Mechanism Design in Facility Location Games

Doctoral Consortium

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ABSTRACT

Algorithmic game theory is a research field integrating game theory and algorithm design. The major target is to design good algorithms in strategic environments. In this thesis, I intend to study a well-established problem in algorithmic game theory called facility location games. In the most classic setting, the government plans to build a facility on a street where some strategic agents with private information live. Agents want to be as close as possible to the facility. The objective of the government is to collect the agents' information and use a mechanism to decide where to build the facility so that agents will not gain by reporting false information and certain objective values are approximately optimized.

In recent years, many extensions to the original facility location games are proposed and studied by the researchers. This thesis studies three different kinds of facility location games, and designs truthful mechanisms with good performances, which will offer new insight on extending the classic facility location games.

KEYWORDS

Facility location games; Mechanism design; Optimality; Approximability; Truthfulness

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1 INTRODUCTION

Facility location game (FLG) is a typical problem in computer science and economics [8, 10, 12]: the government plans to build a public facility (e.g., a library or a park) to serve some self-interested agents located in some area. Each agent reports her private location to the government. Taking the reported information as input, a mechanism run by the government outputs a location where the facility is to be opened. Every agent wants to be as close as possible to the single facility, and takes the distance to the facility as her own cost. The adopted mechanism is publicly known beforehand, and thus an agent may misreport her location if she can reduce her cost by changing the location of the facility. To avoid such misreporting, the concept of strategy-proofness (or called truthfulness) is introduced: a deterministic mechanism is strategy-proof, if no agent can benefit from misreporting, regardless of what other agents do. A deterministic mechanism is group strategy-proof, if no group of agents can misreport such that each agent in this group can gain. A randomized mechanism, which returns a probability distribution over locations, is also considered. Strategy-proofness can be defined with respect to truthful-in-expectation if no agent can reduce her expected cost by misreporting.

Mechanisms are often required to optimize some system objective, e.g., minimizing social cost (the sum of costs of all agents) or minimizing the maximum cost (the maximum cost of all agents). The performance of a mechanism is evaluated by the approximation ratio: a mechanism is called approximate if for every input instance, the objective value for the outcome is no more than times that of an optimal solution. Besides, it is also possible to measure the agents' gain (utility) instead of their loss (cost), and the objectives are similarly defined.

In most studies of FLGs, customers (agents) are strategic players. However, in reality each potential facility might have a private opening cost, and may strategically report it. Motivated by this, this thesis studies three kinds of FLGs: 1) a budgeted facility location game, where facilities are strategic players; 2) a constrained facility location game with candidate locations, where customers are strategic players; 3) a dual-role facility location game, where every agent plays a dual role of facility and customer.

2 FACILITY LOCATION GAMES

Procaccia and Tennenholtz [12] initiate the study of FLGs from the perspective of approximate mechanism design. In their work, strategic customers (or agents) report their locations, and a mechanism maps the reports of customers to the locations for building facilities in order to optimize a social objective function. Since then, the model with strategic customers who report their private information, such as locations and preferences, have been widely studied [1–3, 5–7, 10, 11, 14].

Let k be the number of facilities to be built. Formally, in an instance of a classic k-facility location game, $N = \{1, \ldots, n\}$ is agent set, and each agent $i \in N$ has a private location $x_i \in S$ in a metric space (S, d), where $d : S^2 \to \mathbb{R}$ is the metric. Denote by $\mathbf{x} = (x_1, \ldots, x_n)$ the location profile of agents. A deterministic mechanism f takes the reported agents' location

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profile $\mathbf{x}$ as input, and outputs a facility location profile $\mathbf{y} = (y_1, \ldots, y_k) \in S^k$, that is, selecting k locations in (S, d) for building facilities. A randomized mechanism outputs a probability distribution over S^k. Once given an outcome $\mathbf{y}$, the *cost* of each agent $i \in N$ is defined as the distance to the closest facility, i.e., $c_i(\mathbf{y}) = d(x_i, \mathbf{y}) := \min_{1 \le j \le k} d(x_i, y_j)$.

There are two well-known system objective functions: *social cost* and *maximum cost*. Our goal is to design truthful mechanisms (exactly or approximately) optimizing some system objective at the same time.

3 CONTRIBUTIONS

In this thesis, I focus on designing truthful mechanisms with good performances for the FLGs. Overall, there are three kinds of FLGs.

Budgeted Facility Location Games. Our work [9] investigates the FLGs with payments, where facilities are strategic players. In the game, customers and facilities are located at publicly known locations on a line segment. Each selfish facility has an opening-cost as her private information, and she may strategically report it. Upon receiving the reports, the government uses a mechanism to select some facilities to open and pay to them. Besides, we introduce money into the game and use a budget to limit the ability of the government to select facilities. With respect to a subset of facilities selected by the government, each customer either incurs a connection *cost* equal to the distance to the nearest opened facility, or obtains a *utility* equal to a constant minus that distance. The performance of a mechanism is measured by comparing the objective value of the outcome with that of an optimal solution under the budget constraint which upper bounds the total opening-costs of facilities in the solution. Under a given budget B, which constrains the total payment, we derive upper and lower bounds on the approximation ratios of truthful budget feasible mechanisms under four utilitarian and egalitarian objectives, and study the case when the augmented budget is allowed.

Facility Location Games with Candidate Locations. Our work [13] studies the FLGs with candidate locations from a mechanism design perspective. Suppose there are n agents located in a metric space whose locations are their private information, and a group of candidate locations for building facilities. The authority plans to build some homogeneous facilities among these candidates locations to serve the agents, who bear a cost equal to the distance to the closest facility. Our goal is to design mechanisms for minimizing the total/maximum cost among the agents. In this work, we study the problem of locating one or two facilities in a metric space, where there are n agents and a set of feasible locations for building the facilities. For the single-facility location game under the maximum-cost objective, we give a deterministic 3-approximation group strategy-proof mechanism, and prove that no deterministic (resp., randomized) strategy-proof mechanism can have an approximation ratio better than 3 (resp., 2). For the two-facility location game on a line, we give an anonymous deterministic group strategy-proof mechanism that is $(2n-3)$-approximation for the total-cost objective, and 3-approximation for the maximum-cost objective.

Dual-Role Facility Location Games. Our extension work of [4] studies a monetary FLG, where every agent plays a dual role of facility and customer. In this game, each selfish agent has a publicly known location in a metric space, and can allow a facility to open at her location. The opening cost is her private information and she may strategically report it. Besides, each agent also bears a service cost equal to the distance to the nearest open facility. We study truthful mechanisms, which, given reports from all agents, output a set of agents whose facilities could open, and payment to each of these agents, such that no agent has an incentive to misreport. The system objective is to minimize (exactly or approximately) the total opening and service cost or the maximum agent cost of the outcome. For the total-cost objective, we give an optimal truthful mechanism which runs in exponential time; when restricted to polynomial-time computation, we prove a small gap between the best known approximation ratios of truthful mechanisms and the pure (nonstrategic) optimization counterparts. For the maximum-cost objective, we provide an optimal truthful mechanism which runs in polynomial time. Moreover, when the total payment cannot exceed a given budget, we calculate the lower bounds and upper bounds on approximation ratios of truthful mechanisms for both objectives.

4 FUTURE WORK

My work till now has addressed some issues on designing truthful mechanisms for three kinds of Facility location games. However, there is scope for more improvements. Further, I aim to focus on more complex settings in FLGs, and study other good properties of mechanisms. More specific details are as follows:

Dynamics in FLGs. I intend to study the multi-stage FLGs on a line, where agents are placed on a line, and each agent arrives at any time and can be served during r consecutive stages. Assume that each facility has a capacity, and we want to locate facilities on each stage to minimizing the total cost of agents and facilities.

Fairness in FLGs. In this part, I want to consider the fairness in FLGs and study the trade-off between fairness and efficiency: the fairness of agents, the fairness of facilities, and the combined fairness of agents and facilities.

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