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Research Article H2Rec: Homogeneous and Heterogeneous Network Embedding Fusion for Social Recommendation

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ABSTRACT

Due to the problems of data sparsity and cold start in traditional recommendation systems, social information is introduced. From the perspective of heterogeneity, it reflects the indirect relationship between users, and from the perspective of homogeneity, it reflects the direct relationship between users. At present, most social recommendation is based on the homogeneity or heterogeneity of social networks. Few studies consider both of them at the same time, and the deep structure of social networks is not extensively exploited and comprehensively explore. To address these issues, we propose a unified H2Rec model to fuse homogeneous and heterogeneous information for recommendations in social networks. Considering the rich semantics reflected by metapaths in heterogeneous information and the wealth of social information reflected by homogeneous information network and a random walk strategy guided by metapaths to generate node sequences in a heterogeneous information network (HIN). Finally, we combine the two parts into a unified model for social recommendation. Extensive experiments on three real-world datasets demonstrate the effectiveness of the proposed model.

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1. INTRODUCTION

With the rapid development of information technology and Internet technology, people have entered the era of information overload from the era of lack of information. The recommendation system and search engines plays an important role in finding the items that target users are interested in from a large amount of data [1]. The traditional methods (such as matrix factorization (MF) [2]) only utilize the user-item rating matrix for recommendation, that is, whether the user has interactive behavior to the items (such as browsing, attention, rating, etc.); its main purpose is to learn an effective prediction function to describe the user-item interaction record (such as the user-item rating matrix). Due to the problems of data sparsity, cold start, and system performance, these problems may have a negative impact on the recommendation results. To solve these problems, researchers integrate the social attribute information of users or items (such as social labels [3], interaction information between users [4], user's individual click behavior [5], and trust relationships between users [6]) as important influencing factors into the traditional recommendation framework and thus propose a social recommendation method [7]. The existing research based on social networks is based on two aspects: social network homogeneity or heterogeneity. It has achieved performance improvement to a certain extent, but it still faces the following problems: how to extensively exploit and comprehensively explore the homogeneous and heterogeneous information in social networks for recommendations.

How to overcome these challenges to further improve recommendation performance has become one of the hot topics in recent studies [8-13]. Although recommender systems have been comprehensively analyzed in the past decade, the study of social-based recommender systems has just started. Aiming at providing a general method for improving recommender systems by incorporating social network information, Ref. [1] proposes a MF framework with social regularization. In view of the exponential growth of information generated by online social networks, social network analysis is becoming important for many Web applications. Following the intuition that a person's social network will affect personal behaviors on the Web, Ref. [8] proposes a factor analysis approach based on probabilistic MF to solve the data sparsity and poor prediction accuracy problems by employing both users' social network information and rating records. In Ref. [6], a social recommendation model, SocialMF, is constructed by using matrix decomposition technology, which integrates trust propagation on the basis of the matrix decomposition model. Although these existing studies have achieved good results, most of them utilize a single type of external relationship, and the deep structure hidden in social networks and rating patterns has not been fully explored.

To mine deeper structural information in social networks, as a promising direction, a heterogeneous information network (HIN), consisting of multiple types of nodes and links, has been

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proposed as a powerful information modeling method [10]. The major challenge of utilizing all different types of information in a HIN is to combine self-rating data with various external information or relationships and learn the importance of each factor. Ref. [11] proposes an MF-based recommendation framework to combine user ratings and various entity similarity matrices defined by meta-path-based similarity functions using the information network. In many scenarios, the entity recommendation problem exists in a HIN environment. Different types of relationships can potentially be used to improve the recommendation quality. In Ref. [12], the author studies the entity recommendation problem in HINs. Specifically, the author proposes combining heterogeneous relationship information for each user differently and aims to provide high-quality personalized recommendation results using user implicit feedback data and personalized recommendation models. To take full advantage of the relationship heterogeneity in information networks, the author first introduces meta-path-based latent features to represent the connectivity between users and items along different types of paths. Social recommendation methods tend to leverage social relations among users obtained from social networks to alleviate data sparsity and cold start problems in recommender systems. It employs simple similarity information of users as social regularization on users, that is to say, it only puts constraints on users without considering the impact of items for recommendation, and the similarity information of users only stems from social relations of related users; it may not work well for dissimilar users. To overcome the shortcomings of social regularization, Ref. [13] designs a novel dual similarity regularization to impose constraints on users and items with high and low similarities simultaneously. With dual similarity regularization, the author further proposes an optimization function to integrate the similarity information of users and items under different semantic metapaths.

Although HIN-based methods have achieved performance improvement to some extent, existing methods mainly learn a linear weighting mechanism to combine path-based similarities or latent factors without considering the implicit direct relationship between homogeneous information. To address these issues, we propose a unified H2Rec model to fuse homogeneous and heterogeneous information for recommendations in social networks. Considering the rich semantics reflected by metapaths in heterogeneous information and the wealth of social information reflected by homogeneous information, the proposed method uses a random walk strategy to generate node sequences in a homogeneous information network and a random walk strategy guided by metapaths to generate node sequences in a HIN. For each metapath, we learn a unique embedding representation for a node by maximizing its cooccurrence probability with neighboring nodes in the sequences sampled according to the given metapath. Finally, we combine the two parts into a unified model for social recommendation. Extensive experiments on three real-world datasets demonstrate the effectiveness of the proposed model. Then, we take multiple embedding fusion as the output of network information embedding. Finally, we extend the classic MF framework by incorporating fused network information embedding. The prediction model and the fusion function are jointly optimized for the rating prediction task.

By integrating the above two parts together, this work presents a novel network information embedding-based recommendation approach, called H2Rec. We present an overall illustration of the proposed approach in Figure 1. Extensive experiments on three real-world datasets demonstrate the effectiveness of the proposed approach. We also verify the ability of H2Rec to alleviate the cold start problem. The key contributions of this paper can be summarized as follows.

- (i) For the first time, homogeneous and heterogeneous information networks are considered at the same time, and the deeper relationships in network information are extensively and comprehensively mined.
- (ii) We propose a novel network information embedding for recommendation models, called H2Rec. H2Rec can effectively integrate various kinds of embedding information in social networks to enhance recommendation performance.
- (iii) Extensive experiments on three real-world datasets demonstrate the effectiveness of the proposed model. Moreover, we reveal that the transformed embedding information from homogeneous and heterogeneous information networks can improve the recommendation performance.

2. RELATED WORK

Recommender systems belongs to a subclass of information system, and its popularity has increased rapidly in recent years. Although they were initially conceived to cover the field of e-commerce, today they have successfully expanded to various scenarios, such as e-government, e-business, e-commerce/e-shopping, e-library, e-learning, e-tourism, e-resource services, and e-group activities. The aim of recommendation system is to provide personalized recommendation for online products and services to overcome the increasingly serious problem of information overload [14]. Traditional recommendation systems mainly include content-based recommendation, collaborative filtering (CF) recommendation and hybrid recommendation [15]. Furthermore, recent surveys like the one conducted by Ref. [16] have enlightened us on the use of fuzzy techniques for supporting recommender systems.

In the literature on recommender systems, early works mainly adopt CF methods to utilize historical interactions for recommendations [17]. Particularly, the MF approach [2,18] has shown its effectiveness and efficiency in many applications, which factorizes the user-item rating matrix into two low-rank user-specific and item-specific matrices and then utilizes the factorized matrices to make further predictions. In related research on recommendation systems, the early recommendation method is mainly CF [17], which uses historical interactions for recommendations. CF technology can be divided into memory-based CF and model-based CF according to whether machine learning is adopted. In model-based CF technology, MF [2] is the most popular because of its excellent scalability and easy implementation. It shows its effectiveness and efficiency in many applications by decomposing the user-item rating matrix into two small matrices containing hidden variables, one representing the user's implicit characteristics and the other representing the implicit features of the item, and then utilizes the factorized matrices to make further predictions. Since CF methods usually suffer from cold start problems, many works attempt to

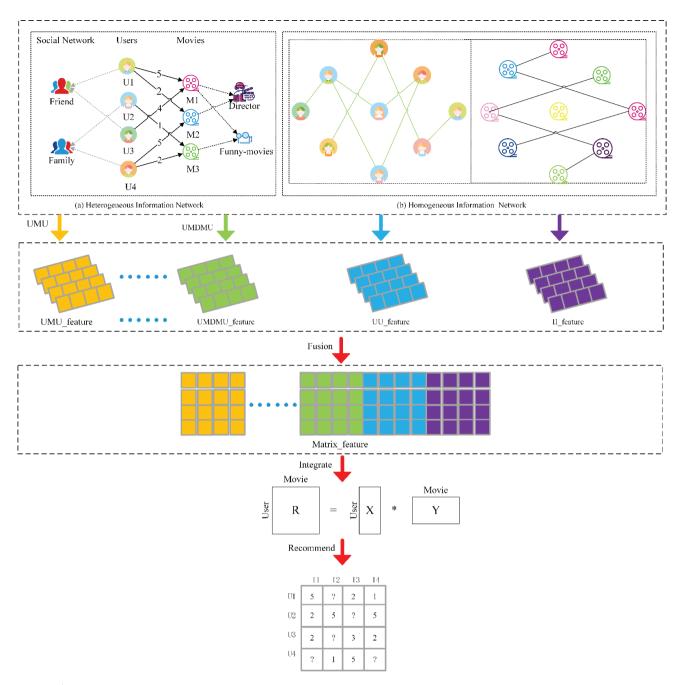


Figure 1 Schematic illustration of the proposed H2Rec approach.

leverage additional information to improve recommendation performance. Ref. [18] considers the information of both ratings and reviews and proposes a unified model to combine content-based filtering with CF for rating prediction tasks. Ref. [6] added a trust propagation mechanism to MF and proposed a recommendation matrix decomposition technology based on trust propagation. An increasing number of studies have begun to use friend relations among users. In Ref. [1], the additional social regularization term ensures that the distance of latent feature vectors of two friends with similar tastes becomes closer. Membership among users is also explored to boost CF [19].

On the other hand, the deep learning-based embedding technique has shown its power in many recommendation tasks with its capability of extracting hierarchical representations from raw data. Ref. [20] showed how to train distributed representations of words and phrases with the skip-gram model. Inspired by the success of word embedding models, Ref. [21] learned item embeddings using the sets of items each user had consumed and further proposed a cofactorization model to jointly decompose the user-item interaction matrix and the itemitem cooccurrence matrix with shared item latent factors. As embedding methods have the capability of extracting hierarchical representations from raw data, many researchers have also tried to incorporate these extracted factors into recommender systems. For example, Ref. [22] presented a novel perspective to address the recommendation task by utilizing network representation learning techniques, which first transforms the adoption records into a k-partite adoption network and then

applies the network embedding approach to learn vertex embeddings. Embeddings for different kinds of information are projected into the same latent space, where we can easily measure the relatedness between multiple vertices on the graph using some similarity measurements. In this way, the recommendation task has been cast into a similarity evaluation process using embedding vectors. To leverage the knowledge extracted from social networking sites for cross-site cold-start product recommendation, Ref. [23] learned both user and item feature representations from e-commerce websites using recurrent neural networks and then developed a featurebased MF approach to leverage the learned user embeddings for cold-start product recommendation. Most existing relation-based CF methods usually utilize information from single or multiple homogeneous networks that only have a single type of node and link. However, the widely available attribute information of users and items is useful for recommendations, which is seldom exploited in this kind of method.

To mine deeper structural information in social networks, as a promising direction, a HIN, consisting of multiple types of nodes and links, has been proposed as a powerful information modeing method. As a unique characteristic of HINs, metapaths not only represent rich semantics but can also be utilized to integrate different types of information in HINs. It is a challenge for the HIN to know how to deeply mine the semantic meanings of the information network. For meta-path-based data mining, researchers have performed much work, including clustering [24], classification [25], and link prediction [26]. Among these research works, the similarity measure in HIN is an important and basic function, so many methods based on path similarity measurement have been proposed. For example, the concept of metapath similarity is introduced in Ref. [27]; it defines a novel similarity measure called Path-Sim that is able to find peer objects in the network (e.g., find authors in a similar field and with similar reputations), which turns out to be more meaningful in many scenarios than random-walk-based similarity measures. Based on the idea that two objects are related if they are referenced by related objects, Ref. [28] proposes a general framework, called HeteSim, to evaluate the relatedness of heterogeneous objects in heterogeneous networks. HeteSim is a path-based relevance measure that can effectively capture the subtle semantics of search paths. Based on the pairwise random walk model, HeteSim treats arbitrary search paths in a uniform way, which guarantees the symmetric property of HeteSim.

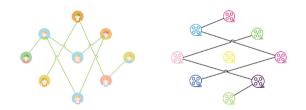
Network embedding has shown potential in structural feature extraction and has been successfully applied to many data mining tasks. DeepWalk [29] is a relatively basic algorithm for network representation learning. It combines random walk and skip-gram to learn the vector representation of nodes in the network (w.r.t the structural feature of the learning graph w.r.t the attribute, and the number of attributes is the dimension of the vector) so that the traditional machine learning algorithm can be used to solve related problems. To embed HINs, Ref. [30] designed a meta-path-based random walk strategy to generate meaningful node sequences for network embedding. Ref. [31] presents a novel idea on network representation learning, termed heterogeneous network embedding (HNE), which jointly considers both the content and relational information. HNE maps different heterogeneous objects into a unified latent space so that objects from different spaces can be directly compared. Ref. [32] proposes a new representation learning framework HIN to vector (hin2vec). The core of this framework is a neural network model that aims to capture rich semantics embedded in HIN by using different types of relationships between nodes. Although these methods can learn network embedding in various heterogeneous networks, the traditional HIN-based recommendation does not consider the direct relationship between nodes implied in homogeneous information networks, which may be very important in some applications. This paper proposes a method based on the integration of homogeneous and heterogeneous networks and discusses its benefits to the recommendation system.

3. PRELIMINARY

The recommendation scenario in social networks is shown in Figure 2, which consists of two central elements: the network diagram of the social relationship between users and the network diagram of the relationship between items.

Theorem 1. (Social Network) [33]: The social network is denoted as $\mathscr{G}_{uu} = (\mathscr{U}, \mathscr{E}_{uu})$, which describes the social relationship between users. \mathscr{U} is the user set and \mathscr{E}_{uu} is the edge set. For any two users $u_i \in \mathscr{U}$ and $u_j \in \mathscr{U}$, if user u_i has a social relationship with user u_j (such as a trust relationship or a friend relationship), there will be an edge $e_{ii} \in \varepsilon_{uu}$ from u_i to u_i ; otherwise, none.

Theorem 2. (Item Relationship Network): The item relationship network is denoted as $\mathscr{G}_{ii} = (\mathscr{I}, \mathscr{C}_{ii})$, which describes the relationship between items. \mathscr{I} is the item set and ε_{ii} is the edge set. For any two items $i_m \in \mathscr{I}$ and $i_n \in \mathscr{I}$, if item i_m is related to item i_n (such as



(a)One typical example of the social network (b)Example of item relation network

Figure 2 Example of the recommendation scenario in a social network.

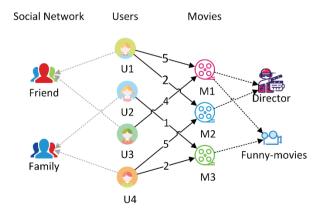


Figure 3 Heterogeneous information network.

having the same attribute relationship), there will be an edge $e_{mn} \in \epsilon_{ii}$ from i_m to i_n ; otherwise, none.

HIN is a special information network, as shown in Figure 3. It contains multiple types of nodes and multiple types of links.

Theorem 3. (HIN) [34]: The HIN is denoted as $\mathcal{G}_{HINs} = (\mathcal{V}, \mathcal{E})$, which consists of an object set \mathcal{V} and an edge set \mathcal{E} . A HIN is also associated with an object type mapping function $\emptyset : \mathcal{V} \to \mathcal{A}$ and a link type mapping function $\psi : \mathcal{E} \to \mathcal{R}$. A and R denote the sets of predefined object and link types, where $|\mathcal{A}| + |\mathcal{R}| > 2$.

Theorem 4. (Network Schema) [8,35]: The network schema is denoted as $\mathcal{S} = (\mathcal{A}, \mathcal{R})$, which is a metatemplate of information network $\mathcal{G}_{HINs} = (\mathcal{V}, \mathcal{E})$, and its object type mapping $\emptyset : \mathcal{V} \to \mathcal{A}$. The link type is mapped to $\psi : \mathcal{E} \to \mathcal{R}$, which is a directed graph defined on the object type \mathcal{A} and the edge relationship \mathcal{R} .

As shown in Figure 4, we show the network schema on the three datasets used in our experiments.

Theorem 5. (Metapath) [27]: Metapath ρ is defined on the network schema $\mathscr{S} = (\mathscr{A}, \mathscr{R})$ and is denoted as a path in the form of a $A_1 \xrightarrow{R_1} A_2 \xrightarrow{R_2} \cdots \xrightarrow{R_l} A_{l+1}$ path. Among them, $A_1 \xrightarrow{R_1} A_2 \xrightarrow{R_2} \cdots \xrightarrow{R_l} A_{l+1}$ describes a composite relationship $R = R_1 \circ R_2 \circ \cdots \circ R_l$ from A_1 to A_{l+1} , where \circ represents the combination operator on the relation.

The HIN contains many types of nodes and links. The meaning of the connection of two nodes of the same type can be changed due to the different connection paths. The metapath is composed of a sequence of relationships of different objects, which defines the complex relationship from the first node to the last node in the sequence path. For example, the path User-Movie-User (UMU) represents the user watching the same movie as the target user, while the path User-Movie-Type-Movie-User (UMTMU) represents the user watching the same type of movie as the target user. In Table 1, we give the definition and semantic description of metapaths under the three data types used in this paper.

4. COMBINED WITH NETWORK EMBEDDING FACTORS

Learning useful representations from highly structured objects is useful for various machine learning applications. In addition to reducing engineering workload, these representations can also lead to greater predictive power. Inspired by the recent progress in network embedding research, we use representation learning methods to extract and represent useful information in the information network for recommendation. Ref. [36] proposed an algorithm framework, node2vec, for graph representation learning. For any given graph, the framework can learn the continuous feature representation of nodes, which can then be used for various downstream machine learning tasks.

4.1. Node2vec Algorithm

4.1.1. Random walk

To generate meaningful node sequences, the key is to design an effective random walk strategy that can capture the complex semantics reflected in the network. Formally, given a source node u, node2vec simulates a random walk with a fixed length l. Let c_i denote the *i*-th node in the random walk, starting from $c_0 = u$. c_i Node is generated by the following distribution function:

$$P\left(c_{i}=x|c_{i-1}=v\right) = \begin{cases} \frac{\pi_{vx}}{Z} & \text{if } (v,x) \in E\\ 0 & \text{otherwise} \end{cases}$$
(1)

where π_{vx} is the unnormalized transition probability between nodes v and x, and Z is the normalizing constant.

4.1.2. Optimization objective

Let f(u) be the mapping function that maps node u to the embedding vector. For each node u in the graph, $\mathcal{N}_S(u)$ is defined as the set of neighboring vertices of node u sampled by sampling strategy S. The goal of node2vec optimization is to maximize the probability of neighboring vertices given each node:

$$max_{f} \sum_{u \in V} \log Pr\left(\mathcal{N}_{S}(u) | f(u)\right)$$
(2)

where $f : \mathcal{V} \to \mathbb{R}^d$ represents a function that maps each node to a *d*-dimensional feature space, and $\mathcal{N}_S(u) \in \mathcal{V}$ represents the neighborhood of node *u*. We can learn the embedding mapping function $f(\cdot)$ by applying stochastic gradient descent (SGD) to optimize this objective.

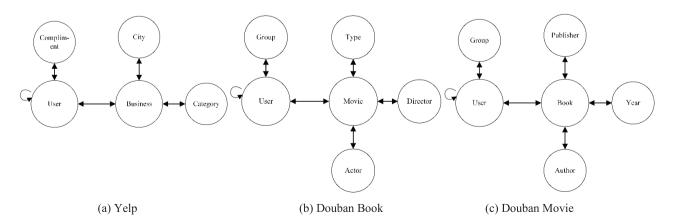


Figure 4 Network schemas of heterogeneous information networks for the three datasets.

	(a) Yelp						
Meta-paths	Semantic Meaning						
UBU	users who review the same business with the target user						
UBCiBU	users who review businesses in the same city with the target user						
UBCaBU	users who review the same type of business with the target user						
BUB	businesses reviewed by the same user with the target business						
BCiB	businesses in the same city with the target business						
BCaB	businesses of the same type with the target business						
	(b) Douban Movie						
Meta-paths	Semantic Meaning						
UU	friends of the target user						
UMU	users who view the same movies with the target user						
UMTMU	users who view the movies having the same types with that of the target user						
UMDMU	Users who watch movies directed by the same director with the target user						
MUM	movies watched by the same user with the target movie						
MDM	movies directed by the same director with the target movie						
MAM	movies played by the same actor with the target movie						
MTM	movies of the same type with the target movie						
	(c) Douban Book						
Meta-paths	Semantic Meaning						
UBU	Users who read the same book as the target user						
UBPBU	Users who read books published by the same publisher as the target user						
UBYBU	Users who read books published in the same year as the target user						
UBABU	Users who read books written by the same author as the target user						
BUB	Books read by the same user as the target book						
BPB	Books published by the same publisher as the target book						
BYB	Books published in the same year as the target book						

 Table 1
 The semantic meanings of different meta-paths for the used three datasets.

4.2. Embedding of Homogeneous Information Networks

4.2.1. Generate node sequence based on random walk

First, we propose a basic recommendation model based on homogeneous networks, which is based on the embedding of connected users (items). Since the output of node2vec can be interpreted as a high-level representation of network nodes, we pretrain node2vec to mine the deep social structure of a given social network \mathscr{G}_{uu} . Because users in social networks usually express their social interests by making different friends, a better understanding of these social networks may help recommendations. Given a social network (item network), we can obtain the neighborhood of node *u* in a fixed-length window, and let $X_{\mathscr{U}} \in \mathbb{R}^d$ denote the neighborhood of user *u* extracted from the given social network.

4.2.2. Node embedding representation

According to node2vec, we can learn the representation of nodes by optimizing Eq. (2). Let em_u^{SN} represent the user node embedding learned in the social network and em_v^{IN} represent the item node embedding learned in the item network.

4.3. Embedding of HINs

4.3.1. Meta-path-based random walk

To generate meaningful node sequences, the key is to design an effective walking strategy that can capture the complex semantics reflected in HINs. Therefore, we adopt a random walk method based on metapaths to generate node sequences. Given a HIN $\mathscr{G}_{HIN} = (\mathscr{V}, \mathscr{E})$ and a metapath $\rho : A_1 \xrightarrow{R_1} A_2 \xrightarrow{R_2} \cdots \xrightarrow{R_l} A_{l+1}$, the walk path is generated according to the following distribution:

$$P\left(n_{t+1} = x | n_t = v, \rho\right)$$

$$= \begin{cases} \frac{1}{|\mathcal{N}^{A_{t+1}}(v)|}, (v, x) \in \mathcal{E} \text{ and } \phi(x) = A_{t+1} \\ 0, \text{ otherwise} \end{cases}$$
(3)

where n_t is the *t*-th node in the walk, v has the type of A_t , and $\mathcal{N}^{A_{t+1}}(v)$ is the first-order neighbor set for node v with the type of A_{t+1} . A walk will follow the pattern of a metapath repetitively until it reaches the predefined length.

4.3.2. Remove noise nodes

Since we are concerned about improving recommendation performance, we only need to focus on learning the effectiveness of users

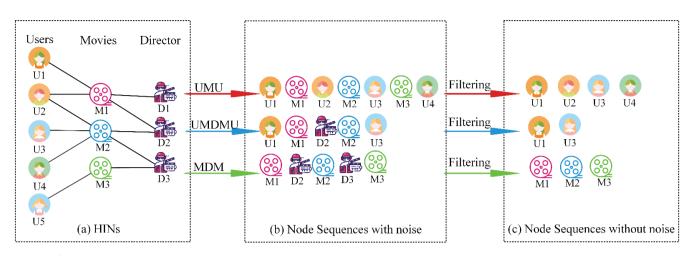


Figure 5 The process of removing noise nodes.

and the items themselves. For other types of nodes, we do not pay much attention to the improvement of our recommendation performance. Therefore, we need to denoise after forming the metapath. In this way, the final sequence we obtain is only composed of user node types or item node types, that is, a sequence composed of homogeneous nodes. The connection between homogeneous nodes is essentially constructed through heterogeneous neighboring nodes. As shown in Figure 5, we show the specific denoising process. To learn the effective representation of users and items, only metapaths whose starting type is user type or item type are considered. In this way, some metapaths can be derived, such as UMU, UMDMU, and MUM; taking the metapath of UMU as an example, a sampling sequence can be generated $u_1 \rightarrow m_1 \rightarrow u_2 \rightarrow m_2 \rightarrow$ $u_3 \rightarrow m_2 \rightarrow u_4$, and the deletion is different from the starting node type node. In this way, we finally obtain a sequence of homogeneous nodes $u_1 \rightarrow u_2 \rightarrow u_3 \rightarrow u_4$.

4.3.3. Node embedding representation

Given the metapath, we can construct the domain node set \mathcal{N}_u of node u in a fixed-length window. Through node2vec, we can learn the representation of nodes with the optimization of Eq. (2). For our HIN model, given a node $v \in \mathcal{V}$, we can obtain a node embedding representation set $\left\{ em_v^{(l)} \right\}_{l=1}^{|\mathcal{P}|}$, where \mathcal{P} represents the element path set and $em_v^{(l)}$ represents the embedding representation of node v.

4.4. Fusion of Homogeneous and Heterogeneous Network Embedding

After learning about the node representations of homogeneous networks and heterogeneous networks, the next step is to fuse these learned node representations. In our model, we designed a function $h(\cdot)$ to fuse embedded information based on a sigmoid function:

$$em_{\nu}^{(V)} \leftarrow h\left(em_{\nu}^{(l+1)}\right)$$

$$= \delta\left(\sum_{l=1}^{\mathscr{P}+1} w_{\nu}^{(l+1)} \delta\left(M^{(l+1)}em_{\nu}^{l+1} + b^{(l+1)}\right)\right)$$
(4)

where $em_v^{(l)}$ represents the embedding representation of node vunder the *l* th metapath in the HIN and $em_v^{(1)}$ represents the embedding representation of node v under the homogeneous information network. $em_v^{(V)}$ represents the embedding representation of the fusion node learned in the information network, which we call the information network embedding; $\delta(\cdot)$ is a nonlinear function, that is, a sigmoid function. $M^{(l+1)}$ represents the transformation matrix and $b^{(l+1)}$ represents the bias vector.

5. THE PROPOSED APPROACH

5.1. Matrix Factorization

CF is one of the most popular and classic technologies in recommender systems. It is based on the assumption that if users have been interested in some items in the past, they may still be enthusiastic about them in the future. CF technology can be divided into memory-based CF and model-based CF according to whether machine learning is adopted. Among the model-based CF technologies, MF [2] is the most common and popular technology. Because of its excellent scalability and easy implementation, to learn potential factors from user rating data, we introduce MF as our basic recommendation framework.

The MF model maps users and items to a joint latent factor space of dimension f so that user-item interactions are modeled as inner products in this space. Correspondingly, for each item *i*, there is a vector $Q_i \in \mathbb{R}^f$ associated with it; for each user *u*, there is a vector $P_u \in \mathbb{R}^f$ associated with it. For a given item *i*, the elements of Q_i measure the extent to which the item possesses those factors, positive or negative. For a given user *u*, the elements of P_u measure the extent of interest the user has in items that are high on the corresponding factors, again, positive or negative. The obtained dot product result $Q_i^T P_u$ captures the interaction between user *u* and item *i*, that is, the user's overall interest in product features. This approximates user *u*'s rating of item *i*, which is denoted by $\hat{R}_{u,i}$, leading to the estimate

$$\hat{R}_{u,i} = Q_i^T P_u \tag{5}$$

where $P_u \epsilon \mathbb{R}^f$ and $Q_i \epsilon \mathbb{R}^f$ represent the potential factors of user *u* and item *i*, respectively, and its objective optimization function is Eq. (6).

$$\mathscr{L} = \sum_{\left\langle u, i, R_{u,i} \right\rangle} \left(R_{u,i} - \hat{R}_{u,i} \right)^2 + \lambda \sum_{u} \left(\|P_u\|_2 + \|Q_i\|_2 \right) \tag{6}$$

5.2. H2Rec Model

After we learned the node embedding representation of the information network, according to the node embedding, we can further obtain the user's embedding representation $\left\{ Em_{u}^{(U)} \right\}_{u \in \mathcal{U}}$ and the item's embedding representation $\left\{ Em_{i}^{(I)} \right\}_{i \in I}$.

5.2.1. Integrating MF with network embedding for recommendation

MF decomposes the user-item rating matrix into two small matrices containing hidden variables, one representing the hidden features of the user and the other representing the hidden features of the item, and then uses the decomposed matrix to make further predictions. In MF, the score prediction of user u for item i is defined in Eq. (5). Combined with the embedded representation of the learned user u and item i, the score prediction function is as follows:

$$\hat{R}_{u,i} = Q_u^T P_i + \alpha E m_u^{(U)^T} \gamma_i^{(I)} + \beta \gamma_u^{(U)^T} E m_i^{(I)}$$
(7)

where $E_u^{(U)}$ and $E_i^{(I)}$ are the fused embeddings, $\gamma_u^{(U)}$ and $\gamma_i^{(I)}$ are userspecific and item-specific latent factors to pair with the network embeddings $E_u^{(U)}$ and $E_i^{(I)}$ respectively, and α and β are the tuning parameters to integrate the three terms.

We blend the fusion function into a MF framework for learning the parameters of the proposed model. The objective can be formulated as follows:

$$\mathcal{L} = \sum_{\substack{\langle u, i, R_{u,i} \rangle \\ +\lambda \sum_{u} \left(\|Q_{u}\|_{2} + \|P_{i}\|_{2} + \|\gamma_{u}^{(U)}\|_{2} + \|\gamma_{i}^{(I)}\|_{2} + \|\rho^{(U)}\|_{2} + \|\rho^{(I)}\|_{2} \right)}$$
(8)

where $\hat{R}_{u,i}$ represents the predicted score, λ is the regularization parameter, and $\rho^{(U)}$ and $\rho^{(l)}$ are the parameters of the function $h(\cdot)$ for users and items, respectively. We use the SGD method to optimize the final target \mathcal{D} . Q_u and P_i are decomposed by Eq. (1), and the other parameters in the model will be updated by the following formula:

$$\gamma_{u}^{(U)} \leftarrow \gamma_{u}^{(U)} - \eta \cdot \left(-\beta \left(R_{u,i} - \hat{R}_{u,i} \right) Em_{i}^{(I)} + \lambda_{\gamma} \gamma_{u}^{(U)} \right)$$
(9)

$$\gamma_i^{(I)} \leftarrow \gamma_i^{(I)} - \eta \cdot \left(-\alpha \left(R_{u,i} - \hat{R}_{u,i} \right) Em_u^{(U)} + \lambda_\gamma \gamma_i^{(I)} \right)$$
(10)

$$\mu_{u,l}^{(U)} \leftarrow \mu_{u,l}^{(U)} - \eta \cdot \left(-\alpha \left(R_{u,i} - \hat{R}_{u,i} \right) \gamma_i^{(I)} \frac{\partial E m_u^{(U)}}{\partial \rho_{u,l}^{(U)}} + \lambda_\rho \rho_{u,l}^{(U)} \right)$$
(11)

$$\rho_{i,l}^{(I)} \leftarrow \rho_{i,l}^{(I)} - \eta \cdot \left(-\beta \left(R_{u,i} - \hat{R}_{u,i} \right) \gamma_u^{(U)} \frac{\partial E m_i^{(I)}}{\partial \rho_{i,l}^{(I)}} + \lambda_\rho \rho_{i,l}^{(I)} \right) \quad (12)$$

where η is the learning rate, λ_{γ} is the regularization for parameters $\gamma_{u}^{(U)}$ and $\gamma_{i}^{(I)}$, and λ_{ρ} is the regularization for parameters $\rho^{(U)}$ and $\rho^{(I)}$.

6. EXPERIMENT AND RESULTS

In this chapter, we use three real datasets (Yelp,¹ Douban Movies² and Douban Book³) to experiment with our H2Rec model to verify the effectiveness of the model.

6.1. Datasets

To verify that our model can demonstrate its effectiveness in different fields, we used three datasets from different fields, including the Yelp dataset from the business field, the Douban book dataset from the book field, and the Douban movie ratings dataset from the movie field. The Yelp dataset records the user ratings of local businesses and contains the social relationship and attribute information of the businesses, including 16,239 users and 14,282 local businesses, and 198,397 rating datapoints from 1 to 5; the Douban Books dataset, which includes 13,024 users and 22,347 books and 792,026 ratings datapoints from 1 to 5; and the Douban movie dataset includes 13,367 users and 12,677 movies and 1,068,278 movie ratings datapoints from 1 to 5. In addition, in Table 2, we provide a detailed description of the three datasets in the experiment.

6.2. Evaluation Metrics

In the recommendation system, there are three main experimental methods to obtain different indicators, namely, offline experiments, user studies, and online experiments. In the application, the recommendation system can be divided into top-N recommendations and scoring recommendations according to the scene. The scoring recommendation is generally calculated by root mean square error (RMSE) and absolute average (MAE). Among them, RMSE has increased the penalties for inaccurate items, and the evaluation has become more stringent. In this experiment, we mainly use RMSE and MAE to evaluate the recommended performance of different models. The metrics MAE and RMSE are defined as follows:

$$MAE = \frac{\sum_{u,i\in T} |R_{u,i} - \hat{R}_{u,i}|}{|T|}$$
(13)

RMSE =
$$\sqrt{\frac{\sum_{u,i\in T} (R_{u,i} - \hat{R}_{u,i})^2}{|T|}}$$
 (14)

where $R_{u,i}$ represents the actual score of user u on item i in the test set; $\hat{R}_{u,i}$ represents the predictive score of user u on item i by the recommendation system; and T is the score set.

¹http://www.yelp.com/dataset-challenge

²http://movie.douban.com

³http://book.douban.com

6.3. Models for Comparison

We consider the following methods to compare:

- PMF [33]: This algorithm is a CF recommendation algorithm based on MF. It explicitly decomposes the rating matrix into two low-dimensional matrices.
- (ii) SoMF [1]: This method aims to provide a general approach for improving the recommendation system by combining

 Table 2
 Statistics of the three datasets of our experiment.

social network information and proposes a MF framework with social regularization.

- (iii) HeteMF [34]: This method proposes a recommendation framework based on MF to combine user ratings and use information networks to define various entity similarity matrices based on meta-path-based similarity functions.
- (iv) SemRec [35]: This is a CF method on a weighted HIN, which is constructed by connecting users with items of

Dataset	Density (%)	Relations (A-B)	Number of A	Number of B	Number of (A-B)	
		User-Business	16239	14284	198397	
Yelp		User-User	10580	10580	158590	
	0.08	User-Compliment	14411	11	76875	
		Business-City	14267	47	14267	
		Business-Category	14180	511	40009	
Douban Book		User-Book	13024	22347	792026	
	0.27	User-User	12748	12748	169150	
		Book-Author	21907	10805	21905	
		Book-Publisher	21773	1815	21773	
		Book-Year	21192	64	21192	
		User-Movie	13367	12677	1068278	
Douban Movie	0.63	User-User	2440	2294	4085	
		User-Group	13337	2753	570047	
		Movie-Director	10179	2449	11276	
		Movie-Actor	11718	6311	33587	
		Movie-Type	12678	38	27668	

Table 3The experiment data of MAE and RMSE on Yelp, Douban Movie, and Douban Book. A smaller MAE or RMSE value indicates a
better performance.

Dataset	Training	Metrics	PMF	SoMF	HeteMF	SemRec	DSR	H2Rec _{HIN}	H2Rec _{HON}	H2Rec
Yelp	90%	MAE	1.0412	1.0095	0.9487	0.9043	0.9054	0.8304	0.8431	0.8213
		RMSE	1.4268	1.3392	1.2549	1.1637	1.1186	1.0861	1.102	1.0556
	80%	MAE	1.0791	1.0373	0.9654	0.9176	0.9098	0.849	0.8418	0.8265
		RMSE	1.4816	1.3782	1.2799	1.1771	1.1208	1.1109	1.1053	1.0642
	70%	MAE	1.1170	1.0694	0.9975	0.9407	0.9429	0.8576	0.859	0.8416
	7070	RMSE	1.5387	1.4201	1.3229	1.2108	1.1582	1.124	1.1302	1.0836
	60%	MAE	1.1778	1.1135	1.0368	0.9637	1.0043	0.8741	0.8687	0.8523
		RMSE	1.6167	1.4748	1.3713	1.238	1.2257	1.1498	1.1405	1.0951
	80%	MAE	0.5741	0.5817	0.5750	0.5695	0.5681	0.5611	0.5616	0.5511
	80%	RMSE	0.7641	0.7680	0.7556	0.7399	0.7225	0.7158	0.7164	0.6889
Douban Movie	60%	MAE	0.5867	0.5991	0.5894	0.5738	0.5831	0.5658	0.566	0.5549
	00%	RMSE	0.7891	0.7950	0.7785	0.7551	0.7408	0.7222	0.7224	0.6935
	400/	MAE	0.6078	0.6328	0.6165	0.5945	0.6170	0.5706	0.5709	0.559
	40%	RMSE	0.8321	0.8479	0.8221	0.7836	0.7850	0.7303	0.7306	0.6995
	20%	MAE	0.7247	0.6979	0.6896	0.6392	0.6584	0.5899	0.5899	0.5737
		RMSE	0.9440	0.9852	0.9357	0.8599	0.8345	0.7626	0.7627	0.7198
Douban Book	80%	MAE	0.5774	0.5756	0.5740	0.5675	0.5740	0.554	0.554	0.5459
		RMSE	0.7414	0.7302	0.7360	0.7283	0.7206	0.702	0.702	0.6796
	60%	MAE	0.6065	0.5903	0.5823	0.5833	0.6020	0.5614	0.5615	0.5528
		RMSE	0.7908	0.7518	0.7466	0.7505	0.7552	0.7117	0.7115	0.6883
	40%	MAE	0.6800	0.6161	0.5982	0.6025	0.6271	0.5768	0.5767	0.565
		RMSE	0.9203	0.7936	0.7779	0.7751	0.7730	0.7361	0.7357	0.7049
	20%	MAE	1.0344	0.6327	0.6311	0.6481	0.6300	0.6381	0.6384	0.6115
		RMSE	1.4414	0.8236	0.8304	0.8350	0.8200	0.8348	0.8358	0.7677

the same level. It can not only flexibly integrate heterogeneous information but also obtain priority and personalized weights that represent user preferences on the path.

- (v) DSR [13]: This method designs a novel dual similarity regularization to impose constraints on users and items with high and low similarities simultaneously.
- (vi) H2Rec_{HIN}: This is a variant of H2Rec that only integrates HNE.
- (vii) H2Rec_{HON}: This is a variant of H2Rec that only integrates homogeneous network embedding.
- (viii) H2Rec: This is the proposed recommendation model based on the embedding fusion of homogeneous and heterogeneous information networks.

6.4. Results

First, we divide the three datasets into a training set and a test set and set the five training ratios of the datasets Douban Movie and Douban Book to {20%, 40%, 60%, 80%}. Since the Yelp dataset is very sparse, its five training ratios are set to {60%, 70%, 80%, 90%}. For each ratio, we randomly generate ten evaluation sets. We average the results as the final performance. The experimental results are shown in Table 3. For conducting a more intuitive analysis, the line chart (see Figure 6) was drawn based on the data from Table 3.

We set parameters for H2Rec as follows: the embedding dimension number d = 64 (We divide d into {16; 32; 64; 128; 256}, 64 is the optimal dimension) and the tuning coefficients $\alpha = \beta =$ 1.0. Following [29], the path length for random walk is set to 40.

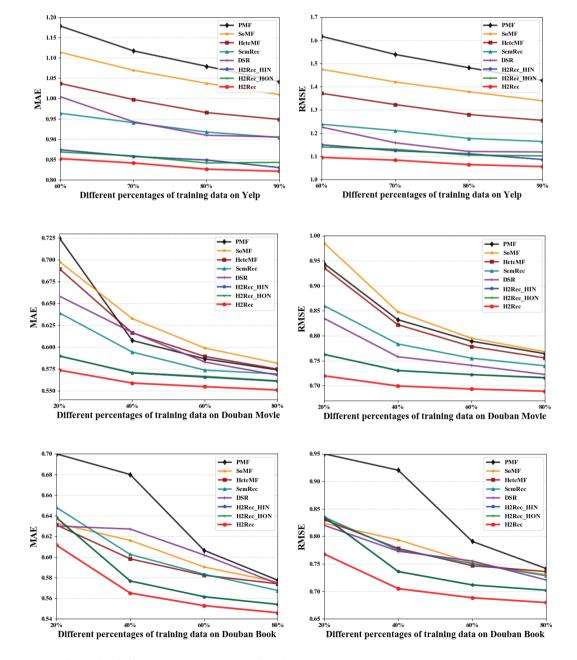


Figure 6 Results of effectiveness experiments on three datasets.

Following [37,38], we set the number of latent factors for all the MF-based methods to 10. Other baseline parameters either adopt the original optimal setting or are optimized by using a validation set of 10% training data. For H2Rec_{HIN} and H2Rec_{HON}, we set the embedding dimension number d = 64 for fairness.

In this study, PMF is regarded as the baseline model due to its being one of the most classic algorithms in the recommendation field. On the whole, compared with the most basic MF model, our model shows an improvement of more than 20%, which proves the effectiveness of adding information network embedding. Compared with other models such as SoMF and HeteMF that use social information or heterogeneous information, H2Rec performs better with regard to RMSE and MAE indicators, proving that the fusion of homogeneous and heterogeneous information network embedding can effectively mine useful information in the information network. However, we can see that for the two evaluation indicators of MAE and RMSE, the performance of all models in the Yelp dataset is worse than that in the other datasets, which is due to the extremely sparse data in the Yelp dataset. The major findings from the experimental results are summarized as follows.

- (1) Compared with the most basic MF model PMF, which does not add any auxiliary information, all other models that add auxiliary information (such as social information, tag information, attribute information, etc.) show better performance, which fully illustrates the effectiveness of adding auxiliary information.
- (2) Comparing the model based on metapath similarity with H2Rec_{HIN}, which only uses HNE and H2Rec_{HON}, which only uses homogeneous network embedding, we can see that the performance of network embedding guided by metapaths is better than that based on metapath similarity.
- (3) In addition, comparing our H2Rec model with its two variant models H2RecHIN and H2RecHON, we find that H2Rec performs better on both evaluation indicators, which shows that the fusion of heterogeneous and heterogeneous networks can more fully utilize the deep relationships hidden in the information network.

7. CONCLUSION

In this paper, we propose a unified H2Rec model to fuse homogeneous and heterogeneous information for recommendations in social networks to extensively exploit and comprehensively explore the homogeneous and heterogeneous information in social networks for recommendations in social networks. Considering the rich semantics reflected by metapaths in heterogeneous information and the wealth of social information reflected by homogeneous information, the proposed method uses a random walk strategy to generate node sequences in a homogeneous information network and a random walk strategy guided by metapaths to generate node sequences in a HIN. Finally, we integrate MF with network embedding for recommendation. Extensive experiments on three real-world datasets demonstrate the effectiveness of the proposed model. Moreover, we reveal that the transformed embedding information from homogeneous and heterogeneous information networks can improve the recommendation performance.

In future work, we will study how to apply deep learning methods (such as convolution neural networks and auto-encoders) to better integrate homogeneous and heterogeneous network embedding.

CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest.

AUTHORS' CONTRIBUTIONS

Yabin Shao established the research direction and content. Cheng Liu conducted the literature review and wrote the entire manuscript. All authors have read and agreed to the published version of the manuscript.

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