

Memento Filter: A Fast, Dynamic, and Robust Range Filter

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Range filters are probabilistic data structures that answer approximate range emptiness queries. They aid in avoiding processing empty range queries and have use cases in many application domains such as key-value stores and social web analytics. However, current range filters do not support dynamically changing and growing datasets. Moreover, several of these designs also exhibit impractically high false positive rates under correlated workloads, which are common in practice. These impediments restrict the applicability of range filters across a wide range of use cases.

We introduce Memento filter, the first range filter to simultaneously offer dynamicity, fast operations, and a robust false positive rate for any workload. Memento filter partitions the key universe and clusters its keys according to this partitioning. For each cluster, it stores a fingerprint and a list of key suffixes contiguously. The encoding of these lists makes them amenable to existing dynamic filter structures. Due to the one-to-one mapping from keys to suffixes, Memento filter supports inserts and deletes and can even expand to accommodate a growing dataset.

We implement Memento filter on top of a Rank-and-Select Quotient filter and InfiniFilter and demonstrate that it achieves a competitive false positive rate and performance with the state of the art while also providing dynamicity. Due to its dynamicity, Memento filter is the first range filter applicable to B-Trees. We showcase this by integrating Memento filter into WiredTiger, a B-Tree-based key-value store, significantly boosting its performance for mixed workloads.

CCS Concepts: • **Theory of computation** → **Bloom filters and hashing**; • **Information systems** → **Unidimensional range search**.

Additional Key Words and Phrases: Range Filter; Dynamic Data Structure; Data Growth; Scalability

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1 INTRODUCTION

What is a Filter? A filter is a compact probabilistic data structure that answers approximate membership queries on a set. Since a filter is space efficient, it is often stored in a higher level of the memory hierarchy, making it fast to query. A filter cannot return a false negative but may return a false positive with some probability known as the false positive rate (FPR), determined by its memory footprint. Filters are ubiquitously used in many application domains to avoid disk reads [19] or network hops [7] when querying for non-existing keys.

Range Filters and Applications. Traditional filters, such as Bloom filters [5], only answer membership queries for a single query key. A range filter, on the other hand, is a filter that answers range emptiness queries over a set S [40]. Given a range $q = [q_l, q_r]$, a range filter

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returns a true positive if there is some key in S that is also in the range q . It returns either a true negative or a false positive otherwise. Range filters are used to avoid processing empty range queries in applications such as social web analytics [14], replication in distributed key-value stores [43], statistics aggregation of time series [27], and SQL table accesses [31]. Previous research has demonstrated the significant performance boost range filters provide in these systems [2, 10, 29, 32, 36, 44, 47].

The Need for Dynamic Range Filters. A static range filter is sufficient for applications with immutable (i.e., non-changing) data. For example, LSM-Trees consist of a set of immutable files to which range filters can be attached to speed up range queries. However, many applications have a dynamic nature, as they must support insertions, deletions, and growing datasets. These include (1) LSM-Trees that use a single global filter to map each key to the file storing it [13, 18, 42, 45], (2) B-Tree indexes [12, 41], and (3) Hybrid Transactional/Analytical Processing (HTAP) systems [6, 25]. Such applications require dynamic range filters that maintain high performance and low FPRs with an increasing dataset size.

State of the Art. All existing range filters [10, 15, 22, 29, 32, 36, 44, 46, 47] resemble Bloom filters and/or create a static model of the data distribution. The Bloom filter-inspired methods [10, 15, 22, 29, 32, 36, 44, 46] hash a key to one or more bits in a bitmap and set those bits to ones, potentially mapping multiple keys to the same bit. These filters fail to support deletions, as changing a bit from one to zero may result in false negative query results. Furthermore, such filters are unexpandable as they provide no obvious way of remapping keys to a larger bitmap [17]. Those range filters that model the data distribution [10, 22, 29, 44, 46, 47] also cannot update their models due to their static layout and the lack of global information about the dataset. Thus, none of the current range filters are dynamic or expandable.

Moreover, many of the above range filters [10, 22, 29, 44, 46, 47] provide no FPR guarantees under common workloads [15, 32] where the query end-points are close to the keys in the set. For example, queries for employee salaries tend to target ranges near the data values rather than unrealistic ranges. The FPR measured in practice may approach 1, rendering the filter useless [15].

Research Challenge. We identify the following research question: *can we design a range filter that simultaneously (1) guarantees a theoretically optimal FPR for any key set and workload, (2) provides fast operations, and (3) supports insertions, deletions, and resizing while maintaining (1) and (2)?*

Core Contributions. We introduce Memento filter, the first range filter providing dynamicity, resizing, fast operations, and an optimal FPR for any workload. It achieves this by taking a new approach to range filtering. Memento filter partitions the key space into equi-width partitions. It then clusters the keys according to this partitioning and maintains a suffix for each key in a Rank-and-Select Quotient filter (RSQF) [39]. It further derives a fingerprint for each cluster based on its partition number. All suffixes in a given partition are stored compactly and contiguously alongside their fingerprint to support cache-efficient parsing. An RSQF employs Robin Hood Hashing [9] to resolve hash collisions, allowing for storing the variable length data of the partitions by pushing colliding filter content to the right. As Memento filter establishes an unambiguous one-to-one mapping from each key to a suffix (i.e., unlike Bloom filter-inspired approaches), it can expand efficiently by remapping these suffixes to a larger filter. Memento filter handles a range query by finding the partitions that intersect the query range. It then compares the key suffixes of those partitions and the suffixes of the query end-points to check if any fall within the specified range.

Additional Contributions:

- We provide the most comprehensive theoretical comparison of range filters to date and show that Memento filter not only matches the state of the art in terms of FPR and performance but also provides dynamicity and expandability.

| Symbol | Definition |
|--------------------|------------------------------------------------------|
| $q = [q_l, q_r]$ | An inclusive query range. |
| u | Size of the key universe. |
| N | The number of keys in the dataset. |
| R | Maximum query range length. |
| ϵ | Target FPR. |
| $h(x)$ | General hash function of an RSQF. |
| $h_f(x)$ | The fingerprint of x resulting from $h(x)$. |
| n | The number of slots of an RSQF. |
| F | The hash table of an RSQF. |
| α | A RSQF's load factor. |
| f | Fingerprint size used in filter. Expressed in bits. |
| r | Size of the mementos. Expressed in bits. |
| $m(k)$ | The memento of a key k . |
| $p(k)$ | The prefix of a key k . |
| ℓ | Average number of keys in a non-empty partition. |
| Memento | The r least significant bits of a key. |
| Prefix | The prefix excluding the r least significant bits. |
| Partition | A partition of the universe defined by a prefix. |
| Vacant Fingerprint | A zero fingerprint used in encoding keepsake boxes. |

Table 1. Definitions of terms and symbols. The table is split into three sections presenting the terms and notation used to describe the general range filtering problem, the RSQF, and Memento filter, respectively.

- We use variable-length fingerprints to create an expandable variant of Memento filter with an optimal and robust FPR, similarly to InfiniFilter and Aleph Filter, among other expandable filters [3, 16, 17].
- We empirically evaluate Memento filter against all major range filters in a static setting. We also conduct the first evaluation of range filters in a dynamic setting.
- We integrate Memento filter with a B-Tree-based key-value store and show that it significantly boosts throughput for dynamic mixed workloads. Memento filter is the first range filter to achieve such a feat.

2 PROBLEM ANALYSIS

This section defines the problem of range filtering and shows that no existing solution simultaneously provides (1) a robust FPR for any workload, (2) fast worst-case performance, and (3) the ability to handle dynamic data.

Definitions and the Theoretical Lower Bound. The problem of range filtering is a generalization of the classic filtering problem. A query takes the form of an interval $q = [q_l, q_r]$ of length at most R . Given a set of keys S coming from a universe of size u , the goal is to ascertain the emptiness of the range, i.e., whether or not $S \cap q \neq \emptyset$ with an FPR of at most ϵ . The top section of Table 1 outlines the terms used to describe the range filtering problem henceforth.

Many applications are subject to *Correlated Workloads* [15, 32]. The range queries of such workloads are close to the keys of the underlying dataset but do not contain keys within them. The reason such queries are commonplace is that users typically issue queries that are informed by the keys in the dataset. For example, a user searching a medical database for patients in a given age group is more likely to issue the query 40-41 years old than 140-141. Many range filters exhibit high

FPRs in the face of correlated workloads, as they only maintain coarse-grain information about the data. The general distance between the queries and the keys is referred to as the *Correlation Degree* of the workload. A *robust* range filter supports any workload with range queries of length at most R , including correlated workloads, without degradation of its FPR.

Goswami et al. [26] prove an information-theoretic memory lower bound of $\log_2(\frac{R}{\epsilon}) - O(1)$ bits per key for any robust range filter supporting range queries of length at most R with an FPR of ϵ . While several non-robust range filters have been proposed that require less memory than this bound, they do so in exchange for a much higher FPR under correlated workloads.

Prior Work. We now describe all existing range filters in a roughly chronological order.

ARF [2] is a range filter employing a binary trie that adapts to the data and query distributions. However, it is superseded by **SuRF** [47], which utilizes succinct tries to compactly encode keys while maintaining ordering information to support range queries. This trie contains the shortest unique prefix of each key to ensure that the final structure does not consume too much memory, causing the number of internal nodes of the trie to depend on the length and distribution of the keys. Fingerprints and key suffixes can be stored in the leaves of the trie for each key, improving support of point and range queries, respectively.

SuRF exhibits a high FPR under correlated workloads since the shortest unique prefixes cannot differentiate between close queries and keys. Moreover, SuRF does not support insertions or deletes due to its succinct encoding scheme, and its query time deteriorates with the length of keys.

Rosetta [32] employs a hierarchy of Bloom filters, each storing key prefixes of a given length. The hierarchy is treated as a segment tree [1] and supports range filtering by checking whether all prefixes in a specified query range are absent. Rosetta achieves a near-optimal FPR by employing a recursive query process that corrects the false positives of an upper-level filter with the help of lower-level filters. During this process, Rosetta checks for $O(\log_2 R)$ sub-intervals as dictated by the segment tree, possibly followed by additional Bloom filter lookups. While the hierarchical structure of Rosetta makes it robust, each query entails probing many bits chosen by hash functions, leading to many random cache misses [22, 36, 46].

REncoder [22, 46] and **bloomRF** [36] improve on Rosetta's speed by encoding the prefixes of a given key in a cache-friendly manner within a single bitmap. Specifically, REncoder breaks Rosetta's segment tree into mini-segment trees and encodes each contiguously in multiple locations. These contiguous encodings give the filter access to the query range decomposition with a single memory access. In contrast, bloomRF generalizes the segment tree to have a larger fanout based on the key length and the dataset size. It further employs a Prefix Hashing scheme with Piecewise Monotonic Hash Functions to improve cache locality. Such a hashing scheme positions neighboring sub-intervals of equal length next to each other in the bitmap, giving simultaneous access to them all using a single cache miss.

While REncoder and bloomRF improve on Rosetta's speed, they still incur several cache misses for queries and do not provide dynamicity or expandability. They also forgo robustness, as encoding all the levels of the segment tree in the same bitmap results in a uniform FPR assignment to all levels. This uniformity significantly decreases filtering for correlated queries since only a limited number of the lower levels of the tree can filter them out.

Proteus [29] combines SuRF with a Bloom filter. The SuRF instance is truncated to contain prefixes of keys up to a given length l_1 and acts as a pre-filter for the Bloom filter, while the Bloom filter stores key prefixes of a fixed length $l_2 > l_1$. This hybrid structure exposes a rich range filter design space with many tradeoffs. Proteus tunes the prefix lengths l_1 and l_2 to minimize the FPR of the hybrid filter, which improves upon both hierarchical filter designs and SuRF. Consequently, Proteus discards prefixes longer than the longest-common prefix of the keys with the queries,

| Filter | Construction | Delete | Range Query (-) | Range Query (+) | Memory Footprint | Robust | Expandable |
|---------------------------|-----------------------------------|------------------------|-------------------------------|---------------------------------|--------------------------------------------------------|--------|------------|
| SuRF | $O(N \log u)$ | - | $O(\log u)$ | $O(\log u)$ | $10 + \frac{10z}{N} + m + o(1)$ | ✗ | ✗ |
| Rosetta | $O(N \log_2(\frac{R}{\epsilon}))$ | - | $O(\log_2(R))$ | $O(\log_2(\frac{R}{\epsilon}))$ | $1.44 \log_2(\frac{R}{\epsilon})$ | ✓ | ✗ |
| REncoder * | $O(Nk)$ | - | k | k | $O(k + \log(\frac{1}{\epsilon}))$ | ✗ | ✗ |
| bloomRF * | $O(N \log(\frac{u}{N}))$ | - | $O(\log(\frac{u}{N}))$ | $O(\log(\frac{u}{N}))$ | $\approx 1.2 \log_2(\frac{R}{\epsilon})$ | ✗ | ✗ |
| Proteus * | $O(N \log(\frac{u}{\epsilon}))$ | - | $O(\log u)$ | $O(\log(\frac{u}{\epsilon}))$ | $\frac{10z}{N} + 1.44 \log_2(\frac{1}{\epsilon})$ | ✗ | ✗ |
| SNARF * | $O(N)$ | $O(\log_2 N)$ | $O(\log_2 N)$ | $O(\log_2 N)$ | $2.4 + \log_2(\frac{1}{\epsilon})$ | ✗ | ✗ |
| Oasis+ * | $O(N)$ | - | $O(\log_2 N)$ | $O(\log_2 N)$ | $\approx 2.4 + \log_2(\frac{1}{\epsilon})$ | ✗ | ✗ |
| Grafitte | $O(N \log_2 N)$ | - | $1 - 2$ | $1 - 2$ | $2 + \log_2(\frac{R}{\epsilon}) + o(1)$ | ✓ | ✗ |
| Memento (Cache Misses) | $O(N)$ | 1 | $1 - 2$ | $1 - 2$ | $\frac{1}{\alpha}(3.125 + \log_2(\frac{R}{\epsilon}))$ | ✓ | ✓ |
| Memento (CPU) | $O(\ell N) \approx O(N)$ | $O(\ell) \approx O(1)$ | $O(\log_2 \ell) \approx O(1)$ | $O(\log_2 \ell) \approx O(1)$ | | | |

Table 2. A comparison of range filters assuming an FPR of ϵ , maximum range query length R , and N keys coming from a universe of size u . For SuRF and Proteus, z refers to the number of internal nodes in the trie, while m denotes the length of the fingerprints stored at the leaves. For REncoder, k refers to the number of hash functions, which we have empirically found to be $O(\log(\frac{1}{\epsilon}))$. For Memento, ℓ is a measure of the local density of the keys which is at most R and thus small. The operation costs are measured in the expected number of random cache misses, and the memory footprint is measured in bits per key. As shown, no existing method supports dynamic key sets, a robust FPR, and fast operations, all at the same time.

causing the structure to lose robustness against correlated queries since it cannot differentiate between close keys and queries. Furthermore, its tuning procedure requires both a sample query set and a static underlying dataset. Queries are also expensive due to the many random cache misses caused by the SuRF instance and the Bloom filter.

SNARF [44] learns from the underlying dataset by creating a linear spline model of the keys' cumulative distribution function. It uses this model to map each key to a bit position in a large bit array and sets it to one. Since this is a monotonic mapping, SNARF answers range queries by scanning the corresponding range of bits in the bit array and returns true if it encounters a one. SNARF saves a substantial amount of memory by compressing the bit array using Golomb or Elias-Fano coding [21, 23, 24]. **Oasis+** [10] employs SNARF's framework and improves upon its learned mapping function by pruning large empty regions of the key space, achieving lower FPRs. It also employs instances of Proteus [29] to answer range queries for select regions of the key space, depending on the data distribution.

However, the FPR of both of these filters suffers under correlated workloads, as such queries tend to always map to a one in the bit array. Moreover, these filters assume complete knowledge of the keys to train the mapping function, meaning they must be constructed on a static dataset. Lastly, they use a logarithmic number of random memory accesses to query the distribution model and use floating-point operations during the process, which slow down the filter and cause precision issues for long keys.

Grafitte [15] implements Goswami et al.'s design of a range filter [26], which uses a locality-preserving hash function to map each key to a bit in a bit array. It compresses the bit array using Elias-Fano coding [21, 23] and handles queries by checking the corresponding range of bits for any set bit. The corresponding range is found by accessing a rank-and-select structure [30, 48] built on top of the Elias-Fano encoding, followed by a binary search [11, 37, 38].

Grafitte is the current state of the art in terms of speed, as it requires up to three random memory accesses to serve a query. Its hashing and coding scheme allows it to achieve the same robustness as Rosetta while enjoying extremely fast queries and a much better FPR vs. memory tradeoff. However,

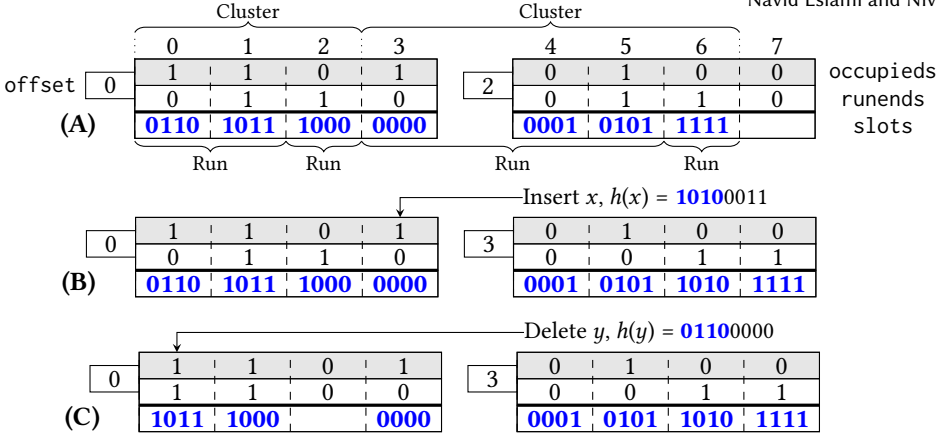


Fig. 1. An RSQF handles hash collisions by pushing fingerprints to the right using Robin Hood Hashing.

the bit array and the hashing function do not allow for insertions, deletions, or expansions without reconstructing the structure from scratch, as with the other Bloom filter-inspired methods.

Summary. Table 2 summarizes the characteristics of existing range filters. The methods annotated with * are heuristic in nature and do not provide strict mathematical bounds on their memory consumption. Therefore, we provide a conservative estimate of their memory footprint based on the experimental data from their respective papers to enable a comparison.

Table 2 shows that existing range filters do not support deletes and cannot expand as more data is inserted. This makes them inapplicable across the wide range of database applications that support range queries over rapidly changing and/or growing data (e.g., from B-tree access in OLTP applications to analytical queries in HTAP systems). Is it possible to design a robust range filter that can accommodate dynamic data while also being competitive in terms of query cost, FPR, and memory footprint?

3 BACKGROUND

This section describes the Rank-and-Select Quotient Filter (RSQF) [39], the base hash table implementation on top of which we build Memento filter. The second section of Table 1 lists the symbols we use henceforth to describe the RSQF and Memento filter. An RSQF is a compact hash table that stores a fingerprint for each key. An RSQF's hash table F consists of n slots, each able to store an f -bit fingerprint. This hash table maps a key x to its *Canonical Slot* using the $\lceil \log_2(n) \rceil$ least significant bits of the key's hash $h(x)$. It further associates a fingerprint $h_f(x)$ to x by taking the following f bits of the aforementioned hash value. For example, considering an RSQF with 16 slots, a key x with a hash of $h(x) = 01100000$ would have Slot 0 as its canonical slot based on the least significant bits of its hash. It will also have $h_f(x) = 0110$ as its fingerprint based on the remaining bits of $h(x)$.

An RSQF resolves hash collisions via Robin Hood Hashing [9], meaning that all fingerprints mapped to a given canonical slot are stored contiguously. To achieve this, fingerprints may shift any other fingerprint they collide with to the right to make space for themselves. A *Run* is defined as the contiguous set of slots storing the fingerprints corresponding to a given canonical slot. A group of contiguous slots occupied by runs, where all but the left-most run are shifted to the right, is known as a *Cluster*. Fig. 1-(A) shows a populated RSQF, along with its runs and clusters. The bold blue text indicates the fingerprints, while the black text indicates the canonical slot addresses.

Metadata. An RSQF associates two metadata bits to each slot to represent where runs begin and end: the *occupieds* and *runends* bits. The *occupieds* bit for slot i indicates whether or not a key

with canonical slot $F[i]$ was inserted into the filter. The `runends` bit indicates whether or not the fingerprint stored at slot $F[i]$ is the last fingerprint in a run. For example, the set `occupieds` bits in Fig. 1-(A) indicate that Slots 0, 1, 3, and 5 are canonical slots, while the set `runends` bits indicate that the runs of these slots end at Slots 1, 2, 5, and 6, respectively.

To improve performance, an RSQF is partitioned into blocks of 64 slots, each augmented with two contiguous 64-bit bitmaps representing the `occupieds` and `runends` bits and an 8-bit `offset` field. A block's `offset` field indicates the number of fingerprints belonging to canonical slots from previous blocks shifted into its slots or subsequent blocks. For example, the `offset` value of the second block in Fig. 1-(A) is equal to 2 because its first two slots contain fingerprints with a canonical slot of 3, which is from the previous block. This field enables an RSQF to skip over many irrelevant slots to find the run of a key.

Locating a Run. Since an RSQF shifts runs to the right, it must search for a canonical slot's run. To this end, an RSQF leverages the fact that each one bit in the `occupieds` bitmap has a corresponding one bit in the `runends` bitmap denoting the end of its run. Therefore, it locates slot $F[i]$'s run by finding the set `runends` bit that matches $F[i]$'s `occupieds` bit. An RSQF starts this process by considering the `offset` field of the associated block. The filter skips `offset` many slots to the right, allowing it to immediately find a slot $F[j]$ containing runs from $F[i]$'s block. Then, it uses specialized CPU instructions to apply efficient rank-and-select operations on the `occupieds` bitmap fragment of $F[i]$'s block and the `runends` bitmap fragment of $F[j]$'s block to quickly find the matching set bits, thus locating the end of $F[i]$'s run.

For example, in Fig. 1-(A), Slot 5's run is located by first skipping `offset=2` slots in the block. Then, using the rank operation on the `occupieds` bitmap, an RSQF realizes that Slot 5's associated `runend` bit comes first among this block's `runend` bits. Armed with this knowledge, an RSQF uses the select operation on the `runends` bitmap fragment, ignoring its first `offset=2` bits, to locate the first set bit. Thus, it finds the end of $F[i]$'s run, i.e., Slot 6.

Queries. A query for a key q starts at the canonical slot of q . If the `occupieds` flag of this slot is zero, an RSQF returns a negative since a run for that slot does not exist. If it is one, its run is located using the procedure described above. An RSQF then searches the discovered run for a fingerprint equal to $h_f(q)$. If one exists, the query returns a positive and a negative otherwise.

Inserts. An RSQF inserts a key x by first locating its run. It then adds the fingerprint $h_f(x)$ to the run by shifting the subsequent runs one slot to the right and updating the filter's metadata accordingly. This shifting procedure may add new runs to x 's cluster. Fig. 1-(B) shows the insertion process of such a key with hash **10100011**. Here, Slot 3's run expands and pushes Slot 0101's run to the right. It also updates the `offset` field of the next block as it shifts slots into it.

Deletes. Similarly to an insert, an RSQF deletes a key y by finding its run and removing from it a fingerprint equal to $h_f(y)$. It then shifts the subsequent slots in the cluster to the left to keep the cluster contiguous. This may split the cluster into smaller clusters. Fig. 1-(C) shows an example of a deletion to fingerprint **0110** at canonical Slot 0. Here, the run shrinks, and the cluster splits into two small clusters of size 1 each.

Iteration. RSQFs support the iteration of their fingerprints via a left-to-right scan. Using the metadata flags, the canonical slot of each fingerprint can be identified. An RSQF can then recover the original hash bits of a key by concatenating its fingerprint to the address of its canonical slot. As we shall see later, iteration serves as the cornerstone of expansion operations.

Allocation. An RSQF supports a load factor of up to $\alpha = 95\%$. Beyond this load factor, the filter's performance deteriorates rapidly since the cluster lengths skyrocket.

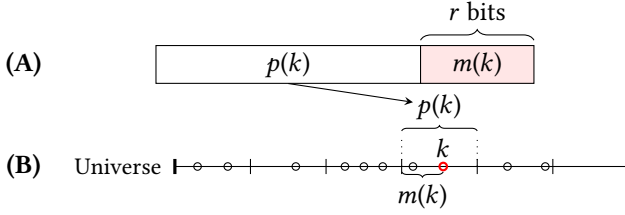


Fig. 2. Each key k is split into a prefix $p(k)$ and a memento $m(k)$. Prefixes partition the key universe and cluster the keys, while mementos denote the positions of keys in their partitions. Each circle in (B) represents a key in the key set.



Fig. 3. Clusters are comprised of runs, where all but the first are shifted to the right. A run contains one or more keepsake boxes. The single cluster in the illustration has three runs, and the runs contain three, two, and one keepsake box, respectively.

Analysis. An RSQF has an FPR of $\epsilon \leq \alpha \cdot 2^{-f}$ since each canonical slot has an average of α fingerprints in its run, and each of those fingerprints matches a query with probability 2^{-f} . The structure has an overall memory footprint of $\frac{1}{\alpha}(2.125 + f)$ bits per key.

4 MEMENTO FILTER

We introduce Memento filter, the first range filter to simultaneously provide a robust FPR, fast inserts, queries, and deletes, and the ability to efficiently expand and contract. Conceptually, Memento filter partitions the key space into equally sized partitions. For each partition with at least one key, it contiguously stores a fingerprint along with fixed-length suffixes of all keys in that partition. We refer to these suffixes as mementos, and we refer to a fingerprint along with its collection of mementos as a keepsake box. Memento filter processes a range query by visiting all intersecting partitions with a matching fingerprint and checking for overlapping mementos in their keepsake boxes. Memento filter guarantees a desired FPR for fixed-length keys and range queries of length up to R . We show how to extend it to support variable-length keys at the expense of robustness and arbitrary range queries at the expense of FPR and speed.

We build Memento filter on top of an RSQF as its use of Robin Hood Hashing allows for storing variable-length keepsake boxes. At the same time, such variable-length keepsake boxes may elongate the runs and clusters of the filter and potentially damage performance. To counteract this, we show how to succinctly encode keepsake boxes and how to traverse them efficiently, leading to 1 and 2 random cache misses for point and range queries, respectively. In Section 6, we show analytically that Memento Filter achieves the same performance as an RSQF by leveraging this succinct encoding. Table 1 provides a list of symbols and definitions used in this section.

Prefixes and Mementos. Memento filter splits each key k into a *prefix* $p(k)$ and a *memento* $m(k)$. A memento is the $r = \lceil \log_2 R \rceil$ least significant bits of k , where R is the maximum range query length that the filter must support. A prefix is the maximal prefix of k not containing the r least significant bits. Fig. 2-(A) provides an example of this split. Based on these prefixes, Memento filter partitions the *key universe* into partitions of length $2^f \geq R$, as depicted in Fig. 2-(B). A partition contains keys with the same prefix. Given this partitioning, a key k 's memento $m(k)$ represents k 's

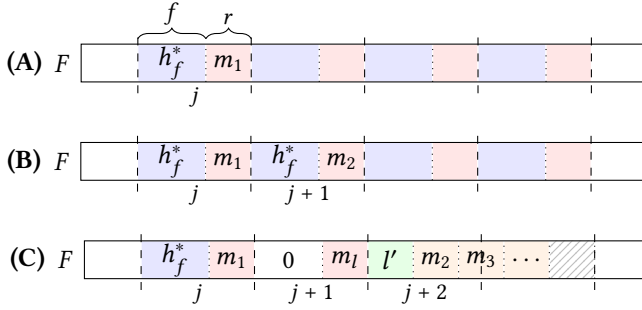


Fig. 4. Each keepsake box is encoded using one of three cases, depending on how many mementos it contains.

position in the partition defined by its prefix $p(k)$. Notice that, as in the design of all range filters (except SuRF), we assume the keys to be fixed-length strings.

Since the maximum query range size R is typically small in comparison to the universe size u , a key's prefix tends to be much longer than its memento, e.g., a 56-bit prefix vs. an 8-bit memento for a 64-bit key. Furthermore, since the partition size 2^r is at least as large as the query length R , any range query intersects with at most two consecutive partitions of the key universe. Thus, Memento filter splits a query range into at most two sub-ranges, each subsumed by a partition. It then answers the query by checking for the inclusion of any mementos in these sub-ranges.

Crucially, the resulting range query performance is workload-agnostic. The intuition is that mementos encode information about each key's least significant bits. Hence, any query's end points can be reliably checked for overlap with the mementos within the matching partitions to ascertain membership with a false positive rate that depends only on the fingerprint length.

Memento filter uses the prefix of the keys to insert them into an RSQF. That is, it computes $h(p(k))$ and $h_f(p(k))$ to map a key k to a canonical slot and to derive a fingerprint, where $h(\cdot)$ and $h_f(\cdot)$ are hash functions defined in Section 3 in the context of an RSQF. Memento filter stores the mementos and the fingerprint of the keys in each partition consecutively and succinctly to maximize cache locality and space efficiency.

Slot Structure. Memento filter allocates slots with a width of $f + r$ bits in its underlying RSQF, as shown in Fig. 4-(A). This allows it to store one fingerprint and one memento in each slot. We omit the occupieds, runends, and offset fields of the RSQF in Fig. 4-(A) to highlight new design elements built on top.

Keepsake Boxes. Since Memento filter hashes the prefix of each key to map it to a canonical slot and to generate a fingerprint, keys from different partitions may map to the same run and may even share a fingerprint due to hash collisions. We use the term *keepsake box* to refer to the union of the keys/mementos with a shared fingerprint in a run. Note that the keys within a keepsake box may come from different partitions due to hash collisions. Fig. 3 shows how there can be multiple keepsake boxes within a run and several runs within a cluster. Clusters and runs are delimited using the RSQF's metadata fields as shown in Section 3. We now focus on encoding and delimiting keepsake boxes in a run.

Encoding of a Keepsake Box. Because keepsake boxes are variable-length, their encoding must represent how long they are to facilitate unambiguous decoding. The simplest solution is to store a counter for each keepsake box that denotes how many mementos it contains. This approach, however, would use excessive space for metadata when the keepsake boxes contain few mementos.

To overcome this challenge, our filter enforces the following invariant: *the keepsake boxes of a run must be stored in non-decreasing order of their fingerprints*. We show how this allows for delimiting

keepsake boxes within a run without using additional metadata. Furthermore, to save space and optimize cache behavior, Memento filter minimizes the number of shared fingerprints stored for a keepsake box.

Consider a keepsake box with canonical slot $F[i]$, fingerprint h_f^* , and a list of l associated mementos $m_1 \leq \dots \leq m_l$. Assume that this keepsake box's fingerprint is stored in $F[j]$, where $j \geq i$ due to hash collisions. It is encoded as follows:

Case (1) $l = 1$: The only memento m_1 is stored with the keepsake box's fingerprint in the same slot of $F[j]$, as shown in Fig. 4-(A).

Case (2) $l = 2$: A fingerprint-memento pair is stored for each key, as depicted in Fig. 4-(B). The smaller memento m_1 is stored with the keepsake box's fingerprint in $F[j]$, while m_2 is stored with a copy of the keepsake box's fingerprint in $F[j + 1]$.

Case (3) $l > 2$: The smallest memento m_1 is stored alongside the keepsake box's fingerprint in $F[j]$ while the largest memento $m_l \geq m_1$ is stored with a zero *vacant fingerprint* in $F[j + 1]$. The decrease in the fingerprint values in the run created by the vacant fingerprint acts as an escape sequence, signaling that the keepsake box has more than two members. The smallest and largest mementos of the keepsake box stored in $F[j]$ and $F[j + 1]$ allow for quickly ruling out the existence of a key range without traversing the entire keepsake box.

The rest of the mementos are encoded as a sorted list in the subsequent slots, as shown in Fig. 4-(C). This sorted list is encoded by first writing down its length $l' = l - 2$ using r bits on average. This length parameter is followed up by the mementos, stored compactly, disregarding alignment. Due to this misalignment, the last slot of this memento list may have unused space. This unused space corresponds to the hatched area in Fig. 4-(C).

Based on the above encoding scheme, Memento filter encodes each key in at most $f + r$ bits. With more dataset skew, Memento filter forgoes storing fingerprints for the keys, and thus the memory it uses for each key approaches r bits. A memento usually comprises one byte to be able to answer short range queries efficiently. At the same time, the fingerprint size tends to be at least one byte to achieve an FPR in the range of 1-10%. This implies that each slot can house at least two mementos. Hence, all keepsake box encodings consume at most one slot per key.

A minor caveat is that keepsake boxes with a fingerprint of zero cannot utilize the vacant fingerprint as an escape sequence in Case (3), as it does not create a decreasing order for them. In such a scenario, Memento filter encodes the keepsake box entirely using Case (1). That is, each memento will have its own fingerprint. We will see that there are no zero fingerprints in the context of an expandable Memento filter, and this corner case will naturally disappear.

Since keepsake boxes are ordered according to their fingerprints, an increase in the fingerprint values signals the start of a new keepsake box, which delimits keepsake boxes in Cases (1) and (2). In Case (3), Memento filter delimits keepsake boxes using the length field l' . Note that keepsake box encodings are considered part of their run. Therefore, all of their slots, except for the final slot of the run, have zero runends bits.

Variable-Length Counter Encoding. A keepsake box encoding in Case (3) uses a length field l' to record the number of mementos in the keepsake box. This length is usually smaller than $2^r - 1$, i.e., the maximum length that r bits can represent. However, l' can also exceed this threshold in the unlikely event of fingerprint collisions of densely populated partitions. To keep the encoding small when the count is small while still supporting the rare event of large counts, we employ a variable-length encoding for l' . To this end, Memento filter reserves the value $2^r - 1$, i.e., r one bits, as a special value for l' and generates an encoding in r -bit chunks.

This encoding is specifically designed to keep the common case of small counts as performant, space-efficient, and general as possible, which is not achieved by traditional encoding schemes.

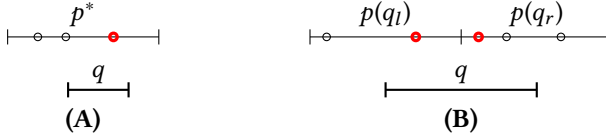


Fig. 5. A range query $q = [q_l, q_r]$ spans (A) one or (B) two partitions. The keys of a partition are represented as circles, and the checked lower bound, largest, and smallest keys are highlighted in bold.

That is, any $l' < 2^r - 1$ is encoded in binary using r bits. For larger lengths, the core idea is to represent l' in base- $(2^r - 1)$. Memento filter achieves this by first writing $\lfloor \log_{2^r-1} l' \rfloor$ copies of the value $2^r - 1$, similarly to unary coding. c of these values signals that there are $c + 1$ digits in l' 's base- $(2^r - 1)$ representation. This “unary code” is followed up with the base- $(2^r - 1)$ representation of l' . Since l' in base- $(2^r - 1)$ cannot have any digit equal to $2^r - 1$, the unary code preceding it is unambiguous and is used to recover c .

For example, if $r = 5$, the number $l' = 30$ is represented as a single r -bit value of $\langle 30 \rangle$. However, given $l' = 31$, its base- $(2^r - 1) = 31$ representation is $\langle 1, 0 \rangle$, which no longer has a single digit. Therefore, l' is encoded to $\langle 31, 1, 0 \rangle$. This encoding has $c = 1$ values of $2^r - 1 = 31$, implying that the base-31 representation of l' has $c + 1 = 2$ digits. The code is then finished by appending l' 's base- $(2^r - 1)$ representation $\langle 1, 0 \rangle$. As l' grows, this base-31 encoding is updated accordingly, e.g., $l' = 32$ is encoded to $\langle 31, 1, 1 \rangle$.

Note that Memento filter still uses at most one slot per memento with this encoding scheme. The reason is that l' is encoded using more than a single r -bit chunk only when it is very large. In this case, the succinct encoding of the long memento list compensates for the extra space required by the length encoding.

Skipping Keepsake Boxes. Memento filter skips over large keepsake boxes with mismatching fingerprints to dramatically improve lookup speed. Since large keepsake boxes are encoded using Case (3), Memento filter uses the list length l' to infer and skip the appropriate number of slots to access the next keepsake box.

Insertions. Memento filter inserts a key x by following the semantics of its underlying RSQF. It first finds x 's canonical slot using $h(p(x))$ and searches for its run. If it finds no such run, it creates one and encodes x 's keepsake box in it. Otherwise, Memento filter iterates over the run and looks for a keepsake box associated with x , skipping the contents of irrelevant keepsake boxes along the way. If there is no keepsake box with a fingerprint matching x 's partition, Memento filter creates a keepsake box and positions it in the run such that the fingerprints maintain a non-decreasing order. Otherwise, $m(x)$ is added to the matching keepsake box, updating the encoding according to the various cases shown in Fig. 4. The insertion procedure may shift the filter's slots to the right, potentially merging several clusters.

Deletions. Memento filter deletes a key y by first finding its keepsake box in its run. It then locates and removes some memento equal to $m(y)$ in the keepsake box. Similarly to how an RSQF handles deletes, Memento filter may have to shift several slots to the left, potentially splitting their cluster into smaller clusters.

Range Queries. As with all range filters, we consider range queries with a maximum length of R . Any such range query $q = [q_l, q_r]$ may intersect with at most two partitions of the key universe since the partitions are of length 2^r and $q_r - q_l + 1 \leq R \leq 2^r$. Thus, it must be the case that $p(q_r) - p(q_l) \leq 1$. That is, the prefixes of the end-points of the query can differ by at most one. Given this observation, Memento filter processes a range query as follows:

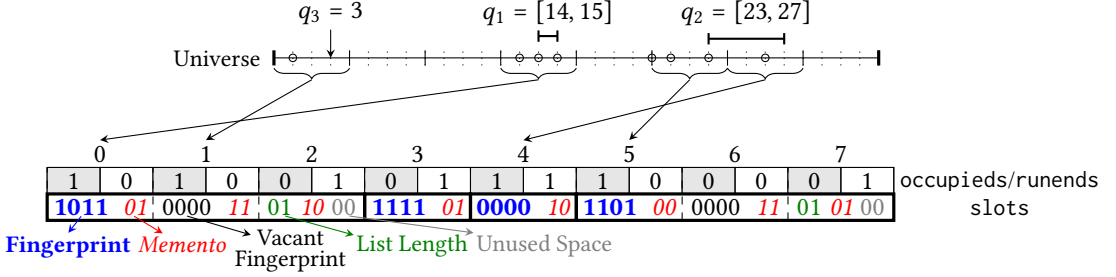


Fig. 6. Memento filter services range queries by finding the keepsake boxes corresponding to the overlapping partitions in the key universe and searching for mementos that fall into the query range. Here, the filter's fingerprint and memento sizes are $f = 4$ and $r = 2$, respectively. Each universe partition is of length 4, and the keys are denoted by circles. Black arrows represent the canonical slot each partition maps to, and runs are delineated using thick lines. All runs begin in their respective canonical slots except for the first partition's run, which is pushed from Slot 1 to Slot 3 due to Robin Hood Hashing.

If $p(q_l) = p(q_r) = p^*$: In this case, the query intersects with a single partition, i.e., the set of keys with a prefix equal to p^* , as depicted in Fig. 5-(A). Memento filter first checks if the canonical slot of the partition p^* is occupied using the occupieds bitmap. If it is not, the query results in a negative, as no keepsake box exists for p^* . If it is, Memento filter searches for a keepsake box with a fingerprint matching $h_f(p^*)$. It returns a negative if such a keepsake box does not exist. If one does exist, $m(q_r)$'s lower bound, i.e., the largest memento in the keepsake box that is less than or equal to $m(q_r)$, is calculated using binary search and is checked for inclusion in the range $[m(q_l), m(q_r)]$. If this range includes the lower bound, the query results in a positive, since it signifies that a potential key of the key set is in the query range. Otherwise, the query returns a negative, as no key is in the range. In Fig. 5-(A), the bold circle is the lower bounding memento of $m(q_r)$. Here, the example query results in a positive since the lower bounding memento lies in the range $[m(q_l), m(q_r)]$, and because the relative ordering of the mementos in a partition matches the ordering of the keys.

Query $q_1 = [14, 15]$ in Fig. 6 is an example of a range query intersecting a single partition. Here, mementos are $r = 2$ bits long, and both end-points have the same prefix $p^* = 3$. Memento filter processes q_1 by hashing the shared prefix to derive a canonical slot address and a fingerprint, which in this example happen to be 0 and 1011, respectively. It then locates the partition's run in Slot 0 and finds a matching keepsake box in the same slot. Finally, it searches the keepsake box for the lower bound of the right end-point's memento $m(q_{2_r}) = 3$. Since Slot 1 is in the same run and has a vacant fingerprint, it signals that the keepsake box is encoded using Case (3). Thus, Memento filter searches the list of mementos stored in Slot 2 (which has a single memento) while also accounting for the mementos in Slots 0 and 1, resulting in a lower bound of 2. As this lower bound equals the memento of the left end-point $m(q_{2_l}) = 2$, Memento filter returns a positive.

If $p(q_l) + 1 = p(q_r)$: This case implies that the query range has two relevant partitions: one covering the left end-point q_l and another covering the right end-point q_r . Fig. 5-(B) illustrates this case. Processing this type of query amounts to checking whether the largest key in the partition of $p(q_l)$ or the smallest key in the partition of $p(q_r)$ is in the query range. One can observe from Fig. 5-(B) that knowledge of these points is enough to answer this range query. This is equivalent to checking whether the largest and smallest mementos of the keys in these partitions are contained in the sub-ranges of the range query, as defined by the universe partitioning.

Hence, Memento filter processes this query by first locating the keepsake box of $p(q_l)$, if it exists. If it does, its largest memento m' is checked for inclusion in $[m(q_l), 2^r - 1]$. If m' is included in the range, Memento filter returns a positive. If m' is not in the range or a keepsake box for $p(q_l)$ does

not exist, the keepsake box of $p(q_r)$ is located. Memento filter then fetches the smallest memento m'' from the keepsake box and checks for its inclusion in $[0, m(q_r)]$. If m'' is in this range, the query results in a positive. Otherwise, it results in a negative. Fig. 5-(B) shows a positive query, as m' and m'' are in their sub-ranges, implying that the red keys are in q .

A crucial property of the keepsake encoding scheme is that the largest and smallest mementos are always either stored in the same slot as the fingerprint or in its subsequent slot. Therefore, queries find these extrema in a cache-friendly manner without searching.

Query $q_2 = [23, 27]$ in Fig. 6 is an example of a range query intersecting two partitions. Following the above procedure, Memento filter hashes the prefix of the left end-point $p(q_{2l}) = 5$ to derive the canonical slot address 4 and fingerprint 1101. It then finds the corresponding run in Slot 5 and finds a matching keepsake box in the same slot. As the keepsake box is encoded using Case (3), Memento filter determines the largest memento in it by reading the memento stored alongside the vacant fingerprint without searching the list of mementos starting in Slot 7. Finally, as this largest memento, i.e., 3, equals the memento of the left end-point $m(q_{3l}) = 3$, Memento filter returns a positive. Notice how Memento filter skipped locating the run of the right end-point, as the result of the first probe made the query a positive.

Longer Range Queries. Memento filter also supports longer range queries by checking more partitions in exchange for higher FPR and query times. Here, Memento filter only checks the largest and smallest mementos for the first and last partitions, similarly to the above discussion. Moreover, it only needs to check whether a matching fingerprint exists for the intermediate partitions.

Point Queries. Memento filter processes a point query for a key q by finding its corresponding keepsake box. The query results in a negative if there is no such keepsake box. Otherwise, Memento filter uses binary search to find a memento equal to $m(q)$ in the keepsake box. If it finds one, it returns true. The absence of $m(q)$ in the keepsake box implies that q was not in the key set, and the filter thus returns a negative.

Query $q_3 = 3$ in Fig. 6 is an example of a point query. Here, Memento filter hashes the prefix to compute the partitions' canonical slot address $h(p(q_3)) = 1$ and the fingerprint $h_f(p(q_3)) = 1111$. As the relevant canonical slot is occupied, Memento filter locates its corresponding run, which is in Slot 3. It iterates over the keepsake boxes and sees only one with a matching fingerprint of 1111. It then searches for a memento equal to $m(q_3) = 3$ in the keepsake box and returns a negative result since it finds none.

Notice that the insertion, deletion, and query operations described above are general in the sense that they can be applied in any intermixed order, as no assumptions are made regarding the previously applied operations.

Bulk Loading. Memento filter supports bulk loading by first sorting the keys to be inserted in increasing order of their slot addresses, fingerprints, and mementos, respectively. This ordering enables Memento filter to encode all runs and keepsake boxes via a single left-to-right pass of the filter, maximizing cache efficiency. While this is an $O(N \log N)$ algorithm, it performs better than inserting the keys one by one in $O(\ell N) \approx O(N)$ time (as proven in Section 6), since it incurs sequential rather than random memory accesses, leveraging the hardware prefetcher.

Concurrency. As Memento filter is built on top of the RSQF, it can reuse its concurrency mechanisms [39]. More concretely, Memento filter's underlying RSQF is partitioned into regions of 4096 slots, each with a spinlock. A thread performing an operation locks the region its key's prefix hashes to and its subsequent region before modifying the filter. Locking two consecutive regions allows for thread-safe shifting of slots.

In many cases, the workload is not heavily skewed, and threads will typically map to and lock different regions, thanks to the uniformity of hashing. However, if the dataset is heavily skewed, many threads may want to modify the same keepsake boxes, causing lock contention. Alleviating

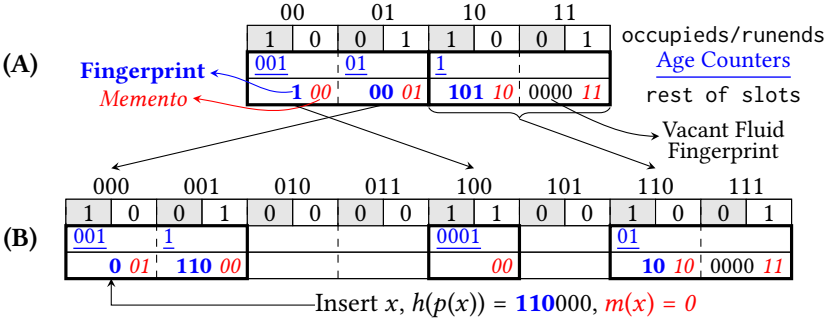


Fig. 7. An expandable Memento filter employs fluid fingerprints comprised of an age counter and a fingerprint. It uses them to remap keepsake boxes during expansions. Runs are delineated using thick lines, and the slot addresses are shown in binary to illustrate how repurposing a fingerprint bit as an address bit affects the keepsake box mappings.

this contention is a promising direction for future work. One potential approach may be buffering blocked inserts and deletes in small, per-thread Memento filters before dumping them into the main filter, similar to [39].

Supporting Variable-Length Keys. Memento filter assumes its keys to be fixed-length strings. Many applications that use range filters operate on numerical data, which are fixed-length binary strings. Memento filter applies to these cases as-is. One can also convert variable-length keys into l^* -bit strings by zero-padding short and truncating long keys, where l^* is the smallest length such that the keys are distinguishable using l^* -bit prefixes. This method strives to support the longest range queries but forgoes robustness due to the truncated suffixes. One can preserve robustness by increasing l^* to keep the bits differentiating the keys and queries.

5 EXPANDABILITY

InfiniFilter. InfiniFilter is an expandable Quotient Filter (a simpler but less efficient version of an RSQF). In InfiniFilter, each slot contains a unary “age counter” of the form $0 \dots 01$ that signals how many expansions ago a key was inserted, along with a fingerprint [17]. During an expansion, a bit from each fingerprint is transferred to its canonical slot’s address, incrementing the age counter and allowing InfiniFilter to uniformly map it to a larger filter with the same slot width as before. We call the concatenation result of an age counter with its fingerprint a “fluid fingerprint.”

Expandable Memento Filter. An expandable Memento filter stores fluid fingerprints instead of standard fingerprints. Fig. 7-(A) shows an example of an expandable Memento filter with a fluid fingerprint and memento length of $f = 4$ and $r = 2$. Slot 00 stores a single-bit fingerprint **1**, along with an age counter of 001, signaling that the key was inserted into the filter two expansions ago.

To delimit and encode keepsake boxes within a run as described in Section 4, they are stored in increasing order of their fluid fingerprints. For example, in Fig. 7-(A), the run at Slot 00 stores two keepsake boxes of length one and puts the one with a fluid fingerprint of 0011 before the other with a fluid fingerprint of 0100.

Since the age counters always have a set bit, fluid fingerprints are never zero. Thus, zero “vacant fluid fingerprints” can always act as escape sequences for encoding keepsake boxes, eliminating the “zero fingerprint” corner case presented in Section 4 for Case (3).

We now describe how Memento filter uses fluid fingerprints to implement its various operations.

Expansions and Contractions. Memento filter expands by allocating a filter two times its size with the same slot width of $f + r$ bits. Recall that the canonical slot address and the fingerprint of

each keepsake box are parts of the same original hash $h(p)$, where p is the prefix of the keepsake box's keys. With this in mind, Memento filter iterates over the old filter, reconstructs the original hash of each keepsake box by concatenating its fingerprint to its canonical slot address, and inserts it into the new filter. The new canonical slot address of a keepsake box is the $\log_2(n) + 1$ least significant bits of its hash, while the fingerprint consists of the remaining more significant bits. Therefore, this process repurposes the least significant bit of each fingerprint to become the most significant bit of the new canonical slot address. It also increments the age counter of the old fingerprints, meaning that the resulting fluid fingerprints retain a length of f bits. Fig. 7-(B) shows an example of such an expansion, followed by the insertion of a new key x with a prefix hash of $h(p(x)) = \mathbf{110000}$ and memento $m(x) = 0$. Analogously, Memento filter contracts by halving the number of slots and transitioning a bit from the addresses to the fingerprints.

Insertions. To maintain a stable FPR, new insertions are made with full-length fingerprints. That is, a key x is inserted by first finding a keepsake box with a *full-length matching fingerprint*. If one exists, $m(x)$ is added to it. Otherwise, Memento filter creates a new keepsake box with a full-length fingerprint for x to minimize the FPR. Fig. 7-(B) shows an example. Even though the new key x has a partially matching fingerprint with the migrated keepsake box in Slot 000, it manifests as a separate keepsake box. Note that an f -bit fluid fingerprint can represent fingerprints of length at most $f - 1$ bits. Thus, each slot and fluid fingerprint of the filter must be one bit wider to maintain the same fingerprint length and FPR as a standard Memento filter.

Deletions. When deleting a key y , Memento filter removes a memento equal to $m(y)$ from the keepsake box with the *longest matching fingerprint*. The reason is that deleting a memento with a shorter associated fingerprint may cause false negatives, as it may have resulted from a hash collision with a different keepsake box.

Queries. Queries are handled as described in Section 4, but Memento filter must probe all keepsake boxes with matching fluid fingerprints for potential mementos. For example, in Fig. 7-(B), a point query with a prefix hash of $\mathbf{110000}$ must check both keepsake boxes at Slots 000 and 001, which have fingerprints $\mathbf{0}$ and $\mathbf{110}$. This does not damage query performance, as partially matching fingerprints are rare, and memory is still accessed sequentially.

Unbounded Expansions. These methods allow Memento filter to expand up to $f - 1$ times, implying that it can grow by a factor of up to 2^{f-1} . For typical fluid fingerprint lengths such as $f = 11$ bits, this translates to Memento filter expanding up to $2^{10} = 1024$ times its original size, which is sufficient for many applications.

However, Memento filter fails to expand more than $f - 1$ times, as the oldest fingerprints run out of bits to sacrifice. Memento filter can overcome this by applying InfiniFilter [17]'s chaining method. Concretely, when a keepsake box's fingerprint is depleted, it is removed from the filter and inserted into a smaller, secondary Memento filter, where the hash is long enough to create a full-length fingerprint. The secondary filter expands until its fingerprints run out of bits, at which point it is added to a chain of filters, and a new secondary filter is created. New insertions always go to the main filter, but deletions and queries must probe all the filters.

Speeding up queries and deletions in this case is an interesting direction for future work. One approach may be to duplicate exhausted fingerprints across the slots that could correspond to it, similarly to what Aleph Filter proposes [16].

Rejuvenation. Memento filter employs InfiniFilter's "rejuvenation" operation. Here, a fingerprint is lengthened during a positive query, whereby the application regains access to the original key and can thus rehash it to derive a longer fingerprint, improving the filter's FPR while also delaying the creation of secondary filters.

6 THEORETICAL ANALYSIS

We show that Memento filter is close to space optimal and that its operation costs are low. The last row of Table 2 summarizes the results of this section.

False Positive Rate. We begin by answering the following question: *Given a prefix p^* , what is the probability P^* that there is a different partition with some prefix $p \neq p^*$ with a matching keepsake box?* The answer to this question is a conservative upper bound on the FPR for both point and range queries, as it does not take into account the *non-robust* filtering the mementos provide.

There are at most $\frac{N}{\ell}$ such partitions with prefix $p \neq p^*$, where N is the number of keys inserted into the filter and ℓ is the average partition size. For p to have a matching keepsake box with p^* , it must have the same canonical slot, i.e., $h(p) = h(p^*)$. If it shares the same canonical slot, it must also share the same fingerprint, i.e., $h_f(p) = h_f(p^*)$. The former has a probability of $\frac{1}{n}$, and the latter has a probability of 2^{-f} . Thus, since these events are independent, the probability that p and p^* have matching keepsake boxes is $\frac{1}{n} \cdot 2^{-f}$. Then, via a union bound on the total possible partitions with prefix $p \neq p^*$, one can see that $P^* \leq \frac{N}{\ell} \cdot \frac{1}{n} \cdot 2^{-f} = \frac{\alpha}{\ell} \cdot 2^{-f}$.

A negative point query for key q can only result in a false positive when there is some partition with prefix $p \neq p(q)$ with the same fingerprint and a memento equal to $m(q)$. The probability ϵ_p of this event is upper bounded by $P^* = \frac{\alpha}{\ell} \cdot 2^{-f}$, since P^* only accounts for the existence of such a partition. Depending on the key distribution, the mementos may provide much better filtering and improve the FPR by a factor of at best 2^{-r} .

Analogously, a negative range query $q = [q_l, q_r]$ can only result in a false positive if there is some partition with a prefix $p \neq p(q_l), p(q_r)$ that has a memento in the target memento ranges. Such a partition exists with probability ϵ_r at most $2P^* = \frac{\alpha}{\ell} \cdot 2^{1-f}$, due to a union bound applied to $p(q_l)$ and $p(q_r)$. The mementos may further improve the FPR by a constant factor. Thus, the overall FPR $\epsilon \leq \max(\epsilon_r, \epsilon_p)$ is at most $2P^* = \frac{\alpha}{\ell} \cdot 2^{1-f}$.

Using this result, one can follow a similar analysis to InfiniFilter [17] and derive a bound of $\epsilon \leq (E + 2) \cdot \frac{\alpha}{\ell} \cdot 2^{-f}$ for the FPR of an expandable Memento filter, where $E \leq \log_2(N)$ is the number of expansions the filter has undergone.

Expected Cluster Length. Let $\beta(\ell)$ be the length of the encoding of a keepsake box with size ℓ , measured in slots. We prove the following bound on the expected cluster length $\mathbb{E}[|C|]$ (see Appendix A):

$$\text{THEOREM 6.1. } \mathbb{E}[|C|] \leq \frac{\alpha\gamma(\ell)}{(1-e^{-\alpha/\ell}) \cdot (1-\alpha) \cdot (\gamma(\ell)-\alpha)}, \text{ where } \gamma(\ell) = \frac{\ell}{\beta(\ell)}.$$

Since $\alpha \leq 0.95$ and $1 \leq \gamma(\ell) \leq 1 + f/r$, the expected cluster size will be $O(\ell)$. This further implies that there is an $O(1)$ number of keepsake boxes in the average cluster. In practice, we have found ℓ to be close to one, implying that Memento filter will have a constant expected cluster length. Theorem 6.1 further demonstrates the excellent scalability of Memento filter with extreme dataset skew. That is, when ℓ is small, $\gamma(\ell)$ will be close to one. Therefore, assuming $\alpha = 0.95$, we have that $\mathbb{E}[|C|] \leq \frac{\alpha}{(1-e^{-\alpha}) \cdot (1-\alpha)^2} \approx 619.64$, which matches a standard RSQF. However, as ℓ increases, $\gamma(\ell)$ tends to $1 + f/r$, which is typically at least 2. In this case, $\mathbb{E}[|C|] \lesssim \frac{2}{1-\alpha} \cdot \ell$, meaning that clusters remain as small as possible. As an example, assuming $f = r$, $\alpha = 0.95$, and $\ell = 7$, we have that $\mathbb{E}[|C|] \lesssim 413.77$, which improves upon an RSQF.

Performance. An insertion into Memento filter locates the target keepsake box and adds the new key's memento to it. In the worst case, this operation will read and shift the entire cluster of the keepsake box. Since the expected cluster length is $O(\ell)$, an insertion also has an expected running time of $O(\ell)$. A deletion follows an analogous procedure and thus has an $O(\ell)$ expected execution time.

A point query locates the appropriate keepsake box by skipping a constant number of keepsake boxes in the cluster and searches for the target memento, requiring a total of $O(\log_2 \ell)$ operations. Range queries access either one or two keepsake boxes. The former case's analysis is identical to the case of point queries. In the latter case, both keepsake box lookups take $O(1)$ operations, and only the largest and smallest mementos are ever accessed for each, which requires $O(1)$ time since they are stored near the keepsake box's fingerprint. Thus, the cost of a range query is $O(\log_2 \ell)$.

Notice that each keepsake box lookup entails only a single random cache miss on average. Probing a keepsake box is done using the already cached memory segments and incurs no further cache misses. Since clusters and keepsake boxes are arranged sequentially, when a cluster becomes too large to fit in a cache line, the resulting extraneous memory accesses and cache misses are all sequential, thus taking full advantage of the hardware prefetcher.

In conclusion, on average, Memento filter will incur a single cache miss for insertions, deletions, and point queries, while range queries are serviced with up to two random cache misses.

Memory Footprint. Each slot in Memento filter is $f + r$ bits long. With the FPR analysis in mind, Memento filter can guarantee an FPR of ϵ with a fingerprint length of $f = 1 + \log_2 \frac{1}{\epsilon}$. Furthermore, to support range queries of length R , r must be at least $\log_2 R$ bits. Taking into account the metadata overhead of the RSQF, a Memento filter with a load factor of α will have a memory footprint of $\frac{1}{\alpha}(3.125 + \log_2 \frac{R}{\epsilon})$. In the case of an expandable Memento filter, since each slot is one bit wider to accommodate the unary age counter, the memory footprint becomes $\frac{1}{\alpha}(4.125 + \log_2 \frac{R}{\epsilon})$.

7 EVALUATION

We compare Memento filter to existing range filters in a standalone setting in Section 7.1 In Section 7.2, we provide experimental results from our integration of Memento filter with WiredTiger, a B-tree based key-value store. We utilize Grafite's benchmark template for our evaluations [15].

Platform. We use a Fedora 39 machine with a single Intel Xeon w7-2495X processor (4.8 GHz) with 24 cores and 48 hyperthreads. It has 64 GBs of main memory, a 45 MB L3 cache, a 48 MB L2 cache, and a 1920 kB L1 cache. It also has two SK Hynix 512 GB PC611 M.2 2280 80mm SSDs, with a sequential read/write performance of up to 3400/2700 MBps and random read/writes of up to 440K/440K IOPS. These SSDs are used in the B-Tree experiments only.

7.1 Standalone Evaluation

Baselines. We conduct experiments over both static and dynamic data. In the static setting, we compare Memento filter with SuRF [47], Rosetta [32], REncoder [22, 46], Proteus [29], SNARF [44], Oasis+ [10], and Grafite [15]. We do not include bloomRF [36] as a baseline as it is closed-source. In the dynamic setting, we only compare the expandable version of Memento filter with Rosetta, REncoder, and SNARF, as other filters do not support incremental updates. We implement Memento filter in C and use the open-source C/C++ implementations of the baselines. All filters are compiled with gcc-13.

We employ the original key suffixes in the leaves of SuRF when considering range query workloads to allow for comparing query end-points at the leaves, and use hash suffixes when considering point query workloads. We allow Rosetta and Proteus to auto-tune their memory allocation with a query sample, showcasing their best performance. We tune Memento filter with a memento size r based on the maximum query size in the workload.

Datasets. We conduct our experiments with the same synthetic and real-world datasets [28, 33] used in previous range filter evaluations [15, 22, 29, 32, 36, 44, 46, 47]:

- **UNIFORM:** 200M 64-bit integers chosen uniformly at random.
- **NORMAL:** 200M 64-bit integers sampled from $\mathcal{N}(2^{63}, 0.1 \cdot 2^{63})$.

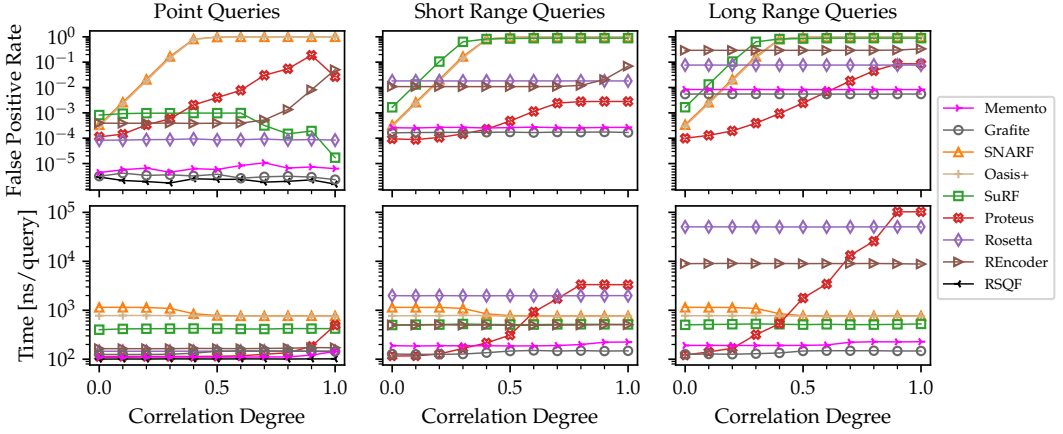


Fig. 8. Most filters (SNARF, Oasis+, SuRF, Proteus, REncoder) exhibit worse FPRs as the workload becomes increasingly correlated and are thus not robust. Some of these filters (SNARF, Proteus) also have varying query times with different correlation degrees. Only Rosetta, Grafite, and Memento filter have a robust FPR guarantee and at the same time, stable query costs.

- BOOKS: Amazon booksale popularity for 200M books.
- OSM: 200M location coordinates from the Open Street Map.

Static Workloads. Following existing works [15, 22, 29, 32, 36, 44, 46, 47], we create a set of 10M range queries of the form $[x, x + R - 1]$, where x is a key from the key universe and R is the range query length. We run separate workloads with point queries ($R = 1$), short range queries ($R = 2^5 = 32$) and long range queries ($R = 2^{10} = 1024$). We choose the starting point x of the queries in one of three ways:

- UNCORRELATED: x is chosen uniformly at random.
- CORRELATED: x is chosen by first considering a randomly chosen key k from the dataset, and sampling from the range $[k, k + 2^{30 \cdot (1-D)}]$, where D is the correlation degree of the workload. By default, we set $D = 0.8$.
- REAL: x is sampled and removed from the underlying dataset.

In all these workloads, we only consider empty query ranges, allowing us to measure the FPR as the ratio of positive results to the query batch size. We also provide a separate experiment detailing filter throughput for positive queries. We only consider the filter query times in our standalone experiments and not the time required to access a slower storage medium.

Experiment 1: Robustness to Correlated Workloads. We evaluate the robustness of the range filters by using the static UNIFORM dataset and a CORRELATED query workload with a varying correlation degree from 0 to 1. All filters are assigned a memory budget of 20 bits per key. The first row of Fig. 8 shows that only Rosetta, Grafite, and Memento filter are unaffected by workload correlation and are thus robust. Both Memento filter and Grafite have better FPRs than Rosetta by up to two orders of magnitude. As shown, Memento filter approximately matches the FPR of Grafite. All other filters exhibit increasing FPRs with more correlation.

Notice that, when considering point queries, SuRF's FPR actually decreases with higher correlation degrees. This is due to SuRF comparing key hashes in this case, which provides much better filtering when the workload is heavily correlated.

The second row of Fig. 8 shows that Grafite and Memento filter are the most efficient range filters in terms of query speed, improving upon all other filters by a factor of at least 4×. Memento filter

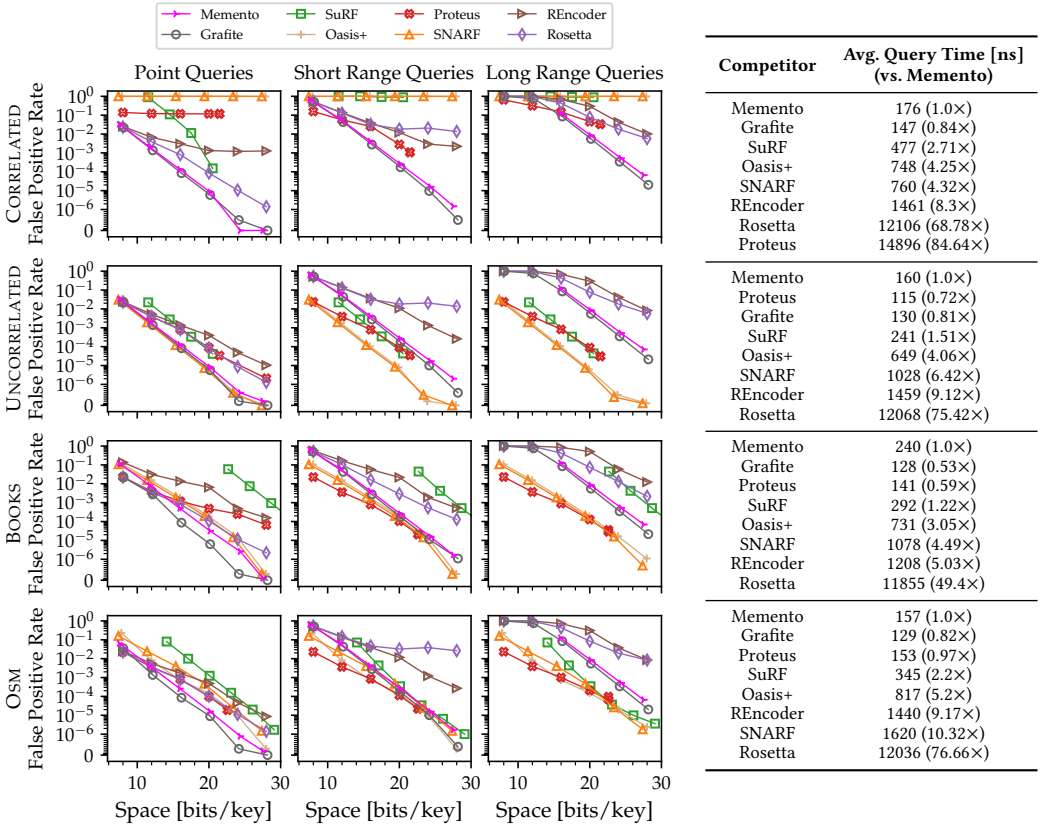


Fig. 9. Memento filter and Grafite provide the best filtering in the case of correlated workloads, the best point filtering in general, and the fastest overall query speed. Even though Memento filter and Grafite are competitive with the state of the art on real workloads, they provide less filtering compared to their heuristic counterparts when considering an uncorrelated workload due to the strong filtering guarantees they provide.

provides faster point queries than Grafite by 20%, while closely matching Grafite’s performance in servicing range queries.

We have included evaluation results for a vanilla RSQF with the same memory footprint in the point query column of Fig. 8. As shown, Memento filter achieves an FPR competitive with a standard RSQF while adding negligible overhead to queries.

Experiment 2: FPR vs. Memory Tradeoff. Fig. 9 shows an FPR comparison of all range filters on synthetic and real-world data. In the synthetic case, we consider the UNIFORM dataset and execute both CORRELATED and UNCORRELATED workloads. For the real workloads, we use the BOOKS and OSM datasets, along with REAL query workloads. Each row of Fig. 9 provides experiment results for a single dataset and workload with varying range sizes, as well as query speed statistics averaged over all range query sizes.

We only provide partial graphs for Memento filter when considering long range queries, since Memento filter requires at least 12 bits to store metadata and a large enough memento in this case. Notice that all other robust range filters exhibit an FPR of 1 below this space threshold, only wasting memory.

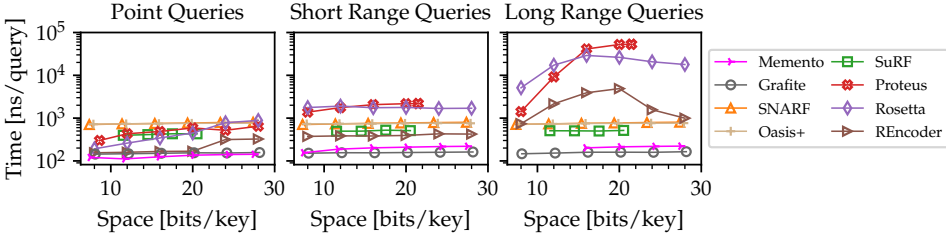


Fig. 10. Grafite and Memento filter provide the best and most stable performance when non-empty queries are involved.

As established before, Grafite and Memento filter have the best FPR when the workload is correlated. Memento filter is competitive with Grafite with only a $1.5\times$ gap in FPR and provides up to 5 orders of magnitude better FPR than Rosetta. Furthermore, Memento filter and Grafite provide the best point filtering across all datasets. However, as the range sizes increase in non-correlated workloads, robust range filters provide less filtering than their heuristic competitors due to their strong FPR guarantees.

In terms of query speed, Grafite and Memento filter provide the best overall performance. Memento filter is slightly slower than Grafite but provides dynamic insertions and deletions in exchange. Even though Proteus is faster than Memento filter and Grafite in the last three rows of Fig. 9, it does not guarantee a robust FPR and is slower when considering correlated workloads.

Experiment 3: Non-Empty Query Performance. Although filters are typically used to reduce slower media accesses, such as network calls and disk reads, they must minimize their added CPU overhead for non-empty queries as well. We thus benchmark filters throughputs on non-empty queries in Fig. 10 by using the UNIFORM dataset and creating query ranges of the form $[x, x + R - 1]$, where x is sampled from $[k - L + 1, k]$ for a randomly chosen key k in the dataset. We also experimented with the NORMAL and REAL datasets, but omit their results as the best filters and their performance remains the same. The results show that Grafite and Memento filter are the fastest to process positive range queries and provide stable performance with varying memory budgets. Memento filter matches Grafite’s performance in processing range queries and has faster point queries by up to 38%.

Experiment 4: Construction Time. Fig. 11 compares the construction times of all range filters with varying dataset sizes. Since the choice of dataset does not influence the construction times of the filters, we use the UNIFORM dataset. We report construction time averages over various memory budgets. The light colors of Fig. 11 used for Rosetta and Proteus indicate the impact of their tuning processes, evaluated with an UNCORRELATED query workload with $\frac{N}{10}$ queries, where N is the number of keys in the dataset.

Memento filter achieves the best construction time in almost all cases, beating its closest competitor by 20%. Memento filter’s bulk loading algorithm can be further optimized by using a multi-threaded sorting algorithm to sort the key hashes. Moreover, since Memento filter is a dynamic filter, it can also be constructed by streaming the keys. Therefore, Memento filter can be constructed in a single pass of the data without the need for sorting, providing significant speedup when the dataset is too large to fit in memory.

Experiment 5: Memento Size Choice. Accurately estimating the maximum range query length R is integral to Memento filter’s performance, as the memento size r is chosen to be $\lceil \log_2 R \rceil$. In practice, however, users may err in estimating R and thus in setting r . Fig. 12 shows how Memento filter’s FPR and query speed vary for different memento size configurations under a memory budget of 20 bits per key. Here, longer mementos imply shorter fingerprints and vice-versa. We consider

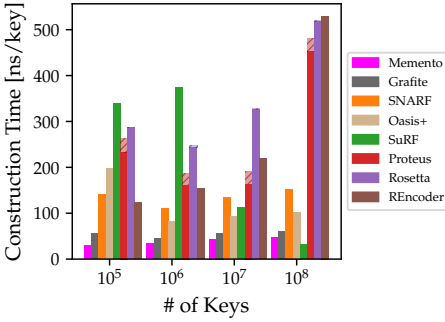


Fig. 11. Memento filter provides the best construction time in almost all scenarios.

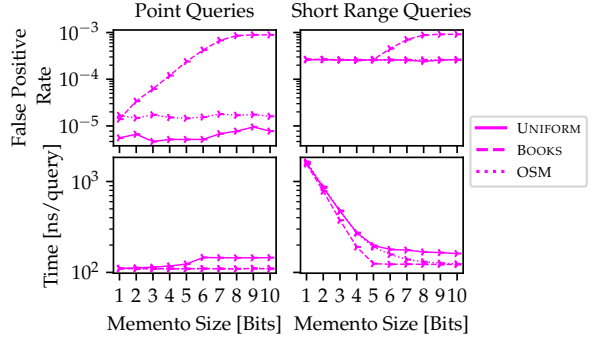


Fig. 12. Memento filter maintains its FPR guarantee if the memento size is too small but exhibits slower queries. If the memento size is too large, Memento filter maintains its original query speed, but its FPR may suffer.

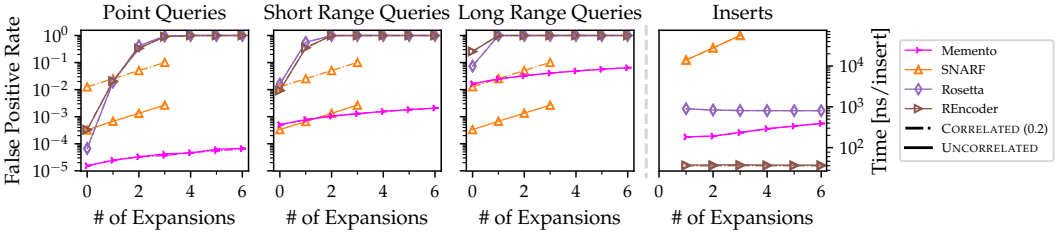


Fig. 13. Memento filter is the only filter that maintains fast insertion times and a low false positive rate across expansions.

various datasets and issue queries from a correlated workload ($D = 0.8$). We also experimented with uncorrelated workloads but have omitted the results, as they are at least as good as the correlated case. The first column considers point queries, while the second column showcases short range queries with $R = 32$. Thus, the optimal memento length r^* in the first column is $r^* = 1$, while for the second column it is $r^* = 5$.

Fig. 12 shows that Memento filter’s FPR does not deviate from the optimal as long as $r \leq r^*$. However, its query time worsens by a factor of 2^{r^*-r} due to the extra lookups, which is proportional to the user’s estimation error of R . In contrast, if $r > r^*$, Memento filter may exhibit a higher FPR depending on the dataset. The reason is that the robust filtering provided by the fingerprints is replaced with the non-robust filtering provided by the mementos. Fig. 12 shows that the FPR does not worsen indefinitely and saturates at a dataset-dependent value. Furthermore, Memento filter maintains its optimal query speed, except for point queries, where it incurs a slight slowdown due to the extra memento comparisons.

We advise practitioners to estimate a lower bound of R as close as possible to the actual value, preserving the excellent FPR guarantee of Memento filter in exchange for slightly slower queries. As filters typically use 1-3 bytes per key in practice, one cannot construct a robust range filter for large R due to the information-theoretic lower bound. For moderate R , having one-byte fingerprints and one to two-byte mementos is common.

Dynamic Workloads. We consider the UNIFORM dataset and construct the filter on a random $\frac{1}{64}$ fraction of the data. We then insert keys into the filter one by one until an *Expansion* occurs, i.e.,

the digested data size doubles. We continue this process until the entire dataset is inserted into the filter. After each expansion, we measure performance statistics by running 10M queries from both UNCORRELATED and CORRELATED workloads.

Experiment 6: Expanding Datasets. We compare Memento filter with Rosetta, REncoder, and SNARF (the only other filters supporting incremental insertions) in a dynamic setting. Fig. 13 plots the FPR and insertion times of these filters across expansions. All filters are constrained to a memory budget of 20 bits per key.

Even though the filters have a similar initial FPR, only Memento filter maintains its FPR guarantee across expansions. It also maintains its excellent insertion speed. Rosetta and REncoder provide no filtering after just three expansions, while SNARF fails to accommodate new insertions efficiently. We do not plot all of SNARF’s performance metrics, as it takes over 5 hours to expand after the third expansion. Even though SNARF still provides better filtering than Memento filter in the face of long and mixed UNCORRELATED range queries, it is worse in all cases as soon as the workload becomes slightly correlated (even with a correlation degree of 0.2).

Memento filter provides better insertion times than Rosetta but is slower than REncoder. Moreover, its insertion throughput is decreasing slightly. This is due to Memento filter expanding when the dataset size doubles, causing a smaller fraction of the filter to fit in the higher levels of cache in exchange for maintaining its FPR.

It is worth noting that only Memento filter is compatible with InfiniFilter’s techniques, as it is a tabular filter. All other range filters utilize bitmaps and Bloom filters in their structures, which makes them unable to expand without rescanning the data from storage.

7.2 B-Tree Evaluation

B-Trees [12, 41] are the de facto standard for file organization and indexing tasks. These structures are search trees that minimize data movement – the main bottleneck of database systems. Similarly to a binary search tree, the internal nodes of a B-Tree partition the search space of the key set into B partitions, where B is dictated by the data movement granule the system offers and the key size. The leaves of the tree contain the entries themselves in sorted order. As entries are added and removed from the tree, it rebalances to maintain robust performance.

Databases use this data structure to achieve efficient random access to keys. They can also scan specific ranges of the entries, as the tree is order-preserving. B-Trees are further optimized using *Buffer Pools*, which cache frequently accessed nodes in main memory to reduce data movement and access latency [20].

B-Trees are ubiquitous in many industrial applications. For example, MongoDB [34], a popular document database, uses a B-Tree-based key-value store called WiredTiger [35] as its backend. However, B-Trees are often subject to workloads with many empty short range queries, comprising up to 50% of their queries. This is observed in several database applications, such as social graph analytics [4, 8]. Thus, B-Trees are a prime example of an application that can significantly benefit from a dynamic range filter.

Due to WiredTiger’s widespread industrial use, we integrate the expandable version of Memento filter with it. We create a single instance of our filter, which is constructed over the entire data. To the best of our knowledge, we are the first to integrate a range filter with a B-Tree, a feat previously impossible due to the dynamicity of B-Trees which necessitates a dynamic/expandable range filter.

Datasets. We conduct our evaluation with subsets of size 100M from the UNIFORM, NORMAL, and BOOKS datasets. In all cases, we store randomly generated 504-byte values in the B-Tree to make for 512-byte key-value pairs.

Workloads. We employ a workload similar to the dynamic workload described in Section 7.1, but initialize the system on a random $\frac{1}{8}$ fraction of the dataset considered. To measure performance

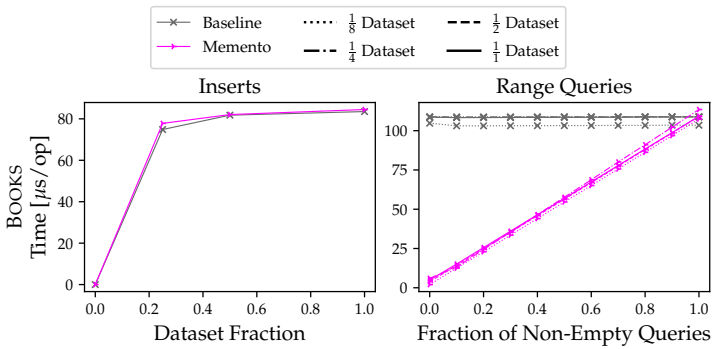


Fig. 14. Memento filter significantly improves the query throughput of WiredTiger when empty queries are present while maintaining the same insertion performance.

statistics, we run 10M mixed range queries of length $1 \leq R \leq 32$, where the left end-point is sampled from the same distribution of the keys, i.e., UNIFORM, NORMAL, and REAL. We vary the percentage of non-empty queries in the workload to provide a clear overall picture of the system’s performance in different scenarios.

Baselines. Since Memento filter is the only dynamic and expandable range filter, we only compare it with a standard instance of WiredTiger. This instance will use all of its allocated main memory for a buffer pool, allowing it to cache many of the B-Tree’s nodes. When integrating the expandable version of Memento filter, we reallocate some of the buffer pool’s memory for a Memento filter with a memory budget of 15 bits per key to draw a fair comparison. We allocate a total memory budget equivalent to 2% of the current dataset size to both instances.

Experiment 7: B-Tree Performance. The results of Fig. 14 show that WiredTiger benefits immensely from Memento filter at all data sizes, achieving faster query processing by $1.9\times$ when 50% of the queries are empty. Note that we only show one plot, as all datasets have similar results. WiredTiger also maintains its query throughput when all queries are non-empty. Furthermore, Memento filter does not significantly affect the insertion times of WiredTiger in both workloads, incurring a minor overhead of $\approx 2.5\%$. Thus, trading off buffer pool memory for a Memento filter in workloads with many empty queries may significantly improve the system’s overall performance.

8 CONCLUSION

We introduced Memento filter, the first dynamic range filter with fast operations and a robust false positive rate guarantee. By encoding keepsake boxes in an RSQF, Memento filter achieved FPRs and performance on par with the state of the art. It further achieved expandability by employing variable-length fingerprints. We argued that Memento filter is the only practical dynamic range filter, and solidified our claim by integrating it with WiredTiger, showing that it significantly boosts range query performance while not hindering insertions. Further exploring the tradeoffs of using Memento filter in a fully functional system, the tradeoffs of partitioning it into smaller filters, and its cacheability are intriguing directions for future work. Additionally, exploring other design choices, such as adaptively partitioning the key universe, will be fruitful.

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A EXPECTED CLUSTER SIZE

PROOF. Consider a randomly chosen cluster C starting in Slot p . We inductively define two series of random variables X_i and Y_i based on C as follows:

- X_1 represents the number of partitions in the key universe mapping to Slot p .
- Y_1 represents the number of keys in the partitions considered in X_1 .
- X_i represents the number of partitions in the key universe mapping to any of the slots in the range $[p + \sum_{j=1}^{i-1} Z_j, p - 1 + \sum_{j=1}^i Z_j]$, where Z_j is the number of slots filled by the keys considered in Y_j . Since $Z_j \leq Y_j$ in Memento filter due to the keepsake box encoding scheme, one can simplify the definition of X_i by instead considering partitions mapping to the range $[p + \sum_{j=1}^{i-1} Y_j, p - 1 + \sum_{j=1}^i Y_j]$ containing more slots, slightly overestimating X_i .
- Y_i represents the number of keys in the partitions considered in X_i .

Observe that $\mathbb{E}[|C|] \leq \mathbb{E}[\sum_{i=1}^{N/\ell} Z_i] \leq \mathbb{E}[\sum_{i=1}^{N/\ell} Y_i] = \sum_{i=1}^{N/\ell} \mathbb{E}[Y_i]$. We thus bound the $\mathbb{E}[Y_i]$ s to bound $\mathbb{E}[|C|]$. First notice that

$$\mathbb{E}[Y_i] = \mathbb{E}[\mathbb{E}[Y_i | X_1, \dots, X_i, Y_1, \dots, Y_{i-1}]] = \mathbb{E}\left[\frac{N - \sum_{j=1}^{i-1} Y_j}{N/\ell - \sum_{j=1}^{i-1} X_j} \cdot X_{i-1}\right] \quad (1)$$

since all partitions in X_i map to the desired range equi-probably and their expected total number of keys $\mathbb{E}[Y_i]$ equals their average size times the number of partitions X_i . Applying the Poisson approximation to the balls and bins problem with partitions as balls and slots as bins, we get

$$\mathbb{E}[X_i | X_1, \dots, X_{i-1}, Y_1, \dots, Y_{i-1}] = \frac{N/\ell - \sum_{j=1}^{i-1} X_j}{n - \sum_{j=1}^{i-1} Z_j} \cdot Y_{i-1} \leq \frac{N/\ell - \sum_{j=1}^{i-1} Y_j}{n - \sum_{j=1}^{i-1} Y_j} \cdot Y_{i-1}. \quad (2)$$

Putting equations 1 and 2 together, we conclude that

$$\begin{aligned} \mathbb{E}[Y_i] &\leq \mathbb{E}\left[\mathbb{E}\left[\frac{N - \sum_{j=1}^{i-1} Y_j}{N/\ell - \sum_{j=1}^{i-1} X_j} \cdot X_i | X_1, \dots, X_{i-1}, Y_1, \dots, Y_{i-2}\right]\right] \\ &\leq \mathbb{E}\left[\frac{N - \sum_{j=1}^{i-1} Y_j}{N/\ell - \sum_{j=1}^{i-1} X_j} \cdot \frac{N/\ell - \sum_{j=1}^{i-1} X_j}{n - \sum_{j=1}^{i-1} Y_j} \cdot Y_{i-1}\right] \\ &\leq \mathbb{E}\left[\frac{N - \sum_{j=1}^{i-1} Y_j}{n - \sum_{j=1}^{i-1} Y_j} \cdot Y_{i-1}\right] \leq \mathbb{E}\left[\frac{N}{n} \cdot Y_{i-1}\right] = \alpha \cdot \mathbb{E}[Y_{i-1}], \end{aligned}$$

further implying $\mathbb{E}[Y_i] \leq \alpha^{i-1} \cdot \mathbb{E}[Y_1]$. Moreover, since $\mathbb{E}[Y_1] = \mathbb{E}[\frac{N}{N/\ell} \cdot X_1] = \ell \cdot \mathbb{E}[X_1]$, we have that $\mathbb{E}[Y_i] \leq \alpha^{i-1} \cdot \ell \cdot \mathbb{E}[X_1]$. Letting the random variable W denote the number of partitions mapped to Slot p and denoting by F the event where Slot p is the first slot in a cluster, we bound $\mathbb{E}[X_1]$ as

$$\begin{aligned} \mathbb{E}[X_1] &= \sum_{i=1}^{N/\ell} i \cdot \Pr(W = i | F) = \sum_{i=1}^{N/\ell} i \cdot \frac{\Pr(F | W = i) \cdot \Pr(W = i)}{\Pr(F)} \leq \sum_{i=1}^{N/\ell} i \cdot \frac{\Pr(W = i)}{\Pr(F)} \\ &\leq \frac{\alpha}{\ell \cdot \Pr(F)} \leq \frac{\alpha}{\ell \cdot \Pr(\text{Slot } p - 1 \text{ empty} \wedge W > 0)} \leq \frac{\alpha}{\ell \cdot (1 - \frac{\sum_i Z_i}{n}) \cdot (1 - e^{-\alpha/\ell})} \\ &\leq \frac{\alpha}{\ell \cdot (1 - \frac{N \cdot \beta(\ell)}{n \cdot \ell}) \cdot (1 - e^{-\alpha/\ell})} = \frac{\alpha \cdot \gamma(\ell)}{\ell \cdot (\gamma(\ell) - \alpha) \cdot (1 - e^{-\alpha/\ell})}. \end{aligned}$$

The last inequality uses Jensen's inequality in conjunction with the concavity of $\beta(\cdot)$. Putting everything together results in $\mathbb{E}[|C|] \leq \sum_{i=1}^N \mathbb{E}[Y_i] \leq \mathbb{E}[X_1] \cdot \ell \cdot \sum_{i=1}^{\infty} \alpha^{i-1} = \frac{\alpha \cdot \gamma(\ell)}{(1 - e^{-\alpha/\ell}) \cdot (\gamma(\ell) - \alpha) \cdot (1 - \alpha)}$. \square

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