Sequential and Swap Mechanisms for Public Housing Allocation with Quotas and Neighbourhood-Based Utilities

Extended Abstract

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ABSTRACT

We consider the problem of allocating indivisible items to agents where both agents and items are partitioned into disjoint groups. Following previous works on public housing allocation, each item (or house) belongs to a block and each agent is assigned a type. The allocation problem consists in assigning at most one item to each agent in a *good* way while respecting diversity constraints. Based on Schelling's seminal work, we introduce a generic individual utility function where the welfare of an agent not only relies on her preferences over the items but also takes into account the fraction of agents of her own type in her own block. In this context, we investigate the issue of stability, and study two existing allocation mechanisms: a sequential mechanism used in Singapore and a distributed procedure based on mutually improving swaps of items.

KEYWORDS

Multiagent Resource Allocation, Diversity Constraints, Distributed Allocation Mechanisms, Computational Social Choice.

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1 INTRODUCTION

Fairly dividing indivisible items among agents is a central problem in multiagent systems. There are often relations connecting both items (*e.g.* spatial or temporal relations [4]) and agents (*e.g.* belonging to the same hierarchical structure, or being of the same type). In public housing allocation problems for instance, agents get assigned to locations (houses), belonging to blocks. They may of course have preferences over those locations, but importantly, this is also a setting where externalities naturally occur [11]: it makes a difference whether your friends, for instance, get assigned to the same block as you. While agents may naturally seek the proximity of other agents of the same type (a phenomenom well-known as *homophily*), the objective might be opposite at the society level. From the designer's perspective, it is indeed often desirable to preserve some diversity. In practice this can be done by imposing some quotas. Recently, several papers have studied variants of these settings [5, 7, 8]. However to the best of our knowledge, none of them addressed a model where agents are motivated (to some extent) by such an homophily bias, while the system has a conflicting diversity objective enforced through a system of quotas. In this paper we undertake the study of such a model.

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2 OUR MODEL AND STABILITY NOTION

We consider an allocation problem involving a set $\mathcal{N}$ of n agents, partitioned into a set T of k types $T_1, \dots, T_k$, and a set $\mathcal{M}$ of m items, partitioned into a set B of l blocks $B_1, \dots, B_l$, where the inequality $|\mathcal{N}| \geq |\mathcal{M}|$ holds; note that it is a realistic assumption, especially when considering the allocation of public goods. We denote by $\mathcal{T}(i)$ the type of any agent $i \in \mathcal{N}$ and by $\mathcal{B}(h)$ the block of any item $h \in \mathcal{M}$. Following the work of Benabbou et al. [2, 3], diversity constraints are here defined using type-block quotas $\lambda_{p,q} \in \mathbb{N}$, with $(p, q) \in [k] \times [l]$, such that $\lambda_{p,q}$ stands for the maximum number of agents of type T_p allowed in block B_q. Without loss of generality, we assume that the inequality $\lambda_{p,q} \leq |B_q|$ holds for all $(p, q) \in [k] \times [l]$ since it is not possible to assign more than $|B_q|$ items in block B_q by definition. We also assume that the inequality $\sum_{p \in [k]} \lambda_{p,q} \geq |B_q|$ holds for all blocks $q \in [l]$ otherwise all allocations satisfying diversity constraints would leave some items unassigned.

Definition 1 (Valid Allocation). *An allocation $A : \mathcal{N} \to 2^{\mathcal{M}}$ is a function that maps every agent $i \in \mathcal{N}$ to a subset $A(i) \subset \mathcal{M}$ of items. An allocation A is* valid *iff:*

(1) $\forall i \in \mathcal{N}, |A(i)| \leq 1$ *(each agent receives at most one item).*
(2) $\forall i, j \in \mathcal{N}$, $A(i) \cap A(j) = \emptyset$ *(agents do not share items).*
(3) $\bigcup_{i \in \mathcal{N}} A(i) = \mathcal{M}$ *(all items are assigned).*
(4) $\forall p \in [k], \forall p \in [l], |\{i \in T_p : A(i) \in B_q\}| \leq \lambda_{p,q}$ *(upper quotas).*

We assume here that the utility $u_i(A)$ that an agent $i \in \mathcal{N}$ derives from an allocation A has two components:

- $u_i^I(A) \in [0, 1]$: an *item-based utility* representing the utility derived by agent i for $A(i)$ the item she receives.
- $u_i^N \in [0, 1] \cap \mathbb{Q}$: a *neighbour-based utility* which is equal to the fraction of agents of type $\mathcal{T}(i)$ assigned to items in block $\mathcal{B}(A(i))$. More formally, it is defined by:

$$u_i^N(A) = \frac{\sum_{j \in \mathcal{N}: A(j) \in \mathcal{B}(A(i))} \mathbb{I}(\mathcal{T}(i), \mathcal{T}(j))}{|\mathcal{B}(A(i))|}$$

where $\mathbb{I}(\mathcal{T}(i), \mathcal{T}(j))$ equals 1 if agents i and j have the same type, and equals 0 otherwise. Then, the utility of agent $i \in \mathcal{N}$ for allocation A is defined by:

$$u_i(A) = u_i^I(A) + \varphi \times u_i^N(A)$$

where $\varphi \in [0, 1]$ is used to define the relative importance of the item-based utility and the neighbour-based utility. This type of utility function thus allows to model agents which are both concerned by the item they obtain, as well as their neighbourhood. We take inspiration from the model of [8] but the differences are important to notice: both our location and neighbourhood relations are partitions among agents, while they are arbitrary (undirected) graphs in [8] – they would thus correspond to collections of cliques in their model. Two types of behaviour are especially interesting:

Definition 2 (Item-Focused and Neighbour-Focused). *An agent is said to be* item-focused *if she only cares about the item she receives (i.e. when $\varphi = 0$). An agent $i \in \mathcal{N}$ is said to be* neighbour-focused *if she only cares about her neighbourhood (i.e. when $\varphi \neq 0$ and $u_i^I(h) = 0$ for all $h \in \mathcal{M}$).*

An instance $\mathcal{I}$ of the public allocation problem with generalized utility function and diversity constraints is a tuple $\mathcal{I} = \langle \mathcal{N}, \mathcal{M}, B, T, u^I, u^N, \lambda, \varphi \rangle$ with $\mathcal{N}, \mathcal{M}, B, T, \varphi$ as defined above, and:

- $\lambda = \langle \lambda_{1,1}, \cdots, \lambda_{k,l} \rangle$ the $[k] \times [l]$ matrix of quotas,
- $u^I = \langle u_1^I, \cdots, u_n^I \rangle$ the item-based utility profile of the agents,
- $u^N = \langle u_1^N, \cdots, u_n^N \rangle$ their neighbour-based utility profile.

When assessing the welfare of the whole society of agents, we rely on the classical utilitarian social welfare: $sw(A) = \sum_{i \in \mathcal{N}} u_i(A)$.

A key property of an allocation is *stability*, in the sense that no individual would like to deviate from the prescribed allocation. In our context, we shall concentrate on the notion of *swap-stability* [1, 11]: it shouldn't be the case that two agents would be happy to swap their items, resulting in a valid allocation. Formally:

Definition 3 (Improving Swap-Deal). *A swap-deal among a pair of agents $(i, j) \in \mathcal{N} \times \mathcal{N}$ is said to be* improving *if and only if $u_i(A(j)) > u_i(A(i))$ and $u_j(A(i)) > u_j(A(j))$.*

From a given allocation A, it may exist some improving swap-deals that lead to an invalid allocation. We thus restrict the set of swap-deals that can be applied from a given allocation as follows:

Definition 4 (Valid Swap-Deal). *A swap-deal among a pair of agents $(i, j) \in \mathcal{N} \times \mathcal{N}$ is* valid *if and only if the resulting allocation satisfies the diversity constraints (i.e., type-block quotas).*

We can now introduce our stability notion.

Definition 5 (Stable Allocation). *An allocation A is* stable *if and only if there is no valid improving swap-deal from A.*

The *Price of Stability* (PoS) is defined as the ratio between the utility of any valid allocation maximizing the utilitarian social welfare and the utility of the best stable valid allocation. We have:

Proposition 2.1. *PoS = 1 when all agents are item-focused, or when all agents are neighbour-focused. PoS > 1 in the general case.*

3 A SEQUENTIAL MECHANISM

The sequential procedure presented in [3] is a simplified version of the Singaporean public housing allocation process: in some random order, the agents sequentially pick the unallocated items that maximize their utilities, while respecting the diversity constraints.

Proposition 3.1. *The sequential mechanism does not always return a valid allocation.*

Proof. Consider an instance with a set of 4 neighbour-focused agents $\mathcal{N} = \{1, 2, 3, 4\}$ partitioned into 2 types $T_1 = \{1, 2\}$ and $T_2 = \{3, 4\}$, a set of 4 items $\mathcal{M} = \{h_1, h_2, h_3, h_4\}$ partitioned into 2 blocks $B_1 = \{h_1, h_2\}$ and $B_2 = \{h_3, h_4\}$, and the following quotas: $\lambda_{1,1} = \lambda_{2,1} = 1$ (at most 1 agent per type in B_1) and $\lambda_{1,2} = \lambda_{2,2} = 2$ (at most 2 agents per type in B_2). When we run the sequential mechanism with agent order $(1, 2, 3, 4)$, nothing prevent the first two agents from picking the two items available in block B_2, which then forces agent 3 to pick an item in block B_1, leaving agent 4 unassigned since her quota is reached in block B_1. In that case, the resulting allocation is not valid as one item remains unassigned. □

Proposition 3.2. *The sequential mechanism does not always return a stable allocation, even when all agents are neighbour-focused. It does return a stable allocation when all agents are item-focused.*

The worst-case error of any algorithm returning a valid allocation, is the ratio between the utility of any valid allocation maximizing the utilitarian social welfare and the utility of the allocation returned by the algorithm. For the sequential mechanism, this error is unbounded in the general case. When $\varphi \neq 0$ and k is a constant, it is upper bounded by $\frac{1+\varphi}{\varphi}k$ and the bound is tight.

4 A SWAP-DEAL MECHANISM

A natural distributed approach in multiagent resource allocation is to start from a valid allocation and let the agents perform bilateral improving swap-deals until they reach a stable outcome [6, 9, 12]. We focus on a simple swap-deal mechanism where at each step, pairs of agents meet randomly and performs a swap-deal if possible.

Proposition 4.1. *The swap-deal mechanism will provably reach a stable outcome.*

The worst-case error of the swap-deal mechanism is similar to that of the sequential mechanism. We then derive the following result on the Price of Anarchy (PoA) [10], which is defined as the largest utility ratio between any valid allocation and any valid stable allocation: it is unbounded in the general case. When $\varphi \neq 0$ and k is a constant, PoA $\leq \frac{1+\varphi}{\varphi}k$ and this bound is tight.

5 CONCLUSION

This paper investigated a model where the agents have an homophily component in their utility function, and there is a society-wide objective to promote diversity through the use of quotas. We show in particular that the simplified version of the sequential mechanism used in Singapore has several drawbacks, among which the lack of swap stability. An easy patch is to let agents swap until a stable allocation is reached – but is there such a guarantee? We show that this is the case, despite the fact that swaps may actually decrease social welfare. In other words, stability comes at a price.

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