Coverage Control under Connectivity Constraints

Extended Abstract

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ABSTRACT

In this paper, centralized and decentralized laws for the coverage problem under connectivity constraints are proposed. We formulated the problem as a continuous optimization task with the inequality constraint that algebraic connectivity must not be less than a small positive number, and solved the problem based on the active set method. This formulation can cause agents to be trapped in undesired local minima; to address this, we added the squared distance between the centroid of the coverage area and that of the agent system to the objective function of the coverage control. To derive the decentralized law, we employed an average consensus estimator that determines the algebraic connectivity and system centroid. When the algebraic connectivity is greater than the threshold, the proposed control laws allow agents to advance along the direction of steepest descent of the objective function. Once the connectivity is equal to the threshold, agents maintain the connectivity while decreasing the objective function by moving along the projection vector of the steepest descent in the direction in which the connectivity is constant. Simulation results confirmed the effectiveness of our proposed laws.

KEYWORDS

Distributed Control; Coverage Control; Connectivity Constraints

ACM Reference Format:
Shota Kawajiri, Kazuki Hirashima, and Masashi Shiraishi. 2021. Coverage Control under Connectivity Constraints: Extended Abstract. In *Proc. of the 20th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2021), Online, May 3–7, 2021*, IFAAMAS, 3 pages.

1 INTRODUCTION

A system containing multiple decision makers is called a multi-agent system. In such a system, each agent attempts to achieve a common goal while cooperating with the other agents [2]. The control of multi-agent systems entails several problems [3], one of which is a coverage problem for optimizing agent positions by minimizing an objective function defined for an area [17]. Although much attention has been paid to coverage control [3], most studies did not consider the communication range of the agents.

It is a relatively tractable task to maintain the connectivity of the communication graph of a system without removing existing edges (called local method [14]); this is because the agents only need to control the distance to the connected agents through a potential function [6, 12, 25]. On the other hand, connectivity maintenance, which allows arbitrary changes to graph topology (called global method), provides increased flexibility to the system but is more challenging [7, 10]. Therefore, numerous studies have been devoted to global method [13, 16, 21–23, 26], although most such studies are not related to coverage control. Among the methods employed in these studies, the control law proposed by Luo and Sycara has the advantage of converging the agent position to the local optimal solution of the coverage problem under connectivity constraints. This law first determines the minimum spanning tree of the communication graph as an optimally connected subgraph. Subsequently, it calculates the control input by solving a quadratic optimization problem under local connectivity constraints with regard to the spanning tree in order to minimize the error from the unconstrained input. However, this requires the calculation of the minimum spanning tree, whose implementation is complex [19]. Moreover, this method does not consider the case in which agents are trapped in a poor local minimum owing to the constraints.

In this study, we adopt a more straightforward approach to solve this problem and provide a simple control law. We formulate the constraints as inequality constraints using algebraic connectivity, which is an indicator of the strength of the ties between graph vertices. The following contributions are made through this paper.

(1) Formulation of the coverage problem under global connectivity constraints as a continuous optimization problem with an inequality constraint.
(2) Proposal of both centralized and decentralized laws for the above problem based on the active set method [18]
(3) Modification of the objective function of the coverage problem to prevent the agents from being trapped in a poor local minimum

2 PROBLEM STATEMENT

We assume a system consisting of $N \in \mathbb{N}$ homogeneous agents moving in a plane, and assume that an arbitrary agent can always communicate with another agent at a distance less than a threshold $R \in \mathbb{R}_+$, where $\mathbb{R}_+$ denotes a set of non-negative real numbers. Under these assumptions, we formulate the coverage problem under connectivity constraints as the following continuous optimization problem with an inequality constraint.

$$\min_{p} J'(p) \quad \text{subject to} \quad \lambda_2(p) \geq \varepsilon, \tag{1}$$

where

$$J'(p) = k_1 \int_Q \min_{i \in \mathcal{V}} \|q - p^i\|^2 \phi(q) dq + k_2 \left\| \sum_{i \in \mathcal{V}} \frac{p^i}{N} - C(Q) \right\|^2. \tag{2}$$

Proc. of the 20th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2021), U. Endriss, A. Nowé, F. Dignum, A. Lomuscio (eds.), May 3–7, 2021, Online.

$p = [p^1 \, p^2 \cdots p^N] \in \mathbb{R}^{2N}$ denotes agent positions, $\lambda_2(p) \in \mathbb{R}_+$ denotes the second-smallest eigenvalue (algebraic connectivity [8]) of a weighted graph Laplacian of a communication graph (R-disk graph [4]) of the system, ε denotes a small positive real number, $\mathcal{V}$ denotes an index set of the agents, k_1 and k_2 denote positive real numbers, $Q \subseteq \mathbb{R}^2$ denotes a bounded region to be covered, $\phi : \mathbb{R}^2 \to \mathbb{R}_+$ denotes a weighting function, and $C(Q) \in \mathbb{R}^2$ denotes the centroid of Q when ϕ is regarded as a density function. The inequality $\lambda_2(p) \geq \varepsilon$ denotes the connectivity constraints and requires a communication graph to be connected. Strictly speaking, $\lambda_2(p) > 0$ is sufficient to ensure connectivity. We introduced ε so that the constraint would include an equality constraint, and we can employ a technique based on the active set method. The first term on the right hand side of equation (2) is a well-known coverage function for deploying agents to an area [5]. The second term is introduced to bring the centroid of the system closer to that of region Q. This modification prevents the agents from being trapped in poor local minima, which is caused by connectivity constraints.

3 METHOD

A centralized control law $\dot{p}^i = u^i \in \mathbb{R}^2$ for problem (1) is expressed as follows.

$$u^i = \begin{cases} u_1^i & \lambda_2 > \varepsilon \text{ or } \left\langle u_1^i, n^i \right\rangle > 0, \quad (3a) \\ u_1^i - \left\langle u_1^i, n^i \right\rangle n^i & \text{otherwise,,} \quad (3b) \end{cases}$$

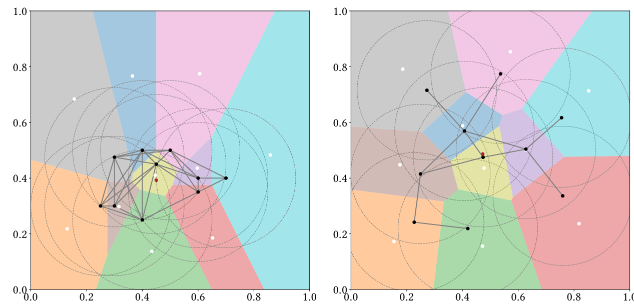
where λ_2 can be calculated from the positions of all agents, $\langle , \rangle$ denotes the inner product of two vectors, and u_1^i denotes the steepest descent direction of J' as follows.

$$u_1^i = -k_1 \mathcal{M}(Q^i(p))(p^i - C(Q^i(p))). - \frac{k_2}{N}\left(\sum_k \frac{p^k}{N} - C(Q)\right), \quad (4)$$

where $Q^i(p) \subseteq \mathbb{R}^2$ denotes a Voronoi region corresponding to agent i, $\mathcal{M}(Q^i(p)) \in \mathbb{R}^2$ denotes the mass of $Q^i(p)$ when ϕ is regarded as a density function, $n^i \in \mathbb{R}^2$ denotes a unit vector directed to $\frac{\partial \lambda_2}{\partial p^i} \in \mathbb{R}^2$, and $\frac{\partial \lambda_2}{\partial p^i}$ can be expressed using $A_{ij} \in \mathbb{R}_+$, which is an element in the i-th row and j-th column of the adjacency matrix of the communication graph [24]. In this study, A_{ij} is specified as a continuous and differentiable function with respect to position, as follows:

$$A_{ij} = \begin{cases} \frac{1}{2}\left(1 + \cos\left(\pi \frac{\|p^i - p^j\|^2}{R^2}\right)\right) & \text{if} \quad \|p^i - p^j\| < R \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

Now, we discuss the moving pattern of agents when employing equation (3) as the control law. We assume that $\lambda_2 > \varepsilon$ initially holds true, and that the control cycle is sufficiently short. First, equation (3a) is employed, and the agents move while decreasing J' because u_1^i is directed towards the steepest descent direction. Subsequently, λ_2 reaches ε, and the control law switches to equation (3b), which is the projection of u_1^i in the direction orthogonal to n^i, in order to decrease J' while ensuring that λ_2 is unchanged. Therefore, equation (3) converges agent positions to the local minima of the problem (1). The discussion above indicates that the control law is optimal in the sense that no agents can decrease the objective function by only modifying their positions when they stop moving.



(a) 0 **step** **(b)** 4×10^5 **steps**

Figure 1: Agent position and Voronoi diagram

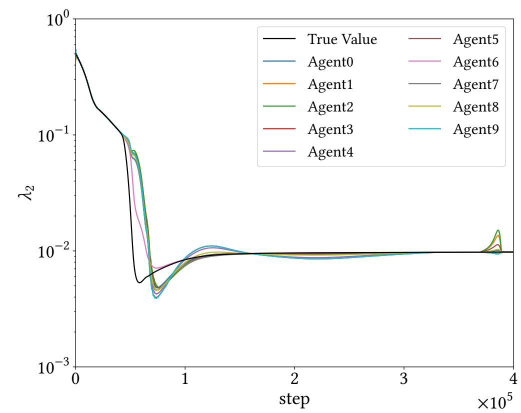


Figure 2: Algebraic connectivity λ_2

The extension of the centralized control law (3) to a distributed law is achieved by calculating λ_2 and $\frac{1}{N}\sum_k p^k$ in a distributed manner. Distributed estimation of λ_2 is presented in [1, 7, 11, 15, 20, 24], and we employ the method proposed by Yang et al. $\frac{1}{N}\sum_k p^k$ can be estimated using an average consensus estimator. In this study, we employ a proportional-integral-average consensus estimator [9], which we modify to suppress the variation of the estimated value owing to changes in the topology of the communication graph. For simplicity, we assume that the agents know the number of agents N and that the agents whose Voronoi regions are in contact with each other can exchange positions via multi-hop communication and conduct Voronoi tessellation.

To demonstrate the effectiveness of the proposed control law, we conducted a simulation. The weighting function is specified as $\phi(q) = 1$ for all $q \in Q$. Note that equation (3) is slightly modified to adjust the effect of the term $\left\langle u_1^i, n^i \right\rangle n^i$. Simulation results for the decentralized control are shown in Figure 1 and 2. In Figure 1, the black circles denote the agents, the white circles denote the centroids of the Voronoi regions, the dashed circles denote the communication range, and the gray lines denote the edges of the communication graph. Figure 2 shows time-series plots of the true and estimated values of λ_2. It was found that the agents deploy over the coverage region while maintaining connectivity (i.e., $\lambda_2 > 0$ always).

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