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A Chaotic Basis for Neural Coding

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Abstract. Recent neurobiological data has demonstrated that some neurons communicate with each other via the timing of individual spikes. The possibility of a neural code based on time-structured spike trains is a departure from established theories based on rate coding. Precisely how these time-structured spike trains communicate information is still open for debate. In this paper we consider the possibility that these spike trains communicate discrete internal neuronal states that are generated from the stabilised orbits of a chaotic attractor.

1 Introduction

Previous research has demonstrated that deterministic chaos can play a central role in neural information processing [1, 5]. Methods of chaos control have been used to stabilise Unstable Periodic Orbits (UPOs) that represent the dynamic memory states of a chaotic neural network. The advantages of this have been discussed elsewhere [3, 2, 4]. This previous work has used neuron models whose output represents an average firing rate. Although this is a common approach in neural modelling, recent experimental evidence suggests that firing rates are inadequate for describing neuronal activity. For example, behavioural experiments have shown that the time it takes a subject to react after the onset of a stimulus is too short for neurons to determine their response based on average firing rates [11]. Many recent theories of neural coding suggest that information is conveyed not in firing rates but in the relative timing of neuronal spikes, referred to as *temporal coding* [7, 11]. Furthermore, experimental results have shown that cortical neurons in vitro and noisy integrate-and-fire model neurons driven by periodic currents accurately reproduce temporally structured spike trains which can support a spike-time neural code [10].

This paper considers the relationship between chaos and temporal coding in spiking neural networks. The approach used here is to investigate neuronal models that have internal dynamics governed by a chaotic attractor and an output consisting of threshold-triggered constant-amplitude spikes. In the absence of control, the internal chaotic dynamics ensure that a chaotic spike train

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is generated by such neurons. In other words, the timing of the spikes is aperiodic, indicating that the neuron is in a transient state. UPOs can then be stabilised on the internal dynamics of the neuron using time delayed feedback control. This results in a periodic spike train from the neuron determined by the particular orbit that has been stabilised. Section two of this paper describes a chaotic spiking model which incorporates delayed feedback control and presents some preliminary results obtained from the model. Section three considers how UPO based neural codes can be decoded using multiple delay connections.

2 Chaotic Spiking Model

This model is based on the Spike Response Model presented in [6], commonly used to model networks of spiking neurons. The following equations define system behaviour:

$$u_i(t) = \sum_{t_i^{(f)} \in \mathcal{F}_i} \eta_i(t - t_i^{(f)}) + \sum_{j \in \Gamma_i} \sum_{t_j^{(f)} \in \mathcal{F}_j} w_{ij} \varepsilon_{ij}(t - t_i^{(f)}) + \sum_{t_i^{(c)} \in \mathcal{G}_i} \xi_i(t - t_i^{(c)}) \tag{1}$$

where $u_i(t)$ is the activation of neuron i, $t_i^{(f)}$ firing time of neuron i, $\mathcal{F}_i$ is the set of all firing times of neuron i, Γ_i is the set of neurons with synaptic connections to neuron i, $\eta_i(\cdot)$ is the kernel which implements the refractory period of the neuron, w_{ij} is the weight of the connection from neuron j to neuron i, $\varepsilon_{ij}(\cdot)$ is the kernel which implements the post synaptic potential generated in neuron i by an impulse from connected neuron j, $t_i^{(c)}$ it the time of a internal chaotically driven impulse in neuron i, $\mathcal{G}_i$ is the set of chaotic impulses for i, and $\xi_i(\cdot)$ is the kernel which models the response to the chaotic impulses.

Neuron i will fire whenever its activation $u_i(t)$ reaches a threshold value ν. After firing, the kernel η_i generates the refractory response of the neuron, which is defined by $\eta_i(x) = -\eta_0 \mathcal{H}(x) e\left[-x/\tau_r\right]$ where η_0 is the amplitude of the refractory response, $\mathcal{H}$ is the Heaviside step function, and τ_r is the decay time constant of the response. The kernel $\varepsilon_{ij}(\cdot)$ which implements the post synaptic after-potential is defined by $\varepsilon_{ij}(x) = \mathcal{H}(x) exp\left[-x/\tau_{ij}\right]$ where τ_{ij} is the decay time constant for the post synaptic after-potentials. The kernel which models the response to the chaotic impulses is defined by $\xi_{ij}(x) = \mathcal{H}(x) exp\left[-x/\tau_c\right]$ where τ_c is the decay time constant for the response to chaotic impulses.

An efficient discrete-time version of this model was implemented using first-order recursive digital filters to approximate the three kernel types:

$$u_i(n) = -\left(u_i^{(t)}(n) + \nu\right) + \sum_{j \in \Gamma_i} u_{ij}^{(f)}(n) + u_i^{(c)}(n) \tag{2}$$

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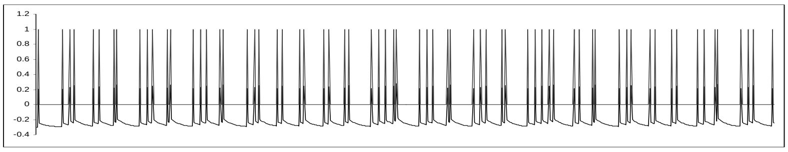

Figure 1: A chaotic sequence of spikes generated by the SPIKE system

The refractoriness of the neuron is modelled by a dynamic threshold implemented using the filter $u_i^{(t)}(n) = \eta_0 y_i(n) + r_i^{(t)} u_i^{(t)}(n-1)$ which is summed with the static threshold ν. The output of neuron i is represented by $y_i \in [0,1]$. Synaptic input is modelled by $u_{ij}^{(f)}(n) = w_{ij} y_j(n) + r_{ij}^{(s)} u_{ij}^{(f)}(n-1)$. The output of the filter for the chaotic impulses is given by $u_i^{(c)}(n) = \beta z_i(n) + r_i^{(c)} u_i^{(c)}(n-1)$. Each filter uses a relaxation factor with time constant μ defined by $r^{(x)} = exp[-T/\mu_x]$ in which T is the size of the time step for the simulation.

The set of chaotic impulses $\mathcal{G}_i$ was generated using the Aihara equation [1] with added delay-feedback control and with $t_i^{(c)}(0)$ set to a value chosen randomly from the attractor:

$$g_i(n+1) = \omega t_i^{(c)}(n) - \alpha f(t_i^{(c)}(n)) + a \tag{3}$$

$$t_i^{(c)}(n) = g_i(n) + k(g_i(n) - t_i^{(c)}(n-\psi) \tag{4}$$

ω, α and a are the constants of the Aihara equation, $k(\leq 0)$ is the control constant and ψ is the time delay. Figure 1 shows a sample of output from a chaotically driven spiking neuron with no delay-feedback control ($k = 0$).

When delayed feedback control is applied to the chaotically driven spiking neuron (i.e. $k < 0$), it is driven towards a periodic firing pattern. The period of the pattern is determined by the value of the delay variable ψ. The graphs in Figure 2 show the spike trains generated by feedback control initiated at $t = 0.153$ using delay $\psi = 2$ (Figure 2(a)) and $\psi = 3$ (Figure 2(b)): The first elicits a period 5 response, the second a period 22 response.

The periodic signal produced by the stabilisation of a UPO in the dynamics of a neuron can form the basis of neural coding. In essence, the UPO is an internal state of the neuron that is communicated by a unique periodic spike train to other neurons. Since chaos is the basis of this behaviour, the neuron has a theoretically infinite number of UPOs available to it, although there is no guarantee that all of them can be stabilised. This gives the neuron a rich set of internal states at its disposal together with a vocabulary of spike trains with which states can be communicated to other neurons.

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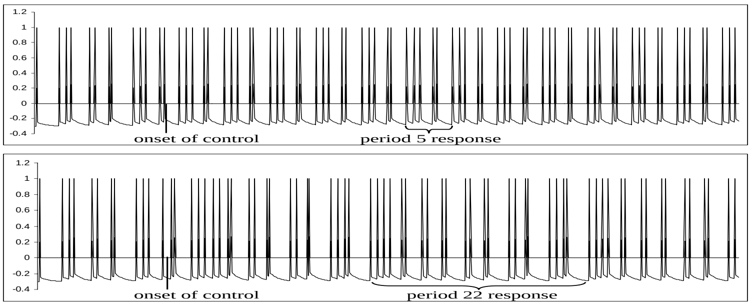

Figure 2: Two periodic spike trains obtained by applying delay feedback control with (a) $\psi = 2$ and (b) $\psi = 3$

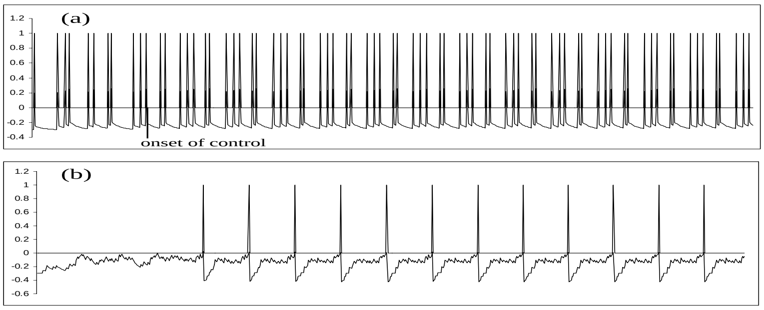

Figure 3: The first experimental run

2.1 Decoding

The time-structured spike trains generated by stabilising UPOs on a chaotic attractor can be *decoded* using multiple time-delayed synaptic connections [4]. Delays in the signaling between neurons is inherent in natural neuronal systems. A time structured sequence of spikes can be recognised by the receiving neuron if the time delays in these synaptic connections are such that each spike in the sequence has a coincidental effect on the potential rise at the axon hillock of the receiving neuron.

A network of two spiking neurons was constructed to demonstrate the use of multiple time delay connections in decoding time-structured spike sequences. Neuron A was chaotic, with delayed feedback control being applied at $t = 0.127$. Neuron B was a passive spiking neuron (i.e. it is not chaotically driven). Five time-delay connections are made from A to B, with the following delays: $0.004, 0.007, 0.008, 0.015, 0.019$. The lengths of the transmission delays

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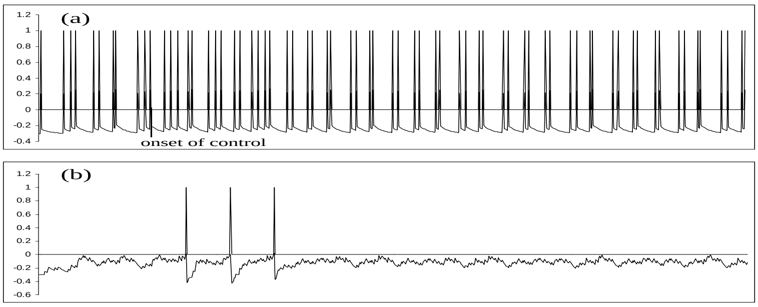

Figure 4: The second experimental run

are such that the all five of the spikes in the periodic orbit in Figure 2(a) will have a coincidental effect on neuron B. This model was subject to two experimental runs. In the first run, the delay on the feedback control ψ was set to 2, stabilising a period five spike response from neuron A (Figure 3(a)). Neuron B did not fire during the initial chaotic sequence, but began to fire periodically once the control had been applied to neuron A (Figure 3(b)), showing that neuron B had 'decoded' the period 5 spike train. In the second run, the delay on the control was set to 5, which stabilised a period 10 orbit on neuron A. Neuron B produced three spikes during the transient phase after the control had started (because it detected three period 5 spike trains in the transient which matched the delays on the connections). Once the period 10 orbit was established in neuron A, neuron B produced no further spikes, demonstrating its selective sensitivity to the particular time-structure of the period 5 spike train generated by neuron A with $\psi = 2$.

2.2 Conclusion

Some of the periodic spike trains which are stabilised by delay feedback control are long (e.g. the period 22 orbit in Figure 2(b)). This would be a significant drawback for real-time applications where fast responses are needed if it was necessary to wait for the whole period of the spike train to be completed before it can be decoded. However, since the inter stimulus intervals of these periodic trains are generated from a chaotic attractor, then by definition the interval between any two spikes in the train is unique to that train. In other words, two consecutive spikes of a stabilised UPO spike train are sufficient to uniquely identify the whole of that spike train.

A question at the heart of many discussions about neural coding is whether information is conveyed solely by the rate at which a neuron fires (rate coding), or whether the precise time of a spike relative to other spikes is the conveyor of information (time-structured coding). Evidence for both have been found in natural neuronal systems [8, 9]. The UPO based model presented above is a unique contribution to the time-structured theory of neural coding. Current research in to neural coding has not yet properly considered the possibility that the time-structured spike trains of neurons are generated from the stabilised orbits of a chaotic attractor. The advantage of this approach is that such neurons would have a very large (theoretically infinite) number of discrete states (UPOs), each described by a unique time-structured spike train.

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