# A Global Multi-Sided Market with Ascending-Price Mechanism

Extended Abstract

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## ABSTRACT

We present an ascending-price mechanism for a multi-sided market with a variety of participants, such as manufacturers, logistics agents, insurance providers, and assemblers. Each deal in the market may consist of a combination of agents from separate categories, and different such combinations are simultaneously allowed. This flexibility lets multiple intersecting markets be resolved as a single global market. Our mechanism is obviously-truthful, strongly budget-balanced, individually rational, and attains almost the optimal gain-from-trade when the market is sufficiently large.

## KEYWORDS

Multi-Sided Markets; Truthful Auctions; Strong Budget Balance

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## 1 INTRODUCTION

The aim of this paper is to automatically arrange the trade in complex multi-lateral markets. As an example, consider a market for a certain kind of laptop computer, and assume for simplicity that it is made of only two components, e.g. CPU and RAM. Even in this simplified market, there may be several different categories of traders: 1. Buyers, who are interested in a laptop; 2. Laptop producers, who produce whole laptops; 3. CPU producers; 4. RAM producers; 5. Constructors, who construct a laptop from its parts; 6. Transporters, who take a laptop and bring it to an end consumer. A deal in this market can take one of two forms:

- A buyer buys a laptop from a laptop-producer, and asks a trasporter to transport it to his/her place. This involves traders of categories 1, 2 and 6.
- A buyer buys a CPU, a RAM and a construction service, and has the final product transported. This involves traders of categories 1, 3, 4, 5 and 6.

In each category there may be many different traders, with potentially different utilities for participating in a deal. Typically, the value of a buyer is positive and the value of a producer or service-provider is negative. The main questions of interest for automatically arranging the trade is *who* will trade and *how much* they will pay (or receive). The answers to these questions should satisfy several natural requirements:

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(1) *Individual rationality (IR)*: no agent should lose from participating: the amount paid by a trading agent should be at most as high as the agent's value (if the value is negative then the agent should receive money). A non-trading agent should pay nothing.

(2) *Weak budget balance (WBB)*: the total amount paid by all agents together should be at least 0, so that the market manager does not lose money. A stronger requirement called *strong budget balance (SBB)* is that the total amount be exactly 0, so that the market manager does not take away money from the market, as this might drive traders away.

(3) High *gain-from-trade (GFT)*: the GFT is the sum of values of all agents actively particiating in the trade. For example, suppose a certain buyer values a laptop at 1000, the laptop-producer values it at -700 (the cost of production is 700), the CPU and RAM producer and constructor value their efforts at -200 each, and the transporter values the deal at -50 (the cost of transportation is 50). Then, the GFT from a deal involving categories 1, 2, 6 is $1000 - 700 - 50 = 250$, and the GFT from a deal involving categories 1, 3, 4, 5, 6 is $1000 - 200 - 200 - 200 - 50 = 350$. Maximizing the GFT implies that the latter deal is preferred.

(4) *Truthfulness*: the agents' values are their private information. We assume that the agents act strategically to maximize their utility (assumed to be their value minus the price they pay). Truthfulness means that such a utility-maximizing agent reports his/her true valuation. A stronger requirement called *obvious truthfulness* [4] is that, for each agent, the lowest utility he may get by acting truthfully is at least as high as the highest utility he may get by acting non-truthfully.

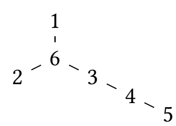
***Previous work.*** The study of truthful market mechanisms started with Vickrey [8]. He considered a market with only *one category* of traders (buyers), where the famous *second-price auction* attains all four desirable properties: IR, WBB, maximum GFT and truthfulness.

When there are *two caterogies* of traders (buyers and sellers), the natural generalization of Vickrey's mechanism is no longer WBB — it may run a deficit. Moreover, Myerson and Satterthwaite [6] proved that *any* mechanism that is IR, truthful and maximizes the GFT must run a deficit. The way out of this impossibility paradox was found by McAfee [5]. In his seminal paper, he presented the first *double auction* (auction for a two-category market) that is IR, WBB, truthful, and *asymptotically* maximizes the GFT. By asymptotically we mean that its GFT is at least $(1 - 1/k)$ of the optimal GFT, where $k$ is the number of deals in the optimal trade. Thus, when $k$ approaches infinity, the GFT approaches the optimum.

McAfee's mechanism has been extended in various ways Gonen et al. [2], Segal-Halevi et al. [7]. Particularly relevant to our setting is the extension by Babaioff and Nisan [1], in which there are *multiple categories* of traders, arranged in a *linear supply chain*. In their model, there is a single *producer* category, a single *consumer* category, and several *converter* categories. Each deal must involve a single producer, a single consumer, and a single agent of each converter category. In our laptop example, their model covers either a market with the chain 1,2,6 or a market with the chain 1,3,4,5,6, but not a market where both chains are possible. For this model, they present an auction mechanism that is IR, WBB, truthful, and attains asymptotically-optimal GFT.

Recently, Gonen and Segal-Halevi [3] considered a multiple-category market in which, like Babaioff and Nisan [1]'s market, all deals must be of the same structure, which they call a "recipe". Their recipes are more general than the linear supply chains of [1], since they are not restricted to a producer-converters-consumer structure. They present auctions that are IR, SBB, truthful and asymptotically-optimal, but only for a single-recipe market.

***Our contribution.*** We study markets with multiple kinds of supply-chains which we call "recipes". In a general multi-recipe market, computing the optimal trade — even without strategic considerations — is NP-hard. In this paper, we focus on a special case in which the optimal trade can be computed in polynomial-time the case in which the agent categories can be arranged in a tree, and each recipe is a path from the root to a leaf of that tree. Our laptop market corresponds to the following tree:




We present an ascending mechanism for such markets. Our mechanism is IR, SBB, obviously-truthful, and its expected GFT is asymptotically-optimal — approaches the optimum when the optimal number of deals in all recipes approaches ∞. Our current mechanism extends [3] only in the setting of *binary* recipes, in which each category participates in each recipe either zero or one times.

## 2 OVERVIEW OF OUR MECHANISM

Our ascending-price auction is a randomized sequential mechanism. The general scheme is presented as Algorithm 1. The auctioneer maintains a price $p_g$ for each category $g \in G$. When the auction starts, every price $p_g$ is initialized to at most $-V$. This guarantees that, initially, all agents in all categories are "in the market" (namely, willing to trade in the current prices). We denote the subset of $N_g$ of agents currently in the market by $M_g$.

The auctioneer chooses a subset of the prices, and increases each price in this subset by a single unit. After each increase, the auctioneer asks each agent in turn, in a pre-specified order (e.g. by their index), whether their value is still higher than the price. An agent who answers "no" is permanently removed from the market. After each increase, the auctioneer computes the sum of prices of the categories in each recipe. When this sum increases to 0, the auction ends and the remaining agents trade in the final prices.

The main challenge in fleshing out this scheme is to decide *which* prices to increase each time. We must ensure that the sum of prices $\sum_{g \in G} p_g \cdot r_g$ remains the same for all recipes $\mathbf{r} \in R$, such that the price-sum crosses 0 for all recipes simultaneously. The process of selecting which prices to increase is shown at Algorithm 2.

It is a recursive algorithm: if the tree contains only a single category (a root with no children), then of course this category

**Algorithm 1** Ascending prices mechanism — recipe-tree.

**Input:** A market $N$, a set of categories $G$ and a recipe-tree $R$.
**Output:** Strongly-budget-balanced trade.
1. *Initialization:* Let $M_g := N_g$ for each $g \in G$.
Determine initial price-vector $\mathbf{p}$:
For each non-leaf $g$, set $p_g := -V$;
For each leaf $g$, set $p_g := -V \cdot (\textsc{MaxDepth} - \textsc{Depth}(g) + 1)$;
2. Using Algorithm 2, select a set $G^* \subseteq G$ of categories.
3. For each $g^* \in G^*$, ask each agent in $i \in M_{g*}$ whether $v_i > p_{g*}$.
(a) If an agent $i \in M_{g*}$ answers "no", then —
remove $i$ from $M_{g*}$ and go back to step 2.
(b) If all agents in $M_{g*}$ for all $g^* \in G^*$ answer "yes", then —
for all $g^* \in G^*$, let $p_{g*} := p_{g*} + 1$.
(c) If after the increase $\sum_{g \in G} p_g \cdot r_g = 0$ for some $\mathbf{r} \in R$,
then go on to step 4;
else go back to step 3.
4. Determine final trade using Algorithm 3.

**Algorithm 2** Given a recipe-tree, find a set of prices to increase.

**Input:** A set of categories $G$,
a set of remaining traders $M_g$ for all $g \in G$,
and a recipe-tree $R$ based on a tree $T$.
**Output:** A subset of $G$ denoting categories
whose price should be increased.
0. *Initialization*: For each category $g \in G$, let $m_g := |M_g|$ = the number of agents of $N_g$ who are in the market.
1. Let $g_0$ be the root category. Let $c_{g_0} := \sum_{g' \text{ is a child of } g_0} m_{g'}$.
2. If $m_{g_0} > c_{g_0}$ [or $g_0$ has no children at all],
then return the singleton $\{g_0\}$.
3. Else ($c_{g_0} \geq m_{g_0}$), for each child $g'$ of $g_0$:
Recursively run Algorithm 2 on the sub-tree rooted at $g'$;
Denote the outcome by $I_{g'}$.
Return $\bigcup_{g' \text{ is a child of } g_0} I_{g'}$.

is selected. Otherwise, either the root category or its children are selected for increase. The selection is based on the number of agents of each category $g$ who are currently in the market. If the number of traders remaining in $g_0$ is larger, then the price selected for increase is the price of $g_0$; Otherwise (if the number of traders remaining in all children of $g_0$ together is larger or equal), the prices to increase are the prices of children categories: for each child category, Algorithm 2 is used recursively to choose a subset of prices to increase, and all returned sets are combined. The resulting subset contains one price for each path from root to tree, so if all prices in the subset are increased simultaneously by one unit, then the price-sum in all recipes increases simultaneously by one unit.

The properties of our mechanism are summarized below.

Theorem 1. *Algorithm 1 is universally strongly-budget-balanced, individually-rational and obviously truthful.*

Theorem 2. *The expected GFT of the ascending-price auction of Section 2 is at least* $1 - 1/k_{\min}$ *of the optimal GFT, where* $k_{\min}$ *is the smallest positive number of deals of a single recipe in the optimal trade,* $k_{\min} := \min_{\mathbf{r} \in R, k_{\mathbf{r}} > 0} k_{\mathbf{r}}$.

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