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Implementing Interval Linear Equations Systems for Enhanced Circuit Analysis

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ABSTRACT

The present study is centered around the deployment of interval linear equations systems in circuit analysis. In the domain of circuit theory, each circuit, constituted by components such as resistance, inductance, and capacitance, can be mathematically represented as a system of linear equations. In the context of electrical circuits, interval representations of current or voltage are considered more informative than single precise values. This is attributed to factors including fluctuating environmental conditions, current variations, tolerances in electrical elements, and power harmonic leakage. The integration of interval linear equations systems becomes crucial in accommodating these variables. We propose an algorithm using a new type of arithmetic operations and pairing technique on intervals for the interval solution of interval linear equations systems. We provide a numerical example to highlight the usefulness of the suggested approach. We also discuss an electrical circuit problem under an uncertain environment by using the proposed algorithm and interval arithmetic operations.

1. INTRODUCTION

Nearly any real-world problem can be modelled as a system of linear or non-linear equations. In many cases, precise parameter values are elusive, known only through estimation or within certain bounds. Interval analysis provides valuable tools and methodologies for solving linear and nonlinear systems of equations in situations where data imprecision or uncertainties exist. Notably, the significance of interval arithmetic in matrix computations was first suggested by Hansen and Smith [1]. This has since inspired an array of research by scholars such as Alefeld et al. [2, 3], Moore et al. [4], Ganesan and Veeramani [5, 6] and Goze [7], who have explored interval arithmetic and interval matrices extensively.

In the context of circuit analysis, systems of linear equations are often employed to solve for unknown currents. Yet, real-world conditions challenge the assumption that a voltage source consistently generates an exact voltage, as minor fluctuations in voltage output are unavoidable. Similar variability applies to other electrical elements within circuits, thus necessitating the treatment of circuit components like voltage sources and resistances as uncertain entities. Closed and bounded intervals provide a practical method for representing these uncertain parameters. When faced with these uncertainties, current flow within a circuit can be accurately encapsulated using a system of interval linear equations.

Numerous authors, including Alefeld et al. [8], Beaumont [9], Siahlooei [10], Hansen [11, 12], Sainz et al. [13], Neumaier [14], Ning and Kearfott [15], Rani [16], Corsaro and Marino [17], and Lodwick and Dubois [18], have explored solutions to systems of fuzzy and interval linear equations.

The examination of interval linear algebra by Surya et al. [19-21] within a precise algebraic framework has been influential. Using the concept of equivalence classes, they successfully defined the notions of a field and a vector space over a field. Cazarez-Castro et al. [22] presented a fuzzy differential equations approach to model the uncertainty of initial conditions for the proportional derivative closed-loop control of a direct current motor. Studies by Rahgooy et al. [23], Rahman and Rahman [24], and Srinivas and Rao [25] have examined the application of the system of fuzzy linear equations in circuit analysis. Furthermore, Yazdi et al. [26] have explored fuzzy circuit analysis through fuzzy differential equations with fuzzy variables. Jesuraj et al. [27], Devi and Ganesan [28], and Sahoo [29] have solved electrical circuits using various methods, including fuzzy Sumudu transform and fuzzy differential equations, respectively. Diffellah et al. [30] have discussed the applications of interval analysis in electric circuit theory.

In this paper, a novel method is proposed for determining the unknown current flowing in given planar circuits under uncertain conditions. The uncertain parameters are represented as closed and bounded intervals. We introduce a new set of arithmetic operations and pairing techniques on intervals to determine the current flow in electrical circuits. Additionally, an interval-based linear system is introduced and its application is demonstrated through a numerical example.

2. BASICS ON INTERVALS

Here are the basic concepts and notions regarding intervals, as recalled from reference [19]: We define the set $I\mathbb{R}=\{\tilde{a}=[a^L, a^U]: a^L \le a^U \text{ and } a^L, a^U \in \mathbb{R}\}$. This set comprises all the closed and bounded intervals. When a^L equals a^U, we refer to $\tilde{a}$ as a degenerate interval. These intervals can be represented as ordered pairs $\langle m, w\rangle$ which are defined as follows: Given an interval $\tilde{a}$, $m(\tilde{a})$ as $\frac{a^U+a^L}{2}$ and $w(\tilde{a})$ as.$\frac{a^U-a^L}{2}$ Consequently, $\tilde{a}$ can be uniquely expressed as $\langle m(\tilde{a}), w(\tilde{a})\rangle$. Consequently, $\tilde{a}$ can be uniquely expressed as $\langle m(\tilde{a}), w(\tilde{a})\rangle$. Conversely, if you have $\langle m(\tilde{a}), w(\tilde{a})\rangle$., you can determine a^L and a^U as follows: $m(\tilde{a})-w(\tilde{a})=a^L$ and $m(\tilde{a})+w(\tilde{a})=a^U$ for the interval $\tilde{a}$. Therefore, given $\langle m(\tilde{a}), w(\tilde{a})\rangle$, you can uniquely recover the interval $[a^L, a^U]$.

Note 1. If midpoint of $\tilde{a}$is zero then $\tilde{a}$ is called as a zero interval. Otherwise, it is non-zero interval. If midpoint of $\tilde{a}$ is positive then $\tilde{a}$ is called as a positive interval.

2.1 Interval arithmetic operations

For any two intervals $\tilde{a}=\langle m(\tilde{a}),w(\tilde{a})\rangle$ and $\tilde{b}=\langle m(\tilde{b}),w(\tilde{b})\rangle$ and $*\in\{+,-,\times,\div\}$, the arithmetic operations are defined as follows [19]:

$$\tilde{a}*\tilde{b}=\langle m(\tilde{a}),\ w(\tilde{a})\rangle*\langle m(\tilde{b}),\ w(\tilde{b})\rangle$$
$$=\langle m(\tilde{a})*m(\tilde{b}),\ \max\{w(\tilde{a}),w(\tilde{b})\}\rangle$$

Note 2. Division is possible when midpoint of the denominator interval is non-zero.

2.2 Interval matrix theory [19]

An interval matrix is a matrix where each element is defined as a closed and bounded interval of real numbers, as opposed to a single exact value. Intervals, in this context, are sets of numbers encompassing all values falling within a specified range. The purpose of employing intervals within matrix representations is to account for uncertainty or imprecision in the individual elements of the matrix. This allows for a more flexible and robust way of modelling data or systems in situations where precise values are not known or cannot be determined.

An interval matrix

$$\tilde{A}=\begin{pmatrix}\tilde{a}_{11} & \dots & \tilde{a}_{1n}\\ \dots & \dots & \dots\\ \tilde{a}_{m1} & \dots & \tilde{a}_{mn}\end{pmatrix}=(\tilde{a}_{ij})$$

where, each $\tilde{a}_{ij}=[a_{ij}{}^L, a_{ij}{}^U]$. Midpoint matrix of $\tilde{A}$, $m(\tilde{A})$ is defined as the matrix of midpoints of every corresponding entries of $\tilde{A}$. Similarly, width matrix $w(\tilde{A})$ is defined as the matrix of widths of every corresponding entry of $\tilde{A}$.

Let $\tilde{\boldsymbol{x}}=(\tilde{x}_1,\tilde{x}_2,\dots,\tilde{x}_n)^t$ be an interval vector in $I\,\mathbb{R}^n$ and $m(\tilde{\boldsymbol{x}})=(m(\tilde{x}_1),m(\tilde{x}_2),\dots,m(\tilde{x}_n))^t$, $w(\tilde{\boldsymbol{x}})=(w(\tilde{x}_1),w(\tilde{x}_2),\dots,w(\tilde{x}_n))^t$ are midpoint vector and width vectors of $\tilde{\boldsymbol{x}}$ respectively.

2.3 Interval matrix operations

For $\tilde{A},\tilde{B}\in I\,\mathbb{R}^{(m\times n)}$, $\tilde{\boldsymbol{x}}\in I\,\mathbb{R}^n$, where $*\in\{+,-,\times,\div\}$ and $\tilde{\alpha}\in I\,\mathbb{R}$, we define [19]:

$$\tilde{A}*\tilde{B}=\left\langle m(\tilde{A})*m(\tilde{B}),\max\left\{\min_{w(\tilde{a}_{ij})\neq 0} w(\tilde{A}),\min_{w(\tilde{b}_{ij})\neq 0} w(\tilde{B})\right\}\right\rangle.$$

Note 3. Matrix division is possible when midpoint of the denominator matrix is invertible.

3. SYSTEM OF INTERVAL LINEAR EQUATIONS

Consider a system of "*m*" interval linear equations in "*n*" unknown intervals:

$$\left.\begin{array}{l}\tilde{a}_{11}\tilde{x}_1+\tilde{a}_{12}\tilde{x}_2+\dots\dots+\tilde{a}_{1n}\tilde{x}_n\approx\tilde{b}_1\\ \tilde{a}_{21}\tilde{x}_1+\tilde{a}_{22}\tilde{x}_2+\dots\dots+\tilde{a}_{2n}\tilde{x}_n\approx\tilde{b}_2\\ .\\ .\\ .\\ \tilde{a}_{m1}\tilde{x}_1+\tilde{a}_{m2}\tilde{x}_2+\dots\dots+\tilde{a}_{mn}\tilde{x}_n\approx\tilde{b}_m\end{array}\right\}$$

where, $\tilde{a}_{ij}$ and $\tilde{b}_i$ are in $I\,\mathbb{R}$.

The above system is called (*m*×*n*) interval linear system. If $\tilde{b}_i\approx\tilde{0}$ for each *i*, then the above system is called the homogeneous system of interval linear equations. Otherwise, it is called a non-homogeneous system of interval linear equations.

The matrix form of the above system is $\tilde{A}\tilde{\boldsymbol{x}}\approx\tilde{B}$. That is:

$$\begin{pmatrix}\tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} & . & . & . & \tilde{a}_{1n}\\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} & . & . & . & \tilde{a}_{2n}\\ . & . & . & . & . & . & .\\ . & . & . & . & . & . & .\\ \tilde{a}_{m1} & \tilde{a}_{m2} & \tilde{a}_{m3} & . & . & . & \tilde{a}_{mn}\end{pmatrix}\begin{pmatrix}\tilde{x}_1\\ \tilde{x}_2\\ .\\ .\\ \tilde{x}_n\end{pmatrix}\approx\begin{pmatrix}\tilde{b}_1\\ \tilde{b}_2\\ .\\ .\\ \tilde{b}_m\end{pmatrix}$$

where, $\tilde{\mathbf{x}}=(\tilde{x}_1,\tilde{x}_2,\dots,\tilde{x}_n)^t$ and $\tilde{B}=(\tilde{b}_1,\tilde{b}_2,\dots,\tilde{b}_m)^t$.

4. ALGORITHM FOR FINDING THE SOLUTIONS OF SYSTEM OF INTERVAL LINEAR EQUATIONS

Given the interval linear system $\tilde{A}\tilde{\mathbf{x}}\approx\tilde{B}$.

Step 1: Compute the solution of the midpoint system $m(\tilde{A})m(\tilde{\mathbf{x}})=m(\tilde{B})$.

Step 2: Let the solution be $m(\tilde{\mathbf{x}})=(m(\tilde{x}_1),m(\tilde{x}_2),\dots.m(\tilde{x}_n))^t$.

Step 3: The pairing number for the interval solution is obtained by the Max-Min principle as follows:

Let $\alpha = \max\left\{\min_{w(\tilde{a}_{ij})\neq 0} w(\tilde{A}), \min_{w(\tilde{b}_{ij})\neq 0} w(\tilde{B})\right\}$. Here $\alpha \succ 0$ unless when $\tilde{A}$ and $\tilde{B}$ degenerate into crisp matrices.

Step 4: The solution to the interval linear system $\tilde{A}\tilde{\mathbf{x}} \approx \tilde{B}$ is:

$$\tilde{\mathbf{x}} = \langle m(\tilde{\mathbf{x}}), \alpha \rangle = \begin{pmatrix} [m(\tilde{x}_1) - \alpha, m(\tilde{x}_1) - \alpha] \\ [m(\tilde{x}_2) - \alpha, m(\tilde{x}_2) - \alpha] \\ . \\ . \\ [m(\tilde{x}_n) - \alpha, m(\tilde{x}_n) - \alpha] \end{pmatrix}.$$

Note: Pairing number is a positive real number which is judiciously evaluated using the given data and which aids the conversion of a real number into unique interval number. For example, if 3 is a real number to be associated with an interval, we calculate the pairing number using the specified method. Suppose the pairing number is 2 then 3 is associated with an interval ⟨3,2⟩=[3-2, 3+2]=[1,5].

Example 4.1 Consider an interval linear system $\tilde{A}\tilde{x} \approx \tilde{B}$ discussed by Ning et al. [15], where $\tilde{A} = \begin{pmatrix} [3.7,4.3] & [-1.5,-0.5] & [0,0] \\ [-1.5,-0.5] & [3.7,4.3] & [-1.5,-0.5] \\ [0,0] & [-1.5,-0.5] & [3.7,4.3] \end{pmatrix} \in \mathrm{I}\mathbb{R}^{n\times n}$ and $\tilde{B} = \begin{pmatrix} [-14,0] \\ [-9,0] \\ [-3,0] \end{pmatrix}$.

Solution: The midpoint system of the above interval system is $m(\tilde{A})m(\tilde{x}) = m(\tilde{B})$, where $m(\tilde{A}) = \begin{pmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix}$ and $w(\tilde{A}) = \begin{pmatrix} 0.30 & 0.50 & 0 \\ 0.50 & 0.30 & 0.50 \\ 0 & 0.50 & 0.30 \end{pmatrix}$ respectively. Also $m(\tilde{\mathrm{B}}) = \begin{pmatrix} -7 \\ -4.50 \\ -1.50 \end{pmatrix}$ and $w(\tilde{\mathrm{B}}) = \begin{pmatrix} 7 \\ 4.50 \\ 1.50 \end{pmatrix}$ respectively.

Solving this midpoint system: $\begin{pmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix}\begin{pmatrix} m(\tilde{x}_1) \\ m(\tilde{x}_2) \\ m(\tilde{x}_3) \end{pmatrix} = \begin{pmatrix} -7 \\ -4.50 \\ -1.50 \end{pmatrix}$ by Gauss elimination method, we get the solution of the midpoint system as $m(\tilde{\mathbf{x}}) = \begin{pmatrix} \text{-2.2232} \\ \text{-1.8929} \\ \text{-0.8482} \end{pmatrix}$. The pairing number for the interval solution is obtained by the Max-Min principle as follows:

$$\text{Let} \quad \alpha = \max\left\{\min_{w(\tilde{a}_{ij})\neq 0} w(\tilde{A}), \min_{w(\tilde{b}_{ij})\neq 0} w(\tilde{B})\right\}$$

$$= \max\left\{ \begin{matrix} \min_{w(\tilde{a}_{ij})\neq 0} \begin{pmatrix} 0.30 & 0.50 & 0 \\ 0.50 & 0.30 & 0.50 \\ 0 & 0.50 & 0.30 \end{pmatrix} \\ \min_{w(\tilde{b}_{ij})\neq 0} \begin{pmatrix} 7 \\ 4.50 \\ 1.50 \end{pmatrix} \end{matrix} \right\}$$

$$= \max\{0.30, 1.50\} = 1.50$$

The interval solution of the interval linear system is:

$$\tilde{\mathbf{x}} = \langle m(\tilde{\mathbf{x}}), \alpha \rangle = \left\langle \begin{pmatrix} \text{-2.2232} \\ \text{-1.8929} \\ \text{-0.8482} \end{pmatrix}, 1.50 \right\rangle$$

$$= \begin{pmatrix} \text{[-3.7232, -0.7232]} \\ \text{[-3.3929, -0.3929]} \\ \text{[-2.3482, 0.6518]} \end{pmatrix}.$$

Comparison study: Ning et al. [15] derived the solution set $\begin{pmatrix} [-6.38, 0] \\ [-6.40, 0] \\ [-3.40, 0] \end{pmatrix}$ by using the interval Gaussian elimination method together with existing interval arithmetic. Also, by using Hansen's method [12], they obtained the solution set (wider box) $\begin{pmatrix} [-6.38, 1.12] \\ [-6.40, 1.54] \\ [-3.40, 1.40] \end{pmatrix}$ and using their own method, obtained the solution set (much wider box) $\begin{pmatrix} [-6.38, 1.67] \\ [-6.40, 1.54] \\ [-3.40, 2.40] \end{pmatrix}$. The intriguing part is that by applying the proposed algorithm, we obtain a solution set $\begin{pmatrix} \text{[-3.7232, -0.7232]} \\ \text{[-3.3929, -0.3929]} \\ \text{[-2.3482, 0.6518]} \end{pmatrix}$ which is better to the prior solution sets. Also, this solution set won't affect the crisp solution set which will significantly reduce vagueness.

5. AN APPLICATION ON ELECTRIC CIRCUIT

The mesh current method is a network analysis technique in which arbitrarily assigned mesh current directions are used to solve for unknown currents and voltages using Kirchhoff's voltage law and Ohm's law. It can typically solve a circuit with fewer unknown variables and simultaneous equations. Due to some uncertain situations, the electrical elements are represented by closed and bounded intervals.

Kirchhoff's Voltage Law (KVL) is a fundamental principle in electrical circuit theory. It asserts that the total sum of voltage changes, which includes voltage drops (from higher potential to lower potential) and voltage rises (from lower potential to higher potential), around any closed path or loop within an electrical circuit must always equal zero. In other words, the algebraic sum of these voltage changes is conserved and obeys the principle of energy conservation. This law is an essential tool in analyzing and solving electrical circuits, helping to understand how voltage behaves within a closed loop in accordance with the conservation of energy.

Consider an electric circuit:

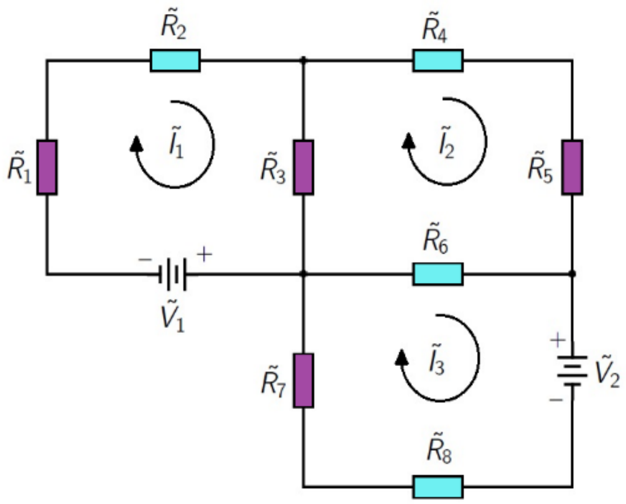

Figure 1. Electric circuit

where, the values of electric resistance are: $\tilde{R}_1 = \tilde{R}_3 = \tilde{R}_5 = \tilde{R}_7 = [1497.5, 1502.5]\Omega = \langle 1500, 2.5\rangle \Omega$, $\tilde{R}_2 = \tilde{R}_4 = \tilde{R}_6 = \tilde{R}_8 = [798.5, 801.5]\Omega = \langle 800, 1.5\rangle \Omega$ and the voltages are $\tilde{V}_1 = [11,13]\,\text{V} = \langle 12, 1\rangle\,\text{V}$ and $\tilde{V}_1 = [22.5,\ 25.5]\,\text{V} = \langle 24, 1.5\rangle\,\text{V}$.

In the realm of mesh analysis, the application of Kirchhoff's Voltage Law involves the utilization of mesh currents. These mesh currents are systematically allocated to individual meshes and preferably oriented in a clockwise direction. The KVL is then sequentially applied to each mesh, leveraging the principle that the voltage drops across a resistor, when traversed by a current I is given by IR.

In this particular method, the approach involves equating the voltage drops across the resistors in accordance with the direction of the mesh currents to the voltage increases experienced across the voltage sources within the electrical circuit.

Regarding mesh 1, in the diagram, the voltage drops across the resistors labelled as $\tilde{R}_1$, $\tilde{R}_2$ and $\tilde{R}_3$ are expressed as $\tilde{I}_1\tilde{R}_1, \tilde{I}_1\tilde{R}_2$ and $(\tilde{I}_1 - \tilde{I}_2)\tilde{R}_3$ respectively. Notably, the distinction arises because the current through $\tilde{R}_3$ in relation to $\tilde{I}_1$ is $(\tilde{I}_1 - \tilde{I}_2)$. Furthermore, the voltage rise from the voltage source is represented as $\tilde{V}_1$.

$\therefore$ The mesh equation for mesh 1 is:

$$\begin{aligned}
&\tilde{I}_1\tilde{R}_1 + \tilde{I}_1\tilde{R}_2 + (\tilde{I}_1 - \tilde{I}_2)\tilde{R}_3 \approx \tilde{V}_1 \\
&\Rightarrow\ \tilde{I}_1\tilde{R}_1 + \tilde{I}_1\tilde{R}_2 + \tilde{I}_1\tilde{R}_3 - \tilde{I}_2\tilde{R}_3 \approx \tilde{V}_1 \\
&\Rightarrow\ \tilde{I}_1(\tilde{R}_1 + \tilde{R}_2 + \tilde{R}_3) - \tilde{I}_2\tilde{R}_3 \approx \tilde{V}_1 \\
&\Rightarrow \langle 1500, 2.5\rangle \tilde{I}_1 + \langle 800, 1.5\rangle \tilde{I}_1 \\
&+\langle 1500, 2.5\rangle \tilde{I}_1 - \langle 1500, 2.5\rangle \tilde{I}_2 \approx \langle 12, 1\rangle \\
&\Rightarrow \langle 1500 + 800 + 1500,\ \ \max\{2.5, 1.5, 2.5\}\rangle \\
&-\langle 1500, 2.5\rangle \tilde{I}_2 \approx \langle 12, 1\rangle \\
&\Rightarrow \langle 3800, 2.5\rangle \tilde{I}_1 - \langle 1500, 2.5\rangle \tilde{I}_2 \approx \langle 12, 1\rangle
\end{aligned} \tag{1}$$

It's worth noting that in the context of mesh analysis in electrical circuits, certain coefficients in the mesh equations have specific meanings.

Let $\tilde{R}_1 + \tilde{R}_2 + \tilde{R}_3$, the co-efficient of $\tilde{I}_1$ is the sum of the resistances of the resistors in mesh 1. This total resistance is commonly referred to as the "self-resistance" of the mesh 1. Also $-\tilde{R}_3$, the co-efficient of $\tilde{I}_2$ is negative, it signifies the resistance of the resistor that is shared by two adjacent meshes, typically mesh 1 and mesh 2. This resistance is known as the "mutual resistance."

The negative sign associated with mutual resistances in mesh equations is a result of the fact that the mesh currents in different meshes often flow in opposite directions through mutual resistors. As a consequence, these resistances are represented with negative signs in the equations. This simplifies the formulation of the mesh equations, making it more convenient than directly applying Kirchhoff's Voltage Law (KVL) to the circuit.

$\therefore$ The mesh equation for mesh 2 is:

$$\begin{aligned}
&\tilde{I}_2\tilde{R}_4 + \tilde{I}_2\tilde{R}_5 + (\tilde{I}_2 - \tilde{I}_3)\tilde{R}_6 + (\tilde{I}_2 - \tilde{I}_1)\tilde{R}_3 \approx \tilde{0} \\
&\tilde{I}_2\tilde{R}_4 + \tilde{I}_2\tilde{R}_5 + \tilde{I}_2\tilde{R}_6 - \tilde{I}_3\tilde{R}_6 + \tilde{I}_2\tilde{R}_3 - \tilde{I}_1\tilde{R}_3 \approx \tilde{0} \\
&-\tilde{I}_1\tilde{R}_3 + \tilde{I}_2(\tilde{R}_3 + \tilde{R}_4 + \tilde{R}_5 + \tilde{R}_6) - \tilde{I}_3\tilde{R}_6 \approx \tilde{0} \\
&\Rightarrow \langle -1500, 2.5\rangle \tilde{I}_1 + \langle 4600, 2.5\rangle \tilde{I}_2 \\
&+\langle -800, 1.5\rangle \tilde{I}_3 \approx \langle 0, 0\rangle
\end{aligned} \tag{2}$$

Also, the mesh equation for mesh 3 is:

$$\begin{aligned}
&-\tilde{V}_2 + \tilde{I}_3\tilde{R}_8 + \tilde{I}_3\tilde{R}_7 + (\tilde{I}_3 - \tilde{I}_2)\tilde{R}_6 \approx \tilde{0} \\
&\tilde{I}_3\tilde{R}_8 + \tilde{I}_3\tilde{R}_7 + \tilde{I}_3\tilde{R}_6 - \tilde{I}_2\tilde{R}_6 \approx \tilde{V}_2 \\
&-\tilde{I}_2\tilde{R}_6 + \tilde{I}_3(\tilde{R}_6 + \tilde{R}_7 + \tilde{R}_8) \approx \tilde{V}_2 \\
&\Rightarrow \langle -800, 1.5\rangle \tilde{I}_2 + \langle 3100, 2.5\rangle \tilde{I}_3 \approx \langle 24, 1.5\rangle
\end{aligned} \tag{3}$$

Hence the given electric circuit is represented as the system of interval linear equations:

$$\begin{aligned}&\langle 3800,2.5\rangle \tilde{I}_1+\langle -1500,2.5\rangle \tilde{I}_2\approx\langle 12,1\rangle\\&\langle -1500,2.5\rangle \tilde{I}_1+\langle 4600,2.5\rangle \tilde{I}_2+\\&\langle -800,1.5\rangle \tilde{I}_3\approx\langle 0,0\rangle\\&\langle -800,1.5\rangle \tilde{I}_2+\langle 3100,2.5\rangle \tilde{I}_3\approx\langle 24,1.5\rangle\end{aligned}\tag{4}$$

Matrix form of the above system is $\tilde{A}\tilde{I}\approx\tilde{B}$:

$$\Rightarrow\begin{pmatrix}\langle 3800,2.5\rangle & \langle -1500,2.5\rangle & \langle 0,0\rangle\\ \langle -1500,2.5\rangle & \langle 4600,2.5\rangle & \langle -800,1.5\rangle\\ \langle 0,0\rangle & \langle -800,1.5\rangle & \langle 3100,2.5\rangle\end{pmatrix}\begin{pmatrix}\tilde{I}_1\\ \tilde{I}_2\\ \tilde{I}_3\end{pmatrix}$$
$$\approx\begin{pmatrix}\langle 12,1\rangle\\ \langle 0,0\rangle\\ \langle 24,1.5\rangle\end{pmatrix}$$

The midpoint system of the above interval system is $m(\tilde{A})m(\tilde{I})=m(\tilde{B})$, where $m(\tilde{A})=\begin{pmatrix}3800 & -1500 & 0\\ -1500 & 4600 & -800\\ 0 & -800 & 3100\end{pmatrix}$ and $w(\tilde{A})=\begin{pmatrix}2.5 & 2.5 & 0\\ 2.5 & 2.5 & 1.5\\ 0 & 1.5 & 2.5\end{pmatrix}$ respectively. Also $m(\tilde{\mathrm{B}})=\begin{pmatrix}12\\ 0\\ 24\end{pmatrix}$ and $w(\tilde{\mathrm{B}})=\begin{pmatrix}1\\ 0\\ 1.5\end{pmatrix}$ respectively.

Solving the midpoint system $m(\tilde{A})m(\tilde{I})=m(\tilde{B})$ by using Gauss elimination method, we get the solution for midpoint system as $m(\tilde{I})=\begin{pmatrix}0.0042929\\ 0.0028753\\ 0.0084840\end{pmatrix}$.

That is $m(\tilde{I}_1)=0.0042929\text{ A}=4.2929\text{mA}$, $m(\tilde{I}_2)=0.0028753\text{ A}=2.8753\text{mA}$ and $m(\tilde{I}_3)=0.0084840\text{ A}=8.4840\text{ mA}$.

The pairing number for the interval solution is obtained by the Max-Min principle as follows:

$$\text{Let}\quad \alpha=\max\left\{\min_{w(\tilde{a}_{ij})\neq 0} w(\tilde{A}),\ \min_{w(\tilde{b}_{ij})\neq 0} w(\tilde{B})\right\}$$
$$=\max\left\{\min_{w(\tilde{a}_{ij})\neq 0}\begin{pmatrix}2.5 & 2.5 & 0\\ 2.5 & 2.5 & 1.5\\ 0 & 1.5 & 2.5\end{pmatrix},\ \min_{w(\tilde{b}_{ij})\neq 0}\begin{pmatrix}1\\ 0\\ 1.5\end{pmatrix}\right\}$$
$$=\max\{1.5,1\}=1.5$$

The interval solution of the interval linear system is:

$$\tilde{I}=\langle m(\tilde{I}),\alpha\rangle=\left\langle\begin{pmatrix}4.2929\\ 2.8753\\ 8.4840\end{pmatrix},1.5\right\rangle$$
$$=\begin{pmatrix}[2.7929,\ 5.7929]\\ [1.3753,\ 4.3753]\\ [6.9840\ 9.9840]\end{pmatrix}.$$

Therefore, the mesh currents of the above electric circuit are:

$$\tilde{I}_1=[2.7929,\ 5.7929]\text{mA}$$
$$\tilde{I}_2=[1.3753,\ 4.3753]\text{mA}$$
$$\text{and } \tilde{I}_3=[6.9840\ 9.9840]\text{mA}.$$

Remark: The system of interval linear equations in the above example was formulated with the underlying assumption that the individual entries are susceptible to errors or uncertainties. This assumption forms the basis for the solution approach employed in handling such uncertainties.

Specifically, if we replace the interval representations in the solution with their respective midpoints, the resulting solution will naturally converge to the solution of a standard (crisp) system of linear equations when there are no errors or uncertainties present. In other words, if the errors are zero, and there is no imprecision associated with the entries, the interval-based solution will yield the same result as the conventional system of linear equations. This demonstrates that the interval-based approach is a flexible method for dealing with uncertainty and imprecision, capable of converging to traditional solutions when circumstances warrant it.

6. CONCLUSIONS

The current fluctuations that are unavoidable in real life prevent us from knowing the precise values of the measured quantities. At most, we characterize the parameters as intervals of uncertainty. In this paper we have provided the interval terminology-based notions for electrical circuit theory. The description of networks leads to systems of equations with interval parameters. We have presented a new method for computing the interval voltages and interval currents. We have shown that the proposed algorithm using a new type of arithmetic operations and pairing techniques on intervals provides better solution than by using previous approaches.

Numerical illustrations are given to support the theory. The proposed algorithm is efficient for large systems also.

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