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Input Arrival-Time-Dependent Decoding Scheme for a Spiking Neural Network

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Abstract. Spiking neurons model a type of biological neural system where information is encoded with spike times. In this paper, a new method for decoding input spikes according to their absolute arrival times is proposed. The output times, which are responses to different input patterns, can differentiate these input patterns uniquely. Features of Spiking Neural Networks (SNN) such as actual spike input time and synaptic weights are utilized. Only a limited number of neurons are needed to implement the decoding scheme.

1 Introduction

Spiking neurons model a type of biological neural system where information is encoded with spike times. A neural spike is a discrete event within a continuous time frame with spatio-temporal properties. It has been shown theoretically that spiking neural networks that convey information by individual spike times are computationally more powerful than neural networks with sigmoidal activation function neurons [4]. Neural network architectures based on spiking neurons that encode information in individual spike times have yielded, amongst others, a supervised classifer [1], a self-organizing map [7] and a network for unsupervised clustering [5].

In this paper, a simple input arrival-time-dependent decoding scheme for a spiking neuron with dynamically changeable membrane potential and synapse weights is considered. It is shown that this scheme has a wide decoding range and can be implemented using only a limited number of spiking neurons. This decoding method has some good features including fast and wide-range decoding.

2 Model of a Spiking Neural Network

The spiking neural network model employed is based on the Spike Response Model (SRM) [3]. Input spikes come at times $t_1 ... t_n$ into the input synapses of

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a neuron as shown in Figure 1.

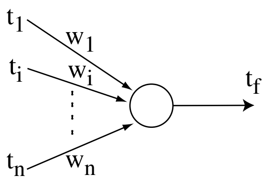

Figure 1: Neuron with excitatory t_i inputs and output t_f.

The neuron generates a spike at time t_f when the internal membrane potential $x_f(t)$ crosses a threshold potential ϑ from below at time $t_f = min\{t : x_f(t) \geq \vartheta\}$. The threshold potential ϑ is assumed to be constant for the neuron. The relationship between input spikes and the internal state variable $x_f(t)$ can be described as follows:

$$x_f(t) = \sum_{i \in \Gamma_f} k_i.w_i.\alpha(t - t_i) \tag{1}$$

k_i is a dynamic variable which represents the peak of the spike response function (Equation 2), w_i is the synaptic weight, t_i is the input spike arrival-time, and $\alpha(t)$ is the Spike Response Function defined as follows:

$$\alpha(t) = \frac{t}{\tau} e^{1-\frac{t}{\tau}} \tag{2}$$

τ represents the membrane potential decay time constant. The height of the Post-Synaptic Potential (PSP) is modulated by parameter k_i and synaptic weight w_i.

3 Input Arrival-Time-Dependent Decoding

The proposed scheme decodes absolute input times. It is composed of three parts as shown in Figure 2(C). The first part (Figure 2(A)) of the model is used for decoding input spike times according to their Inter-Spike Interval (ISI) times, i.e. the times between two consecutive input spikes arrive at two different input synapses. Spike times, which represent a certain input pattern, are decoded into output spike times. The input pattern spikes arrive at times $t_1.....t_n$, with some minimum time resolution Δt, into the input synapses. The ISI1 block (Figure 2(A)) consists of two units: a) a neuron with excitatory inputs; and b) an Excitatory Post Synaptic Potential (EPSP) unit. The EPSP unit updates the dynamic variable k in Equation 1 after every synaptic input according to the following equation:

$$k_i = \frac{\beta}{t_i}, \quad k_1 = 1 \quad , \quad i = 2, 3, ..., n \tag{3}$$

In Equation 3, β is a small constant and i refers to the temporal order of the input spikes, not the spatial number of an input synapse.

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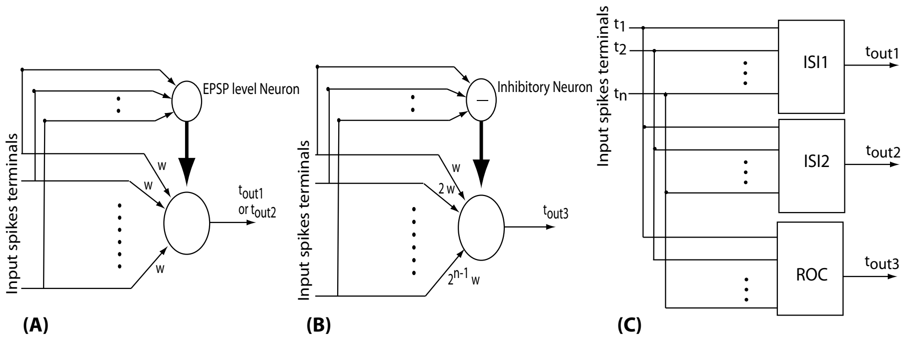

Figure 2: (A) ISI1 or ISI2 blocks (B) Rank Order Coding (ROC) block (C) The combined decoder system.

Equation 3 shows that the value of k_i is inversely proportional to the input spike time t_i. The ISI1 block fires output spikes at certain times which can be utilized to distinguish patterns whose order of input spikes are the same but for which the actual spike times may be different. For instance, two patterns P_A and P_B with spike times $\{t_1^A = 1, t_2^A = 2, t_3^A = 3, t_4^A = 4\}$ and $\{t_1^B = 1.5, t_2^B = 2, t_3^B = 2.5, t_4^B = 3\}$ can be distinguished by the output spike time t_{out1}.
The ISI1 block output times are contained within a range of time, called the output time window. Within this output time window, outputs can represent inputs according to their ISI times. The synaptic weight values may be initially set to be identical, as shown in Figure 2(A).
The ISI2 block (Figure 2(A)) has a construction and function similar to ISI1 block except the parameter k_i is defined as follows:

$$k_i = \beta * t_i, \quad k_1 = 1 \quad , \quad i = 2, 3, ..., n \tag{4}$$

Using a combination of the ISI1 and ISI2 blocks produces a one-to-one correspondence between inputs and outputs (refer to Appendix A).
As these two blocks can distinguish different patterns if and only if their input spikes come in the same order but with various different arrival times, it is necessary to have another part which distinguishes the order of arrival of the spikes comprising a pattern. Rank Order Coding (ROC) [6] is a suitable approach to distinguish the order of input spike arrivals. The ROC block is composed of two units, as shown in Figure 2(B). One of these units utilizes an excitatory neuron and the other unit an inhibitory-like neuron with a special function. The weight values must be distinct in this block in order to produce distinct output times. The shortest neuron activation time will result only when the input spikes arrive in synaptic weight order.
Modifications in the original ROC block proposed in [6] are necessary to account for cases in which two or more input spikes arrive at the same time. In such a case, the inhibitory neuron must be able to recognize that two or more

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inputs have arrived simultaneously and compensate accordingly.
Real-time decoding of inputs is possible when the ISI1, ISI2 and ROC units work simultaneously. Unique output spike combinations at t_{out1}, t_{out2} and t_{out3} will be produced for all input patterns.
In the ISI1 block, all output spike times t_{out1} must be larger than the last input spike time t^{max} of all the input patterns to be learned. This means that the patterns to be learned must be known a priori in order to know the time range of the input spikes for all the input patterns. Furthermore, all input spikes have equal importance, so all spikes representing a pattern must be utilized to determine the neuron membrane potential.
The proposed ISI1 block decoding scheme works as follows:

1. Each input pattern l is represented by a set of input spike times $P_l[t_1....t_i....t_n]$, where $t_i \in R^+$, n is the number of input spikes defining the pattern.

2. The weights associated with each input synapse in the ISI block are initially identical.

3. For each input pattern check the neuron output firing time t_{out1}:
   if $t_{out1} > t^{max}$ go to step 4
   if $t_{out1} < t^{max}$ decrease the synaptic weight values a little, repeat this step again for the same pattern.

4. Step 3 is repeated for all input patterns until $t_{out1} > t^{max}$.

The same scheme is used for ISI2 block in a similar way to get $t_{out2} > t^{max}$.
An appropriate choice of threshold ϑ and initial weights must be found in order for this decoding scheme to work properly.

4 Application of the Decoding Scheme

Sound localization was used as an interesting and useful application of the proposed decoding scheme. The azimuth and elevation angles were to be deduced from sound input data[1]. Sound localization was thought to be an appropriate application because it can utilize the Interaural Time Differences (ITD). ITD is defined as the difference between the arrival times of a sound signal to each ear. In the proposed decoding scheme, the sound siganl itself can be used directly without complex modifications such as those needed in the (HRTF) approach [2].
Sensors representing right (R), left (L), front (F), back (B), above (AB) and below (BL) were placed in their appropriate positions as shown in Figure 3(A). The reception time of a sound at a sensor was determined by the first incoming audio signal which exceeded a pre-determined sound level. Depending on the sound source location with respect to the six sensors, spike arrival time will be different from one sensor (input) to another. The decoder, Figure 3(B),

[1]Simulations were done using Matalb® version 6.5 Release 13.

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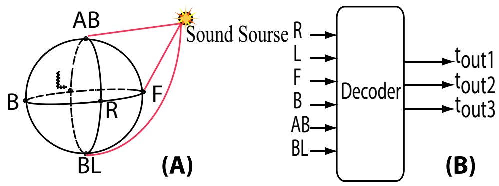

Figure 3: (A) Distribution of sensors (B) Sound localization unit.

will generate a set of output spikes for each input pattern. Echo effects were neglected in this application. The set of $\{t_{out1}, t_{out2}, t_{out3}\}$ times represent the sound source location.
To test the reliability of the decoding scheme, some samples which were not learned were used. It was found that the decoder worked well even with the new (not learned) sound source locations. The output spike times t_{out1} and t_{out2} increased (or decreased) within an appropriate output spike time firing range. t_{out3} time did not change as long as the order of input spike arrival times was unchanged; t_{out3} was not affected by the actual input spike arrival times. As a result, the three output times $\{t_{out1}, t_{out2}, t_{out3}\}$ could be used to determine the position of a sound source quite accurately[2].

5 Conclusions

As shown in this paper, SNN can process actual temporal signals in very close to real-time in applications such as sound localization. Other applications in which temporal information could be used include the decoding of ECG signals with some preprocessing of data. The proposed scheme, in spite of its simplicity, achieved good results in the application described above by effectively utilizing inter-spike arrival time information.

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[2] If the outputs of either or both ISI1 or ISI2 produce output spikes at nearly the same time for different input patterns, the patterns can be more clearly distinguished by allowing a little extra time after the arrival of the last input spike. This allows more time for the outputs to diverge.

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Appendix A

The one-to-one mapping of inputs to outputs of the decoder will be proved. Assume that the potential function in the two ISI blocks has a sufficiently long time constant so that $\alpha(t)$ of Equation 2 can be considered to work simply as a linear function. It then follows that Equation 1 can be re-written as:

$$x(t) = \frac{t}{\tau} \sum_{i=1}^{n} k_i.w_i.u(t - t_i) \tag{5}$$

In Equation 5, $u(t)$ is the Heaviside function. The slope of the function represented by Equation 5 is $\frac{1}{\tau}\sum_{i=1}^{n} k_i.w_i$ at $t \to \infty$. The slope is dependent on the value of k_i assuming τ and w_i are constants. Assume s_i represents the slope of the potential function after the arrival of input spike t_i.

To prove that no coincident potential values are produced for different input patterns after the last input spike has arrived, it is sufficient to show that the slopes of two different input patterns cannot be equal. The following cases cover all the worst case input pattern combinations. Assume two different input patterns P_A and P_B have the same spike orders but different spike times $P_A = \{t_1^A, t_2^A, ..., t_{n-1}^A, t_n^A\}$ and $P_B = \{t_1^B, t_2^B, ..., t_{n-1}^B, t_n^B\}$. If the last spike inputs have the relation $t_n^A > t_n^B$ and $s_{n-1}^A > s_{n-1}^B$, the potential functions of P_A and P_B may intersect at some later time after the last input spike, i.e., $s_n^A < s_n^B$ and then $t_{out1}^A = t_{out1}^B$, for the ISI1 block; however, for the same patterns P_A and P_B the ISI2 block makes the internal potential functions diverge ($s_n^A > s_n^B$) and thus $t_{out2}^A \neq t_{out2}^B$. If $t_n^A < t_n^B$ and $s_{n-1}^A > s_{n-1}^B$, the potential slopes may intersect at some later time ($s_n^A < s_n^B$ and then $t_{out2}^A = t_{out2}^B$) for the ISI2 block, while the ISI1 block would make the internal potentials diverge ($s_n^A > s_n^B$) and thus $t_{out1}^A \neq t_{out1}^B$. If $t_n^A = t_n^B$ and $s_{n-1}^A > s_{n-1}^B$, the ISI1 and ISI2 blocks would produce $s_n^A > s_n^B$ and thus $t_{out1}^A \neq t_{out1}^B$. If $t_n^A = t_{n-1}^B$ and $s_{n-1}^A > s_{n-1}^B$, then ISI1 block would produce $t_{out1}^A \neq t_{out1}^B$; Furthermore if $t_n^A = t_{n-1}^B$ and $s_{n-1}^A < s_{n-1}^B$, then ISI2 block would produce $t_{out2}^A \neq t_{out2}^B$. Thus, all possible spike input sequences produce unique combination of outputs at t_{out1} and t_{out1} of the ISI1 and ISI2 blocks which can be used to recognize a particular input sequence.