarckian repair method and exploitative mutation strategies, including best/1, best/2, best/1, current-to-best/1, current-to-pbest/1, and rand-to-pbest/1.

Although the previous study [39] reported the *poor* performance of P-Co for *numerical* black-box optimization, our results show the *excellent* performance of P-Co for *mixed-integer* black-box optimization. In contrast, P-SHA is one of the state-of-the-art PCMs in DE [39]. P-SHA has also been used in state-of-the-art DE algorithms, including a number of L-SHADE-based algorithms [2, 40]. However, as shown in Figures 1–3, P-SHA perform well on the bbob-mixint suite only at the early stage of the search. As seen from Table 2, P-SHA performs the best in only one case. This observation suggests that replacing P-SHA with P-Co, P-CoBi, or P-j may improve the performance of L-SHADE_{ACO} [23].

On the best configuration. Although we focus on PCMs, it is 415 interesting to discuss which configuration performs the best. Table 3 shows the top 3 out of 160 DE configurations on the bbob-mixint suite for each *n*, where the 160 configurations include the 9 PCMs and NOPCM, 8 mutation strategies, and the two repair methods $(10 \times 8 \times 2 = 160)$. In Table 3, a tuple represents a DE configuration that consists of a PCM, mutation strategy, and repair method. Similar to the above discussion, as seen from Table 3, the configurations including P-CoBi and P-Co perform well for low dimensions. In contrast, the configurations including P-j and P-Sin show the best performance for high dimensions. Although the Lamarckian repair method is included in most of the top three configurations, our results show that the Baldwinian repair method is suitable for P-Sin and P-j, especially for high dimensions. Thus, there is no clear winner between the two repair methods. Interestingly, the classical rand/1 and rand/2 strategies are included in 9 out of the top 18 configurations. Since the best/1 and best/2 strategies are not found in Table 3, we can say that too exploitative mutation strategies are not suitable. This may be because the bbob-mixint functions have many plateaus from the point of view of DE due to the use of the rounding operator. Although the behavior analysis of DE is beyond the scope of this paper, it is an avenue for future research".

Answers to RQ1

Although most previous studies (e.g., [20, 22, 24, 25]) did not use any PCM, our results demonstrated the importance of PCMs in DE for mixed-integer black-box optimization. We found that the best PCM significantly depends on the combination of the mutation strategy and repair method. Some of our results are inconsistent with the results shown in [39] for numerical blackbox optimization. For example, we demonstrated that P-Co performs significantly better than other PCMs on the bbob-mixint suite, especially when using the Lamarckian repair method and exploitative mutation strategies. In contrast, our results show the unsuitability of P-SHA, one of the state-of-the-art PCMs, for mixed-integer black-box optimization.

4.2 Comparison with CMA-ES

As mentioned in Section 1, the previous studies [8, 29, 42] demonstrated that the extended versions of CMA-ES outperform DE with no PCM (DE-scipy [42] described later). However, the results in Section 4.1 show that the use of PCM can significantly improve the

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Figure 4: Comparison of P-j and P-CoBi with the three CMA-ES variants on the 24 bbob-mixint functions with $n \in \{10, 80, 160\}$.

performance of DE. Thus, it is interesting to compare DE with a suitable PCM with the CMA-ES variants.

We consider the comparison with the following three extensions of CMA-ES: CMA-ES-pycma [42], CMA-ESwM [8], and cmaIH1e-1 [29]. Both CMA-ES-pycma and cmaIH1e-1 are the pycma [12] implementations of CMA-ES with simple integer handling. However, the pycma version of cmaIH1e-1 is newer than that of CMA-ES-pycma. Although the previous study [29] investigated three versions of CMA-ES, its results showed that cmaIH1e-1 was the best performer among them. CMA-ESwM is the CMA-ES with margin [9], which uses a lower bound on the marginal probability for each integer variable. In addition, we consider the SciPy version of DE (DE-scipy [42]). We used the benchmarking results of the four optimizers provided by the COCO data archive (https://numbbo.github.io/data-archive).

Figure 4 shows the comparison of two DE algorithms with P-j and P-CoBi with the three CMA-ES variants on the 24 bbob-mixint functions for $n \in \{10, 80, 160\}$. Here, the benchmarking data of CMA-ES-pycma and cmaIH1e-1 for n = 160 are not available. In Figure 4, P-j uses the rand/2 strategy and Baldwinian repair method, and P-CoBi uses the rand/1 strategy and Lamarckian repair method. As shown in Table 3, P-CoBi and P-j with these configurations perform the best for n = 5 and n = 160, respectively. Figure S.17 shows the results for all n, where it is similar to Figure 4.

As shown in Figure 4, for all *n*, P-j and P-CoBi are outperformed by the CMA-ES variants for smaller budgets. In contrast, P-j and P-CoBi perform significantly better than the CMA-ES variants for larger budgets. Although the configuration of P-CoBi is suitable for low dimensions, it outperforms the CMA-ES variants even for $n \ge$ 80. As expected, the performance of P-j and P-CoBi is significantly better than that of DE-scipy for high dimensions.

Figure S.18(a)–(e) show the comparison on the five function groups for n = 80, respectively. As expected, P-j and P-CoBi perform well on the separable functions $(f_1, ..., f_5)$. In addition, P-CoBi shows the best performance on the functions with high conditioning $(f_{10}, ..., f_{14})$ and multimodal functions with weak global structure $(f_{20}, ..., f_{24})$ at 10⁴ function evaluations. P-j also performs the best the functions with low conditioning $(f_6, ..., f_9)$ at 10⁴ function evaluations. Similar to Figure 4, for any function group, the CMA-ES

variants outperform P-j and P-CoBi within about 10^3n function evaluations. In addition, cmaIH1e-1 is the best performer on the multimodal functions with adequate global structure ($f_{15}, ..., f_{19}$). Thus, no optimizer dominates others on any function at any time. These observations suggest that automated algorithm selection [18, 19] with an algorithm portfolio consisting of DE and CMA-ES is a promising approach for mixed-integer black-box optimization.

Answers to RQ2

Our results show that DE algorithms with suitable PCMs (P-j and P-CoBi) can perform significantly better than the three CMA-ES variants with integer handling on the bbob-mixint suite, especially for high dimensions and larger budgets of function evaluations. However, we observed that the DE algorithms are generally outperformed by the CMA-ES variants for smaller budgets of function evaluations and some particular bbob-mixint functions. This complementarity between DE and CMA-ES suggests a promising possibility of automated algorithm selection.

4.3 How does P-SHA fail?

Despite the high performance of P-SHA for numerical black-box optimization, the results in Section 4.1 indicate the poor performance of P-SHA on the bbob-mixint suite. This section investigates the behavior of P-SHA to find out why it did not work well.

Figure 5 shows some analysis results of P-SHA with the rand/1 strategy and Baldwinian repair method on f_3 (the separable Rastrigin function) with n = 80. Since f_3 is separable, it is easy for DE to solve f_3 . Nevertheless, P-SHA found the optimal solution in only 3 out of 15 runs. P-SHA also shows the third worst performance. For the sake of reference, Figure S.19 shows the results of P-JA, which found the optimal solution in all 15 runs.

Figure 5(a) shows the error value $|f(\mathbf{x}_{bsf}) - f(\mathbf{x}^*)|$ of the best solution found so far \mathbf{x}_{bsf} by P-SHA in a typical single run. As seen from Figure 5(a), the improvement of \mathbf{x}_{bsf} stops at about 92 000 function evaluations.

Figure 5(b) shows a diversity value (div) and the number of individuals with the same objective value (nsame) as the best individual $\mathbf{x}_{best} = \underset{\mathbf{x} \in \mathcal{P}}{\operatorname{argmin}} f(\mathbf{x})$ in the population \mathcal{P} for each iteration.

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Figure 5: Analysis results of a typical single run of P-SHA on f_3 with n = 80.

Here, we calculated the div and nsame values of \mathcal{P} as follows: div $(\mathcal{P}) = \frac{1}{\mu} \sum_{\mathbf{x} \in \mathcal{P} \setminus \{\mathbf{x}_{best}\}} \|\mathbf{x} - \mathbf{x}_{best}\|$ and nsame $(\mathcal{P}) = |\{\mathbf{x} \mid \mathbf{x} \in \mathcal{P} \text{ s.t. } f(\mathbf{x}) = f(\mathbf{x}_{best})\}|$. A small div (\mathcal{P}) value means that most individuals in \mathcal{P} are close to the best individual \mathbf{x}_{best} in the solution space. A small nsame (\mathcal{P}) value means that most individuals in \mathcal{P} are at a plateau induced by the rounding operator. Since div and nsame can be calculated only after μ function evaluations, Figure 5(b) starts from μ function evaluations, where $\mu = 100$. On the one hand, as seen from Figure 5(b), the nsame value suddenly becomes 100 at about 92 000 function evaluations. Here, nsame $(\mathcal{P}) = 100$ means that all 100 individuals in \mathcal{P} are in the same plateau. On the other hand, the div value increases after the nsame value becomes 100. These results indicate that individuals in \mathcal{P} explore the solution space even after getting stuck on a plateau in the objective space.

Figure 5(c) shows the 10 elements of the two memories \mathbf{m}_{s} and \mathbf{m}_c for the adaptation of the scale factor s and crossover rate c, respectively. Figure 5(d) also shows the mean of successful s and c values for each iteration. On the separable Rastrigin function, adaptive PCMs generally generate large s and small c values to handle the multimodality and exploit the separability [1, 36, 51]. In fact, as seen from Figure S.19(c), P-JA adjusts m_s and m_c to large and small values during the search, respectively. However, as shown in Figure 5(c), P-SHA adjusts \mathbf{m}_s and \mathbf{m}_c to small and large values until about 92 000 function evaluations, respectively. This kind of parameter adaptation can be found when addressing unimodal functions, e.g., the Sphere function [1, 36, 51]. Thus, this adaptation of s and c in P-SHA fails on f_3 . This failed parameter adaptation causes the stagnation of the search as shown in Figures 5(a) and (b). Interestingly, as seen from Figure 5(c), P-SHA correctly adjusts m_s and m_c to large and small values after about 92000 function evaluations, respectively. Since the population has already stagnated at a plateau after about 92 000 function evaluations, this improvement of parameter adaptation in P-SHA is too late.

A previous study [41] showed the pathological behavior of some adaptive PCMs on functions whose search space characteristics of variables are different from each other. The bbob-mixint functions can be considered to be the same as those kinds of functions investigated in [41] due to the existence of integer variables. In addition, P-SHA has a high tracking performance with respect to successful parameters [38]. These reasons suggest that P-SHA was particularly influenced by the properties of the bbob-mixint functions.

Answers to RQ3

We showed the failed parameter adaptation of the two memories \mathbf{m}_s and \mathbf{m}_c in P-SHA on f_3 (the separable Rastrigin function), which causes the stagnation of the search. Interestingly, the parameter adaptation in P-SHA works well after all individuals in the population reach the same plateau. Although most PCMs can address f_3 easily, P-SHA struggles on f_3 . This is mainly due to the property of the bbob-mixint functions and the high tracking performance of P-SHA.

5 CONCLUSION

We have investigated the performance of the 9 PCMs in DE with the 8 mutation strategies and 2 repair methods on the 24 bbob-mixint functions. Although most previous studies on DE for mixed-integer black-box optimization did not use any PCM, our results show that the use of PCMs can significantly improve the performance of DE (Section 4.1). We have demonstrated that the best PCM depends on the choice of the mutation strategy and the repair method. Unlike the results for numerical black-box optimization reported in [39], our results show that P-SHA is not suitable for mixedinteger black-box optimization. In contrast, we observed that some simple PCMs (e.g., P-Co, P-CoBi, P-j, and P-Sin) work well on the bbob-mixint suite. We have also shown that the DE algorithms with suitable PCMs perform significantly better than the CMA-ES variants with integer handling for larger budgets of function evaluations (Section 4.2). Finally, we have investigated how the parameter adaptation in P-SHA fails (Section 4.3).

We believe that our findings contribute to the design of an efficient DE algorithm for mixed-integer black-box optimization. For example, our results suggest the promise of incorporating either P-Co, P-CoBi, P-j, or P-Sin into a new DE algorithm. Fitness landscape analysis on the bbob-mixint suite is necessary for a better understanding of the behavior of DE algorithms. Decomposed components in this work can be straightforwardly used for automatic configuration [27] of DE. Algorithm selection for mixed-integer black-box optimization is also promising based on our observation of the complementarity between DE and CMA-ES.

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Supplement

Algorithm S.2: The basic DE algorithm with no PCM (NOPCM)

1 $t \leftarrow 1$, initialize $\mathcal{P} = \{\mathbf{x}_1, \dots, \mathbf{x}_{\mu}\}$ randomly, $\mathcal{A} \leftarrow \emptyset$; ² while The termination criteria are not met do for $i \in \{1, ..., \mu\}$ do 3 $\mathbf{v}_i \leftarrow$ Apply differential mutation with *s* to individuals in \mathcal{P} ; 4 $\mathbf{u}_i \leftarrow \text{Apply binomial crossover with } c \text{ to } \mathbf{x}_i \text{ and } \mathbf{v}_i;$ 5 **for** $i \in \{1, ..., \mu\}$ **do** 6 if $f(\mathbf{u}_i) \leq f(\mathbf{x}_i)$ then 7 $\mathcal{A} \leftarrow \mathcal{A} \cup \{\mathbf{x}_i\};$ 8 9 $\mathbf{x}_i \leftarrow \mathbf{u}_i;$ if $|\mathcal{A}| > a$ then Delete randomly selected individuals in \mathcal{A} unless $|\mathcal{A}| < a$; 10 $t \leftarrow t + 1;$ 11

Algorithm S.3: The basic DE algorithm with P-Co

1 $t \leftarrow 1$, initialize $\mathcal{P} = \{\mathbf{x}_1, \dots, \mathbf{x}_\mu\}$ randomly, $\mathcal{A} \leftarrow \emptyset$; ² while The termination criteria are not met do for $i \in \{1, ..., \mu\}$ do 3 $\langle s_i, c_i \rangle \leftarrow \text{Randomly select one from three parameter pairs: } \langle 1, 0.1 \rangle, \langle 1, 0.9 \rangle, \text{ and } \langle 0.8, 0.2 \rangle.$ 4 for $i \in \{1, ..., \mu\}$ do 5 $\mathbf{v}_i \leftarrow \text{Apply differential mutation with } s_i \text{ to individuals in } \mathcal{P};$ 6 $\mathbf{u}_i \leftarrow$ Apply binomial crossover with c_i to \mathbf{x}_i and \mathbf{v}_i ; 7 for $i \in \{1, ..., \mu\}$ do 8 if $f(\mathbf{u}_i) \leq f(\mathbf{x}_i)$ then 9 $\mathcal{A} \leftarrow \mathcal{A} \cup \{\mathbf{x}_i\};$ 10 $\mathbf{x}_i \leftarrow \mathbf{u}_i;$ 11 **if** $|\mathcal{A}| > a$ **then** Delete randomly selected $|\mathcal{A}| - a$ individuals in \mathcal{A} ; 12 $t \leftarrow t + 1;$ 13

Algorithm S.4: The basic DE algorithm with P-Sin

1 $t \leftarrow 1$, initialize $\mathcal{P} = \{\mathbf{x}_1, \dots, \mathbf{x}_{\mu}\}$ randomly, $\mathcal{A} \leftarrow \emptyset$; 2 while The termination criteria are not met do $s \leftarrow \frac{1}{2} \left(\frac{t}{t^{\max}} (\sin(2\pi\omega t)) + 1 \right);$ 3 $c \leftarrow \frac{1}{2} \left(\frac{t}{t^{\max}} (\sin(2\pi\omega t + \pi)) + 1 \right);$ 4 for $i \in \{1, ..., \mu\}$ do 5 $\mathbf{v}_i \leftarrow$ Apply differential mutation with *s* to individuals in \mathcal{P} ; 6 $\mathbf{u}_i \leftarrow$ Apply binomial crossover with *c* to \mathbf{x}_i and \mathbf{v}_i ; 7 **for** $i \in \{1, ..., \mu\}$ **do** 8 if $f(\mathbf{u}_i) \leq f(\mathbf{x}_i)$ then 9 $\mathcal{A} \leftarrow \mathcal{A} \cup \{\mathbf{x}_i\};$ 10 11 $\mathbf{x}_i \leftarrow \mathbf{u}_i;$ **if** $|\mathcal{A}| > a$ **then** Delete randomly selected $|\mathcal{A}| - a$ individuals in \mathcal{A} ; 12 $t \leftarrow t + 1;$ 13

Algorithm S.5: The basic DE algorithm with P-CaRS

1 $t \leftarrow 1$, initialize $\mathcal{P} = \{\mathbf{x}_1, \dots, \mathbf{x}_\mu\}$ randomly, $\mathcal{A} \leftarrow \emptyset$; ² while The termination criteria are not met do **for** $i \in \{1, ..., \mu\}$ **do** $s_i \leftarrow randu[0.5, 0.55]$; 3 $c \leftarrow \text{Randomly select one from } \{0.5, 0.6, 0.7, 0.8, 0.9\};$ 4 for $i \in \{1, ..., \mu\}$ do 5 $\mathbf{v}_i \leftarrow \text{Apply differential mutation with } s_i \text{ to individuals in } \mathcal{P};$ 6 $\mathbf{u}_i \leftarrow \text{Apply binomial crossover with } c \text{ to } \mathbf{x}_i \text{ and } \mathbf{v}_i;$ 7 for $i \in \{1, ..., \mu\}$ do 8 if $f(\mathbf{u}_i) \leq f(\mathbf{x}_i)$ then 9 $\mathcal{A} \leftarrow \mathcal{A} \cup \{\mathbf{x}_i\};$ 10 $\mathbf{x}_i \leftarrow \mathbf{u}_i;$ 11 **if** $|\mathcal{A}| > a$ **then** Delete randomly selected $|\mathcal{A}| - a$ individuals in \mathcal{A} ; 12 13 $t \leftarrow t + 1;$

Algorithm S.6: The basic DE algorithm with P-j

1 $t \leftarrow 1$, initialize $\mathcal{P} = \{\mathbf{x}_1, \dots, \mathbf{x}_\mu\}$ randomly, $\mathcal{A} \leftarrow \emptyset$; 2 **for** *i* ∈ {1, ..., μ } **do** *s*_{*i*} ← 0.5, *c*_{*i*} ← 0.9; 3 while The termination criteria are not met do **for** $i \in \{1, ..., \mu\}$ **do** 4 if randu[0,1] < τ_s then $s_i^{\text{trial}} \leftarrow \text{randu}[0.1,1]$; 5 else $s_i^{\text{trial}} \leftarrow s_i$; 6 if randu[0, 1] < τ_c then $c_i^{\text{trial}} \leftarrow \text{randu}[0, 1]$; 7 else $c_i^{\text{trial}} \leftarrow c_i$; 8 for $i \in \{1, ..., \mu\}$ do 9 $\mathbf{v}_i \leftarrow$ Apply differential mutation with s_i^{trial} to individuals in \mathcal{P} ; 10 $\mathbf{u}_i \leftarrow \text{Apply binomial crossover with } c_i^{\text{trial}} \text{ to } \mathbf{x}_i \text{ and } \mathbf{v}_i;$ 11 **for** $i \in \{1, ..., \mu\}$ **do** 12 if $f(\mathbf{u}_i) \leq f(\mathbf{x}_i)$ then 13 $\mathcal{A} \leftarrow \mathcal{A} \cup \{\mathbf{x}_i\};$ 14 $\begin{aligned} \mathbf{x}_i &\leftarrow \mathbf{u}_i; \\ s_i &\leftarrow s_i^{\text{trial}}, c_i \leftarrow c_i^{\text{trial}}; \end{aligned}$ 15 16 **if** $|\mathcal{A}| > a$ **then** Delete randomly selected individuals in \mathcal{A} unless $|\mathcal{A}| < a$; 17 $t \leftarrow t + 1;$ 18

Algorithm S.7: The basic DE algorithm with P-JA

```
1 t \leftarrow 1, initialize \mathcal{P} = \{\mathbf{x}_1, \dots, \mathbf{x}_\mu\} randomly, \mathcal{A} \leftarrow \emptyset;
 <sup>2</sup> m_s \leftarrow 0.5, m_c \leftarrow 0.5;
 <sup>3</sup> while The termination criteria are not met do
            for i \in \{1, ..., \mu\} do
  4
                   do
  5
  6
                      s_i \leftarrow \text{Randomly select a value from } C(m_s, 0.1);
  7
                   while s_i \leq 0;
                   s_i \leftarrow \min\{s_i, 1\};
  8
                   c_i \leftarrow \text{Randomly select a value from } N(m_c, 0.1);
  9
               if c_i \notin [0, 1] then c_i \leftarrow Replace with 0 or 1 closest to c_i;
 10
            for i \in \{1, ..., \mu\} do
11
                   \mathbf{v}_i \leftarrow Apply differential mutation with s_i to individuals in \mathcal{P};
 12
               \mathbf{u}_i \leftarrow \text{Apply binomial crossover with } c_i \text{ to } \mathbf{x}_i \text{ and } \mathbf{v}_i;
 13
            \Theta_s \leftarrow \emptyset, \Theta_c \leftarrow \emptyset;
14
            for i \in \{1, ..., \mu\} do
15
                   if f(\mathbf{u}_i) \leq f(\mathbf{x}_i) then
 16
                          \mathcal{A} \leftarrow \mathcal{A} \cup \{\mathbf{x}_i\};
 17
                          \mathbf{x}_i \leftarrow \mathbf{u}_i;
 18
                          \Theta_s \leftarrow \Theta_s \cup \{s_i\}, \Theta_c \leftarrow \Theta_c \cup \{c_i\};
 19
            if |\mathcal{A}| > a then Delete randomly selected individuals in \mathcal{A} unless |\mathcal{A}| < a;
20
            if \Theta_s \neq \emptyset and \Theta_c \neq \emptyset then
21
                   m_s \leftarrow (1 - \alpha)m_s + \alpha \operatorname{Lmean}(\Theta_s);
22
                m_c \leftarrow (1-\alpha)m_c + \alpha \mathrm{mean}(\Theta_c); 
23
        t \leftarrow t + 1;
\mathbf{24}
```

Algorithm S.8: The basic DE algorithm with P-SHA

```
1 t \leftarrow 1, initialize \mathcal{P} = \{\mathbf{x}_1, \dots, \mathbf{x}_\mu\} randomly, \mathcal{A} \leftarrow \emptyset;
 k \leftarrow 1;
 3 for i \in \{1, ..., h\} do
 4 m_{s,i} \leftarrow 0.5, m_{c,i} \leftarrow 0.5;
 _{\rm 5}~ while The termination criteria are not met do
            for i \in \{1, ..., \mu\} do
 6
                   r \leftarrow \text{Randomly select a value from } \{1, \dots, h\};
 7
                   do
 8
                     s_i \leftarrow \text{Randomly select a value from } C(m_{s,r}, 0.1);
 9
                   while s_i \leq 0;
 10
                   s_i \leftarrow \min\{s_i, 1\};
11
                   c_i \leftarrow \text{Randomly select a value from } N(m_{c,r}, 0.1);
12
                 if c_i \notin [0, 1] then c_i \leftarrow Replace with 0 or 1 closest to c_i;
13
            for i \in \{1, ..., \mu\} do
14
                  \mathbf{v}_i \leftarrow \text{Apply differential mutation with } s_i \text{ to individuals in } \mathcal{P};
15
               \mathbf{u}_i \leftarrow \text{Apply binomial crossover with } c_i \text{ to } \mathbf{x}_i \text{ and } \mathbf{v}_i;
16
17
            \Theta_s \leftarrow \emptyset, \Theta_c \leftarrow \emptyset;
18
            for i \in \{1, ..., \mu\} do
                  if f(\mathbf{u}_i) \leq f(\mathbf{x}_i) then
19
                         \mathcal{A} \leftarrow \mathcal{A} \cup \{\mathbf{x}_i\};
20
                          \mathbf{x}_i \leftarrow \mathbf{u}_i;
21
                          \Theta_s \leftarrow \Theta_s \cup \{s_i\}, \Theta_c \leftarrow \Theta_c \cup \{c_i\};
22
            if |\mathcal{A}| > a then Delete randomly selected individuals in \mathcal{A} unless |\mathcal{A}| < a;
23
            if \Theta_s \neq \emptyset and \Theta_c \neq \emptyset then
\mathbf{24}
                   m_{s,k} \leftarrow \text{Lmean}(\Theta_s);
25
                   m_{c,k} \leftarrow \text{Lmean}(\Theta_c);
26
                   k \leftarrow k + 1;
27
                  if k > h then k \leftarrow 1;
28
29
           t \leftarrow t + 1;
```

Algorithm S.9: The basic DE algorithm with P-EPS

1 $t \leftarrow 1$, initialize $\mathcal{P} = \{\mathbf{x}_1, \dots, \mathbf{x}_\mu\}$ randomly, $\mathcal{A} \leftarrow \emptyset$; 2 for $i \in \{1, ..., \mu\}$ do $s_i \leftarrow \text{Randomly select one from } \{0.4, 0.5, 0.6, 0.7, 0.8, 0.9\};$ 3 $c_i \leftarrow \text{Randomly select one from } \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\};$ 4 5 while The termination criteria are not met do for $i \in \{1, ..., \mu\}$ do 6 $\mathbf{v}_i \leftarrow$ Apply differential mutation with s_i to individuals in \mathcal{P} ; 7 $\mathbf{u}_i \leftarrow$ Apply binomial crossover with c_i to \mathbf{x}_i and \mathbf{v}_i ; 8 **for** $i \in \{1, ..., \mu\}$ **do** 9 10 if $f(\mathbf{u}_i) \leq f(\mathbf{x}_i)$ then 11 $\mathcal{A} \leftarrow \mathcal{A} \cup \{\mathbf{x}_i\};$ $\mathbf{x}_i \leftarrow \mathbf{u}_i;$ 12 else 13 $s_i \leftarrow \text{Randomly select one from } \{0.4, 0.5, 0.6, 0.7, 0.8, 0.9\};$ 14 $c_i \leftarrow \text{Randomly select one from } \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\};$ 15 **if** $|\mathcal{A}| > a$ **then** Delete randomly selected individuals in \mathcal{A} unless $|\mathcal{A}| < a$; 16 17 $t \leftarrow t + 1;$

Algorithm S.10: The basic DE algorithm with P-CoBi

1 $t \leftarrow 1$, initialize $\mathcal{P} = \{\mathbf{x}_1, \dots, \mathbf{x}_\mu\}$ randomly, $\mathcal{A} \leftarrow \emptyset$; 2 for $i \in \{1, ..., \mu\}$ do $s = \langle s_i, c_i \rangle \leftarrow$ Generate the scale factor and crossover rate by using Algorithm S.11; 4 while The termination criteria are not met do for $i \in \{1, ..., \mu\}$ do 5 $\mathbf{v}_i \leftarrow \text{Apply differential mutation with } s_i \text{ to individuals in } \mathcal{P};$ 6 7 $\mathbf{u}_i \leftarrow \text{Apply binomial crossover with } c_i \text{ to } \mathbf{x}_i \text{ and } \mathbf{v}_i;$ for $i \in \{1, ..., \mu\}$ do 8 if $f(\mathbf{u}_i) \leq f(\mathbf{x}_i)$ then 9 $\mathcal{A} \leftarrow \mathcal{A} \cup \{\mathbf{x}_i\};$ 10 11 $\mathbf{x}_i \leftarrow \mathbf{u}_i;$ else 12 $\lfloor \langle s_i, c_i \rangle \leftarrow$ Generate the scale factor and crossover rate by using Algorithm S.11; 13 if $|\mathcal{A}| > a$ then Delete randomly selected individuals in \mathcal{A} unless $|\mathcal{A}| < a$; 14 $t \leftarrow t + 1;$ 15

Algorithm S.11: The parameter generation method in P-CoBi

1if randu[0, 1] < 0.5 then2 $s_i \leftarrow$ Randomly select a value from C(0.65, 0.1);3else4 $s_i \leftarrow$ Randomly select a value from C(1, 0.1);5if randu[0, 1] < 0.5 then6 $c_i \leftarrow$ Randomly select a value from C(0.1, 0.1);7else8 $c_i \leftarrow$ Randomly select a value from C(0.95, 0.1);9return $\langle s_i, c_i \rangle$;

Algorithm S.12: The basic DE algorithm with P-c

1 $t \leftarrow 1$, initialize $\mathcal{P} = \{\mathbf{x}_1, \dots, \mathbf{x}_\mu\}$ randomly, $\mathcal{A} \leftarrow \emptyset$; $2 \quad q_1 \leftarrow \langle 0.5, 0 \rangle, q_2 \leftarrow \langle 0.5, 0.5 \rangle, q_3 \leftarrow \langle 0.5, 1 \rangle, q_4 \leftarrow \langle 0.8, 0 \rangle, q_5 \leftarrow \langle 0.8, 0.5 \rangle, q_6 \leftarrow \langle 0.8, 1 \rangle, q_7 \leftarrow \langle 1, 0 \rangle, q_8 \leftarrow \langle 1, 0.5 \rangle, q_9 \leftarrow \langle 1, 1 \rangle; q_9 \leftarrow \langle 1, 1 \rangle; q_9 \leftarrow \langle 1, 0 \rangle, q_9 \leftarrow$ 3 **for** *i* ∈ {1, ..., 9} **do** $o_i \leftarrow 0$; 4 while The termination criteria are not met do for $i \in \{1, ..., \mu\}$ do 5 for $j \in \{1, ..., 9\}$ do $\downarrow \tau_j \leftarrow \frac{o_j + \epsilon}{\sum_{k=1}^9 (o_k + \epsilon)}$ 6 7 for $j \in \{1, ..., 9\}$ do 8 9 if $\tau_j \leq \delta$ then $\begin{bmatrix} o_j \leftarrow 0; \\ \tau_j \leftarrow \frac{o_j + \epsilon}{\sum_{k=1}^9 (o_k + \epsilon)}; \end{bmatrix}$ 10 11 for $i \in \{1, ..., \mu\}$ do 12 $\langle s_i, c_i \rangle \leftarrow$ Randomly select one from $\mathbf{q}_1, \ldots, \mathbf{q}_9$ with the propabities τ_1, \ldots, τ_9 ; 13 for $i \in \{1, ..., \mu\}$ do 14 $\mathbf{v}_i \leftarrow$ Apply differential mutation with s_i to individuals in \mathcal{P} ; 15 $\mathbf{u}_i \leftarrow \text{Apply binomial crossover with } c_i \text{ to } \mathbf{x}_i \text{ and } \mathbf{v}_i;$ 16 for $i \in \{1, ..., \mu\}$ do 17 if $f(\mathbf{u}_i) \leq f(\mathbf{x}_i)$ then 18 $\mathcal{A} \leftarrow \mathcal{A} \cup \{\mathbf{x}_i\};$ 19 $\mathbf{x}_i \leftarrow \mathbf{u}_i;$ 20 if $q_j = \langle s_i, c_i \rangle$ then 21 $o_j \leftarrow o_j + 1;$ 22 if $|\mathcal{A}| > a$ then Delete randomly selected individuals in \mathcal{A} unless $|\mathcal{A}| < a$; 23 24 $t \leftarrow t + 1;$

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Figure S.1: Comparison of the nine PCMs and NOPCM with the rand/1 mutation strategy and the Baldwinian repair method on the 24 bbob-mixint functions for $n \in \{5, 10, 20, 40, 80, 160\}$.



Figure S.2: Comparison of the nine PCMs and NOPCM with the rand/1 mutation strategy and the Lamarckian repair method on the 24 bbob-mixint functions for $n \in \{5, 10, 20, 40, 80, 160\}$.



Figure S.3: Comparison of the nine PCMs and NOPCM with the rand/2 mutation strategy and the Baldwinian repair method on the 24 bbob-mixint functions for $n \in \{5, 10, 20, 40, 80, 160\}$.



Figure S.4: Comparison of the nine PCMs and NOPCM with the rand/2 mutation strategy and the Lamarckian repair method on the 24 bbob-mixint functions for $n \in \{5, 10, 20, 40, 80, 160\}$.



Figure S.5: Comparison of the nine PCMs and NOPCM with the best/1 mutation strategy and the Baldwinian repair method on the 24 bbob-mixint functions for $n \in \{5, 10, 20, 40, 80, 160\}$.



Figure S.6: Comparison of the nine PCMs and NOPCM with the best/1 mutation strategy and the Lamarckian repair method on the 24 bbob-mixint functions for $n \in \{5, 10, 20, 40, 80, 160\}$.



Figure S.7: Comparison of the nine PCMs and NOPCM with the best/2 mutation strategy and the Baldwinian repair method on the 24 bbob-mixint functions for $n \in \{5, 10, 20, 40, 80, 160\}$.



Figure S.8: Comparison of the nine PCMs and NOPCM with the best/2 mutation strategy and the Lamarckian repair method on the 24 bbob-mixint functions for $n \in \{5, 10, 20, 40, 80, 160\}$.

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Figure S.9: Comparison of the nine PCMs and NOPCM with the current-to-rand/1 mutation strategy and the Baldwinian repair method on the 24 bbob-mixint functions for $n \in \{5, 10, 20, 40, 80, 160\}$.



Figure S.10: Comparison of the nine PCMs and NOPCM with the current-to-rand/1 mutation strategy and the Lamarckian repair method on the 24 bbob-mixint functions for $n \in \{5, 10, 20, 40, 80, 160\}$.



Figure S.11: Comparison of the nine PCMs and NOPCM with the current-to-best/1 mutation strategy and the Baldwinian repair method on the 24 bbob-mixint functions for $n \in \{5, 10, 20, 40, 80, 160\}$.



Figure S.12: Comparison of the nine PCMs and NOPCM with the current-to-best/1 mutation strategy and the Lamarckian repair method on the 24 bbob-mixint functions for $n \in \{5, 10, 20, 40, 80, 160\}$.

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Figure S.13: Comparison of the nine PCMs and NOPCM with the current-to-*p*best/1 mutation strategy and the Baldwinian repair method on the 24 bbob-mixint functions for $n \in \{5, 10, 20, 40, 80, 160\}$.



Figure S.14: Comparison of the nine PCMs and NOPCM with the current-to-*p*best/1 mutation strategy and the Lamarckian repair method on the 24 bbob-mixint functions for $n \in \{5, 10, 20, 40, 80, 160\}$.



Figure S.15: Comparison of the nine PCMs and NOPCM with the rand-to-*p*best/1 mutation strategy and the Baldwinian repair method on the 24 bbob-mixint functions for $n \in \{5, 10, 20, 40, 80, 160\}$.



Figure S.16: Comparison of the nine PCMs and NOPCM with the rand-to-*p*best/1 mutation strategy and the Lamarckian repair method on the 24 bbob-mixint functions for $n \in \{5, 10, 20, 40, 80, 160\}$.



Figure S.17: Comparison of P-j and P-CoBi with the three CMA-ES variants on the 24 bbob-mixint functions with $n \in \{5, 10, 20, 40, 80, 160\}$.



ture $(f_{15}, ..., f_{19})$

Figure S.18: Comparison of P-j and P-CoBi with the three CMA-ES variants on each function group with n = 80.

 $(f_{20}, ..., f_{24})$



Figure S.19: Analysis results of a typical single run of P-JA on f_3 with n = 80.